

Nielsen

Composite Materials

Properties
as Influenced
by Phase Geometry



Springer

Lauge Fuglsang Nielsen

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With 241 Figures

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Overview

The subject dealt with in this book is the mechanical and physical behavior of composites as influenced by composite geometry. This subject has a high priority in the general study of composite materials. A better understanding of the behavior of natural composites, improvement of such materials, and design of new materials with prescribed properties are just three examples in modern materials research where more knowledge on geometry versus materials property is absolutely necessary.

An analysis of various composite properties versus composite geometries is presented in this book as the result of integrating the results of two sub-studies:

One study is made on composite properties as these can be related to composite geometry in general by so-called geometry functions. The second study is made on geometry functions as these are related to the geometry of specific composites, such as particulate composites, impregnated materials, laminated composites, and composites made by compaction of powders.

In other words, global solutions for composite properties are developed in the first study, which apply for any composite. Final solutions for composites with specific geometries are then obtained from the global solutions introducing specific geometry functions developed in the second study.

Special composite properties considered are stiffness, shrinkage, hygro-thermal behavior, viscoelastic behavior, and internal stress states. Other physical properties considered are thermal and electrical conductivities, diffusion coefficients, dielectric constants and magnetic permeability. Special attention is given to the effect of pore shape on the mechanical and physical behavior of porous materials.

The theories and the methods developed are verified by results obtained from a FEM-analysis presented, and by experimental and theoretical data from the composite literature. A number of examples are presented which illustrate the very decisive influence of the internal geometry on the mechanical and physical properties of composites.

As a spin-off result the composite theory developed is re-organized to become a “diagnostic tool” with respect to quality control of empirical or semi-theoretical prediction methods suggested in the field of composite materials. Aspects of materials design are also considered.

It is emphasized that strength is not considered as a genuine materials property in this book. It is a phenomenon where discontinuities in the materials structure suddenly occur as the result of violating local potentials to carry stress and/or strain for example. As such strength is a “materials property” that can be calculated from stress/strain results obtained in this book. Examples of such strength predictions for composite materials are presented.

Readers Guidance

Roughly speaking the book is divided into two parts. A theoretical part, and a more applicative part, starting at Chap. 10 where the theories developed are simplified, adapted, and generalized for most practice. Readers, who are interested primarily in applications, may start at this chapter. Any problem considered in Chap. 10 and subsequent chapters can be solved using the software package COM-APPL developed for easy composite analysis¹.

Lists of notations and references used are presented at the end of the book. The former list should be consulted frequently. Symbols and notations used in the book are generally explained only at their first appearance in the text.

The following superior concept of notations is emphasized: Whenever needed to distinguish single component properties from composite properties, subscripts P and S refer to property of component P and property of component S respectively while composite property is not subscripted. Usually the subscripts g and k are used to indicate quantities obtained from – or used in deviatoric analysis and in volumetric analysis respectively. Formally these analyses are very often identical. In such cases only the volumetric analysis is presented with deviatoric results referred to by analogy. Alternatively both subscripts k and g are dropped when the feature discussed applies in principles for both volumetric and deviatoric behavior.

A special subscript, Q, is used in conductivity studies to distinguish results obtained in these studies from similar quantities obtained in the analysis of elastic behavior.

A number of auxiliary expressions are presented in appendix sections at the end of the book: Basic information is given on isotropic elasticity and cubical elasticity in Appendix A. A method is presented in Appendix B for the numerical determination of stresses in ellipsoidal particles in isotropic dilute

¹ COM-APPL can be downloaded from <http://www.mat-mek.dk>

suspensions. A generalized version of the so-called SCS-analysis (Self Consistency Scheme) of composite materials is presented in Appendix C. General viscoelastic models are presented in Appendix D. And finally, models are presented in Appendix E for volume compositions of hardening Portland cement paste and concrete.

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Introduction

In the past five decades considerable attention has been devoted to composite materials. A number of expressions have been suggested by which macroscopic properties can be predicted when the properties, geometry, and volume concentrations of the constituent components are known. Many expressions are purely empirical or semi-theoretical. Others, however, are theoretically well founded such as the exact results from the following classical boundary studies:

Bounds for the elastic moduli of composites made of perfectly coherent homogeneous, isotropic linear elastic phases have been developed by Paul [1] and Hansen [2] for unrestricted phase geometry and by Hashin and Shtrikman [3] for phase geometries, which cause macroscopic homogeneity and isotropy.

