STATISTICAL METHODS AND CONCEPTS

M N Das

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PREFACE

In course of my long career of teaching, research and consultancy work in Statistics, I had occasions to work with quite a number of applied workers who are not Statisticians but still employ statistical methods in their work. Some of them desired to have an insight into the Statistical methods and their implications rather than the technique of applying them mechanically. In my attempt to explain to them the concepts and methods, I had to start from fundamentals and build up the concepts and methods with proper link between them so that their inter-connection was not disturbed. To this end, I had to avoid using masking mathematical terms and concepts if the same were unsupported by statistical implications. For example, if words like eigen vectors and eigen values are mentioned these had to be explained in terms of Statistical concepts they already knew.

In course of such discussions I happened to think about various concepts and methods arising from fundamental concepts. These have been presented in a manner more appealing to common sense and not introduced abruptly by definition. These have been suitably linked to ensure inter-relation of different methods and concepts included in each chapter. While presenting various topics I have relied on common sense, experience and intuition and have used minimum of mathematics.

There are seven chapters in the book covering, (i) Probability, (ii) Distribution, (iii) Inference, (iv) Linear Estimation, (v) Multivariate distribution and correlation coefficients, (vi) Least Squares method and analysis of variance and (vii) Non parametric methods. Numerous illustrations have been given to clarify different procedures and concepts. A number of new concepts have also been introduced.

The book is expected to serve the student community and teachers at graduate and post-graduate levels. As sophisticated mathematics has been avoided, the book will be a great help in teaching statistical methods to people in non-statistical departments as well.

Another chapter on computer programming, with complete programs for problems requiring relatively lengthy calculations, as in the case of multiple and partial correlation coefficients, partial regression coefficients and their standard errors when the variables are not few, has been included.

I owe a great deal to Prof. N. Giri, University of Montreal for the help he rendered while writing this book. Dr A. Dey, Senior Professor in IASRI, New Delhi went through the manuscript and made some useful comments which improved the book. My son, Ashish Das also helped me greatly through discussion and useful comments while preparing the manuscript of the book. He also wrote some of the programs included in the last chapter of the book. But for the interest that my wife, Chitra, took the book could not have been completed.

M.N. DAS

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Chapter 1

PROBABILITY

1.1 INTRODUCTION

There are innumerable happenings in nature and in the realm of human activity which are associated with uncertainties. Though rising of the Sun next day can be taken to be certain, appearance of clouds in the sky next morning is not as certain. The sex of a baby to be born some months hence is again not known for certain. Each such happening is necessarily associated with two or more outcomes, because if there is only one outcome of some happening, there can not be any uncertainty about the outcome.

Birth of a baby is an outcome. Tossing a six-faced die is another happening. It is seen that each such happening is associated with several outcomes. For the happening of birth of a baby the outcomes can be a male or a female baby; the outcome can also be a live or a dead baby. Thus, for a happening there can be different types of outcomes and under each type there are several outcomes. For the toss of a die one type of outcomes is the appearance of a face with numbers 1, 2, 3, 4, 5 or 6. There are, therefore, 6 possible outcomes. Again, another type of outcomes can be appearance of a face with an even number. In this case there are only two outcomes *viz.* appearance of an even numbered or an odd numbered face. Given a definition of outcomes associated with some happening in some situation, it may be possible to know what are all the possible outcomes, but it cannot be known for certain which outcome will materialise on any given occassion. Thus, the outcomes are associated with uncertainties. For many happenings such uncertainties of outcomes are not random or haphazard. These are seen to obey some rules or laws. Thus, if a very large number of births is observed, it is expected that the outcome of half the number of births will be male babies and that of the other half will be female babies.

The main aim of the Science of Statistics is to arrive at some conclusions regarding such unpredictable happenings, taking into account the variation among its outcomes and magnitude of uncertainties associated with them.

For scientific investigation of uncertainty of outcomes, it is first necessary to have a measure of the degree of uncertainty. Intuitively, we can compare among the degrees of uncertainty of several outcomes. For example, if we take the outcome of the happening of baby-birth as the birth of a live or a dead baby in a modern well-equipped hospital, we can say with a great deal of confidence or certainty that the baby will be born alive. Thus these two outcomes have different degrees of uncertainty associated with them.

Next question is how to attach quantitative measures to the degree of uncertainty so as to conform to experience regarding confidence to be placed on individual outcomes. When a baby is born we can measure his weight which is another type of outcome. The weight measurement has a scale or unit of measure; it is expressed by positive numbers having certain limits. There are certain rules for combining the weight measurements of two or more babies. Likewise, certain conventions have developed regarding measurement of the degree of uncertainty of outcomes. These conventions are in conformity with experiences and expectations and do not lead to absured consequences or conclusions. Before we actually define a measure of the degree of uncertainty, it is necessary to formalise the definition of some concepts we have already introduced.