The composites dealt with in this book are of the latter type. For two specific situations (later referred to), Hashin [4] and Hill [5] derived exact solutions for the bulk modulus of such materials. Hashin considered the so-called Composite Spheres Assemblage (CSA) consisting of tightly packed congruent composite elements made of spherical particles embedded in concentric matrix shells. Hill considered materials in which both phases have identical shear moduli.

In the field of predicting the elastic moduli of homogeneous isotropic composite materials in general the exact Hashin and Hill solutions are of theoretical interest mainly. Only a few real composites have the geometry defined by Hashin or the stiffness distribution assumed by Hill. The enormous significance, however, of the Hashin/Hill solutions is that they represent bounds which must not be violated by stiffness predicted by any new theory claiming to consider geometries in general.

For a variety of other composites (than Hashin/Shtrikman/Hill) other theoretically well-founded analytical methods have been developed for strictly defined specific phase geometries. Examples are: Ellipsoidal particles in a continuous matrix are examined by Christoffersen [6] and Levin [7]. Other particulate composites are considered in [8–12]. A special particulate composite with compacted spherical particles is examined by Budiansky in [13].

Special fiber reinforced materials are examined by Stang [14], and so-called graded composite materials are considered in [15,16].

Early composite theories based on statistically defined phase geometries are reviewed in [17]. Such approach, using statistical continuum theories, has been further developed by Torquato in [18,19].

If real geometry and theoretically assumed geometry agree with each other excellent results can very often be obtained by the methods just mentioned. Many real composites, however, have geometries, which are substantially different from any of the geometries considered in prediction methods known to day. Composites geometry will change – not only from type of composite to another type – but also in composites individually. First of all, it is very likely that the geometry of material components will vary with phase concentration. This means, for example, that a method for stiffness prediction applying at one concentration is not necessarily the right one to use at another concentration.

This feature is illustrated in Fig. 1.1 showing the influence of porosity on the stiffness of real porous systems such as tile and hardened cement paste. Very often a final critical porosity of 55–75% is approached where stiffness becomes zero. Obviously the critical porosity indicates the extreme state of a continuous process of geometry transformation where the solid phase is increasingly separated and surrounded by an increasing amount of pores. No model with fixed geometry can be used to predict stiffness of porous material. For this reason most relations to day between stiffness of porous materials and porosity are still the empirical expressions developed in [20–22] for

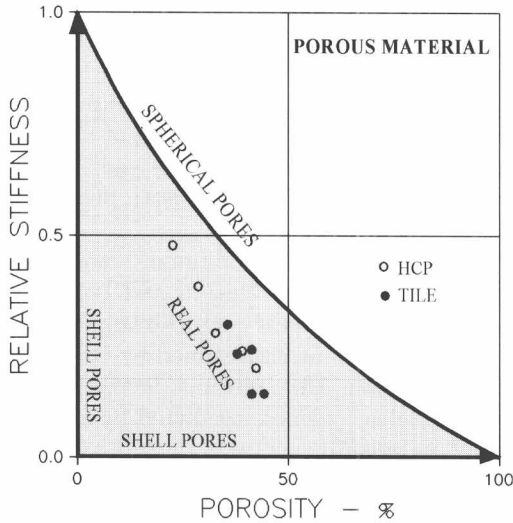


Fig. 1.1. Stiffness of isotropic porous materials as related to pore shapes. Dots are various experimental data reproduced from Chap. 10

example. Only empirical relations are qualified which do not violate the shaded area in Fig. 1.1 bounded by the theory of Hashin [4] previously referred to.

In itself the large number of completely different empirical stiffness expressions suggested for porous materials clearly indicates a need for a more rational research on composite properties versus composite geometry such as reported by the present author in a study [23] on porous materials and impregnated materials.

Change of geometry will influence any mechanical/physical behavior of composites. Stiffness and viscoelasticity (creep and relaxation) will change. Shrinkage and eigenstress-strain (such as hygro-thermal) properties, and heat conductivity are other examples of materials behavior, which will change with geometry.

In order to cope rationally with such changes in composite analysis we must increase our freedom to choose other analytical models than the specific, non-variable ones most often used to day. Valuable progress in materials science can then be achieved in areas, such as a better understanding of the behavior of natural composites, a more rational improvement of such composites, and a rational design of new materials.

Some ideas on how to obtain such freedom are presented by the author in [24] where it is shown that “global” property relations for composites can be established with geometry considered by independent variables, so-called shape functions, which can be studied separately with respect to specific geometries (discrete, continuous, etc).