1.2 SOME DEFINITIONS

1.2.1 Random Experiment

A happening with two or more outcomes is called an experiment.

If the outcomes are associated with uncertainties, the experiment is called random. A random experiment need not always be a planned human activity. It may be natural or any other type of happening.

1.2.2 Events

The outcomes are also called events.

1.2.3 Mutually Exclusive and Exhaustive Events

The possible numerable primary outcomes are mutually exclusive and exhaustive. If, when one outcome occurs no others can occur simultaneously, then the events are said to be mutually exclusive. If there can be no other outcome of a type outside those enumerated, then these events are said to be exhaustive.

EXAMPLE 1.1: Appearances of a head or a tail in the usual coin tossing experiment are two exclusive events, because when a head appears, the tail can not appear simultaneously. These are exhaustive as well because there is no other possible outcome of this type besides these two.

1.2.4 Primary Events

The mutually exclusive and exhaustive events usually constitute primary events.

1.2.5 Derived Events or Event Functions

Two or more primary events can be combined by the conjunction 'or'. Such combined events will be called derived events.

EXAMPLE 1.2: In the experiment of tossing of a die the primary events are the appearance of 1, 2, 3, 4, 5 or 6. A combined or derived event is

(i) the appearance of the face 1 or 2
and (ii) the appearance of faces having even numbers.

If the primary events are denoted by the symbols, A, B, C, etc., then the derived events are written as $A + B$ for A or B. Using notations of set algebra, this derived event is written as $A \cup B$ and is read as 'A union B'. Similarly, the event A or B or C is denoted by $A + B + C$ or $A \cup B \cup C$, a set relation. Another derived event is the intersection of, say, A and B denoted by $A \cap B$. It consists of events present in both A and B.

1.2.6 Degree of Uncertainty of Derived Events

It will be seen that as the primary events are associated with uncertainty, so also are the derived events. Thus, when a rule has been made to attach measures of degree of uncertainty to primary events, there should also be rules to obtain measures of the degree of uncertainty of derived events or some function of derived events. Such rules have been discussed in the latter sections.

1.2.7 Complimentary Events

The events 'Other than A' is the complementary event of A. Complimentary event of A is usually denoted by $\bar{A}$. If appearance of face 1 or 2 in a die experiment is the event A, then appearance of 3, 4 or 5 is $\bar{A}$.

1.2.8 Probability, a Measure of Degree of Uncertainty

A measure of the degree of uncertainty associated with an event is known as probability measure. Such measures are positive and without any scale of measurement like percentages. The measure for a sure event is 1 and that for an impossible event is 0. These can be combined for obtaining a measure for derived events by the rule of addition of the probabilities of the primary events constituting the derived event as discussed afterwards. As the occurrence of any one of all the possible primary events in an experiment is a certainty, the probability measure of this event is 1. If the probability of an event is p, then the probability of its complimentary event is $1-p$. These conventions regarding measure of the degree of uncertainty have been arrived at from experiences gathered from natural happenings and these do not lead to any absurd conclusions. These form the basis of Probability Calculus.

The actual probability measures to be associated with different possible primary events get determined from experience. These can also be suitably assigned values subject to the above conventions. Such assignments of probability measures may be merely theoretical without having anything to do with experience but only to facilitate mathematical manipulation theoretically.

Probability measures assigned through experience of manner of occurrences of equally likely events has given rise to the frequency concept of probability measures. If a random experiment is repeated a very large number of, say, n times, a particular out-come, say, A out of the possible equally likely out-comes will occur a certain number of times, say, m, then we associate the ratio m/n as the probability measure of A. If one watches the sex of a very large number of babies, he may find that in almost half the number of these births male babies were born. Accordingly, from this experience the probability measure associated with the birth of a male baby is given as $\frac{1}{2}$.

This type of measures is called frequency measure of probability. When probability measures are assigned from theoretical considerations, such measures are called theoretic measures of probability. Discussed below are some examples of theoretic measures of probability.

EXAMPLE 1.3: Suppose in a pond there is a certain numbers of fish with known weights on some day. These fish are being caught one by one through a device which can catch only one fish at a time and it is not known in advance which fish is going to be caught. After being caught, a fish may be retained or released back into the pond after inspection.