Aspects of the same problem, how to construct a composite geometry such that prescribed properties can be obtained, have been studied by Milton [25] who introduced the term “inverse homogenization problem” for such composite analysis. Sigmund [26, 27] approaches the problem of inverse homogenization numerically looking at basic porous material structures made by trusses and plates. Milton and Cherkaev [28] provide a basis for studying the problem through analytical studies and construction of so-called extreme materials.

Modern numerical solution techniques such as Finite Element Methods (FEM) have had a tremendous impact on the research on composite materials. These techniques introduced into composite analysis in the 1970es [29, 30] have proved themselves to be very efficient tools in handling composite problems of a complexity (e.g. [31–36]) far beyond what can be treated by analytical means. Recently, numerical methods have also proved their potentials with respect to optimization between shape and properties of structures [37, 38] and between geometry and properties of some special orthotropic composites [39]. Such studies are very useful in the research of optimizing composite geometry in general with respect to composite properties. This feature has been recognized in the works of Sigmund [26, 27] previously referred to.

1.1 Objectives of This Work

In summary, the main objectives of this book are to increase our general understanding of the influence of composite geometry on composite behavior. Some good reasons for increasing our knowledge on geometry versus behavior of composite materials have already been mentioned. Examples are: A better understanding is obtained on the mechanical/physical behavior of natural composites, and a more rational basis is achieved for improving such materials. Geometrical potentials are revealed for the benefit of new materials design with respect to prescribed properties.

As can be noticed from the literature previously cited, other composite researchers agree, that research is necessary on the significance of composite geometry. A number of ways have been applied to approach the problem. Important works have been reported which are based on very strict descriptions and studies (analytical or FEM) of composite geometries, statistically defined (as in [18]), or arranged from basic microstructures (as in [26, 28]). Fine results can be expected from such studies using continuum mechanics on microstructures the geometries of which are basically fixed.

The author's approach presented in this book has another point of departure with respect to "real" composite geometry: It is recognized that varying phase geometries produced by nature or by man can not in general be described (or defined) very precisely. A description must reflect deductions made from experimental studies primarily, including such, which consider technologies used to produce composites.

Basically the methods presented are further developments of the ideas presented in [23, 24, 40] of predicting the properties of any composite material from global expressions with general composite geometries considered by so-called geometry functions. Such functions are presented with specific composite geometries reflected by so-called shape functions. Geometries quantified by these functions are shown to be consistent with the overall composite assumptions previously made with respect to macroscopical homogeneity and isotropy.

Shape functions are developed for a variety of composites including such with geometries previously considered in the literature. Also considered are the somewhat self-defining geometries, which appear in so-called SCS-analysis of composites (Self-Consistency-Scheme).

Special composite problems/properties considered are stiffness, shrinkage, hygro-thermal behavior, viscoelastic behavior, and internal stress states – as well as other physical properties of composites such as thermal and electrical conductivities, diffusion coefficients, dielectric constants and magnetic permeabilities.

The theoretical results obtained are verified by a FEM-analysis made by the author and by theoretical results obtained by other authors. The principal success criteria, however, for the methods developed are that the results

predicted comply with data obtained from experiments on real composites as these data are reproduced from the composite literature.

As a spin-off result the composite theory developed is re-organized to become a “diagnostic tool”, useful in materials design and in quality control of empirical or semi-theoretical prediction methods suggested in the field of composite materials (are such methods consistent with “promises” made with respect to geometry and isotropy).

1.1.1 Summary of Composites Considered

We re-call that the composites primarily considered in this book are perfectly coherent two-phase materials with phase geometries causing macroscopic homogeneity and isotropy. Both phases are isotropically linear-elastic (or -viscoelastic).

Flexible phase geometries primarily are considered which can adjust themselves to form a tight composite. The adjustment can be natural (as in suspensions) or organic (as in bone structures) or it can be the result of compaction (as in sintered powder composites).

As in most literature on composite materials the terms composite, composite material, and two-phase material are used synonymously – unless otherwise indicated as in minor sections of this book where composites in practice do not behave “theoretically”:

When phase geometries are not flexible (such as in composites made of stiff particles in a solidifying matrix as concrete for example) air voids are inevitable at certain concentrations. The two-phase material originally considered becomes a porous two-phase material. In practice such a material can be considered as a normal two-phase composite with a porous matrix. This feature is explained in further details in Sect. 10.4 together with some other composite “defects” (such as incomplete impregnation and incomplete phase contact), which can also be considered introducing some simple phase modifications.