Now, a probability measure to be associated to each fish being caught and retained may be taken as a positive quantity proportional to its weight, or it may be proportional to the inverse of its weight. Either probability measure system may not agree with any type of previous experience but such assignment system may serve some purpose so as to provide guidelines required for certain aspects of fish business.

EXAMPLE 1.4: We have considered below another example of a random experiment with theoretically associated probability measures. Normally, a die has six faces and the appearance of eaeh face in a toss is equally likely, that is, with equal probability. But for a game of chance a manufacturer has prepared a die with six faces but the face areas are unequal and also the different faces have metallic sheets attached so that the probability of appearance of a particular face in a toss is proportional to the number written on the face. As usual the numbers 1, 2, 3, 4, 5 and 6 are written accordingly on the faces. Thus, the probability measure for the event, face 4, is $C.4$ where C is the constant of proportionality to be determined from the fact that the total probability, that is, the probability of any one of the possible primary events is 1. The probability measures for the different primary events are thus

$$1.C,\ 2\,C,\ 3.C,\ 4.C,\ 5.C \text{ and } 6.C,$$

The probability of having *any one* of the faces in a toss is

$$1C + 2C + 3C + 4C + 5C + 6C = 1$$

Hence,

$$C = \frac{1}{21}.$$

Hence, the actual probability measures associated with each of the primary events of this experiment are

$$\frac{1}{21}, \frac{2}{21}, \frac{3}{21}, \frac{4}{21}, \frac{5}{21}, \text{ and } \frac{6}{21}.$$

The probability measure of the derived event constituting the appearance of a face with even number is

$$\frac{2}{21} + \frac{4}{21} + \frac{6}{21} = \frac{12}{21}$$

1.2.9 Generalised Definition of Probability

In earlier sections while defining probability we introduced, (i) random experiments, (ii) mutually exclusive and equally likely primary events, (iii) probability measures associated with the events, (iv) probability measures are positive and not greater than unity, (v) sum of probabilities associated with all the primary events is unity and (vi) probability measure of derived events which are unions of primary events is the sum of the probabilities of the primary events involved in the union.

This definition does not include many random experiments which are encountered in nature and many day-to-day happenings. For example, if we conduct a coin tossing experiment with a biased coin, the two outcomes of head or tail are not equally likely and hence such experiments remain excluded from the foregoing definition. Thus, while defining probability, 'equally likely events' is an unnecessary limitation. Equally likely events no doubt provide with ready probability measure (viz. $1/n$ where n is the total number of primary events) of each of the primary events when n is finite. Again, a finite n is also a limitation that delimits the scope of probability calculus. One can think of many situations where the total number of possible events of random experiments is infinite. For example, if we consider the experiment of baby birth in a hospital and the events as suitable time intervals. The requirement is a measure of probability of birth in any given time interval. Here the number of time intervals can be uncountably large approaching infinity.

This example brings forth another aspect viz. the events need not be discrete, they can be continuous also as time intervals are. In the continous case as the one here, points do not constitute events but only a set of points, viz., an interval constitutes an event. Similarly, suitable lengths of a line can be events but not individual points on a line. Probability to such events is associated to the sets as a whole and not to individual points in them.

These examples amply demonstrate inadequacy of the definition of probability as provided earlier in that this definition does not include many random experiments which must find place in the calculus of probability.

If we now include all those types of random experiments which may produce, (i) non-equally likely events, or (ii) infinite number of events, or (iii) continuous events, association of probabilities to primary events does not become automatic as in the case of 'equally likely' events. This difficulty is overcome by associating probability to the events by some unknown but estimable quantities subject to certain conditions, mainly that (i) these should be non-negative but not greater than one and (ii) the sum of the probabilities associated with all the primary events should add up to unity.

We have now prepared the ground for a formal symbolic generalised definition of probability as below:

A random experiment is associated with outcomes or events. Let collections of these events be called sets. In the discrete case a set may contain even a single event but in the continuous case a set consists of a range of the events. If two sets do not contain any common event, these are called exclusive sets. If there be a number of sets, say, M then these are mutually exclusive sets, if any set is exclusive of any other set, that is, an event contained in a set is not present in any other set. For a random experiment, let there be in all N sets which are mutually exclusive and exhaustive. The sizes of these sets need not be equal. Then these sets form all the primary sets. Here, N can be infinite also. In other words, if there be a number of sets such that each of the events in the experiment is contained in some set or other and no event is included in more than one set, then these sets form all the primary sets. According to this definition of primary sets if out of M primary sets in all, $M/2$ pairs of sets are formed such that no set is included in more than one pair, then these M sets, now in pairs are also $M/2$ primary sets as these are mutually exclusive and account for all the events in the experiment.