Active Filter Design

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Foreword

Many reference works exist on the subject of active filters. These references usually run the gamut from the very theoretical to the "cookbook" variety. As a consequence the engineer is caught between complex theory and minimal analytical feel for practical filter design. Mr. Chen's extensive experience in the theory, design, and application of active filters has allowed him to write a book that resolves this dilemma.

This book treats the subject of active filters in a straightforward fashion. Basic filter theory to complex section cascading are covered and the emphasis throughout is directed toward the practicing engineer. Relevant examples will assist the first time or experienced designer in understanding the art of active filter design.

The applications for filtering in the electronic system world is rapidly expanding. Mr. Chen's book is a much needed and timely addition to the body of knowledge required to apply this technology.

Barry Siegel, group manager
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Preface

This book was originally conceived as an applications-note tutorial written to fill the void existing in the literature of filter theory. It was through a very fortunate set of circumstances, too numerous to mention here, that the tutorial became this book.

There are electronics books available that assume the practicing engineer has no knowledge of filter design and thus provide filtering circuits, charts, and nomographs to allow one to design filters using the "cookbook" approach. At the other extreme many excellent filter theory books can be found that treat the theoretical aspects of filtering with mathematical rigor. These books assume a priori knowledge of filtering theory and leave the reader with no concept of how one applies the theory to design practical electronic filters.

It is hoped that this book will bridge the gap between "cookbook" and theory and further provide the practicing engineer with a comfortable feel of the "how and why" of active filters.

Subjects covered in *Active Filter Design* include a graphic and definitive introduction to filtering terminology; a quantitative discussion covering the subjects of decibels (dB), the Quality factor (Q), transfer functions, and the damping factor (ζ); and the development of the transfer functions of the five basic filters.

The text also explores the practical aspect of filter design and deals with the theoretical aspect of filtering approximations by relating qualitative general and illustrative definitions of the classic Butterworth, Chebychev, Cauer, and Bessel filter approximations. Filter cascading, normalization, frequency transformation, and impedance scaling are also covered.

A selected bibliography has been provided and lists a few references that treat filter design in a highly theoretical manner in addition to references of the "cookbook" style that provide many useful filter design nomographs and charts.

I wish to acknowledge, and am indebted to, James Moyer for his encouragement, to Bill Ersham for allowing James Moyer to fund this activity, and especially to Dana Knight for his technical feedback. Finally, to my wife Helen, in appreciation of her patience and understanding.

Contents

I

Introduction

The intent of this book is to help those who are not familiar with the subject of active filters understand the field adequately enough to delve into the mounds of established literature and applications on the subject.

The text begins by explaining the advantages and disadvantages of passive, active, and digital filters. Chapter III then familiarizes the reader with filtering terminology and Chap. IV begins a quantitative discussion. Subjects covered in Chap. IV include decibels (dB), the Quality factor (Q), transfer functions, and the damping factor. The book continues with a section on a transfer function's poles, zeros, magnitude, phase, and Bode plot. With this knowledge intact, Chap. VI develops the transfer functions of five basic filters: The lowpass, highpass, bandpass, bandreject, and allpass (or phase-shift) filters. This was done to give the reader an intuitive feel as to what type of realizable mathematical expressions yield the various filter types. Chapter VIII then examines the practical aspect of filter design by presenting many active filter circuits, along with their respective transfer functions. The final two chapters deal with the theoretical aspects of filtering approximations and the areas of cascading, normalization, frequency transformation, and impedance scaling.

No effort has been made to extensively cover computer aided design (although there are a few examples in Chap. VIII) since most of the calculations are algebraic in nature and can easily be worked out on a handheld programmable calculator. There also has been no attempt to make this an active filter cookbook, and the subject matter stays clear of the in-depth areas of sensitivity and operational amplifier limitations. However, these subject areas are covered in many of the excellent references listed in the Bibliography (including a section that treats filter design using charts and nomographs).

II

Passive, Active, or Digital Filters? . . . Some Perspective

The following review of the three most commonly mentioned categories of filters—passive, active, and digital—will briefly summarize the forte of each category and how they relate to one another.

Passive filters, which are comprised of resistors, capacitors, and inductors, can filter an extremely broadband of frequencies (from low to fairly high) in the hundreds of megahertz range.

When considering passive filters in low frequency operation cost and space economy tradeoffs come into play because the inductors required for use in these ranges are expensive, of poor or low quality (Q), and are physically large and bulky. At the other end of the spectrum, however, the high frequency limit is constrained solely by the passive component parasitics. In addition, passive filters do not provide power gain and do not require a power supply.

Active filters, on the other hand, are comprised of resistors, capacitors, and an active element—usually the operational amplifier (op amp).

The advantage of the active filter over the passive filter lies in its ability to operate in the lower frequency range exclusive of costly inductors. The high input and low output impedance characteristics of the op amp allow for good isolation of the filter response from variation in source and load impedances. The ease of cascading active filter sections and their ability to provide gain are added desirable features to consider. This is again due to the inherent isolation characteristics of op amps.

Some active filter drawbacks entail a limited frequency response and the additional need of a power supply. Realistically, the high frequency response limitation reflects the state of present day technology applied to the fabrication of op amp integrated circuits.

Finally, it must be realized that at high frequencies the inductors required in passive filters are of better quality, dimensionally smaller, and lower in cost, which removes the cost advantage of using the active filter over its passive counterpart.

Digital filters, unlike the previous two categories, perform the filtering task using various analog and logic elements inclusive of analog-to-digital converters, shift registers, adders, subtractors, multipliers, multiplexers, digital-to-analog converters, and the like.

Like the active filter, the digital filter is best suited for lower frequencies where the cost of bulky inductors make passive filters impractical.

The digital filter far excels its counterparts in the areas where very high order filters and filter multiplexing is required. In high order filter applications the tuning of passive and active filters becomes quite cumbersome, whereas, in digital filtering, tuning is accomplished by reprogramming the coefficients of the mathematic algorithm.

Finally, the digital filter's upper frequency limit is again limited by state-of-the-art technology when applied to and considering the maximum speeds obtainable from the supportive analog and logic elements.

This general overview of the three categories of filters does not go into great detail. However, the only intent of this section was to provide the perspective that active filters are not the final answer to all filtering problems, but are only a segment of the vast and intriguing world of filtering.

III

Basic Filter Concepts

A filter is a network used for separating signal waves on the basis of their frequency. A filter is usually comprised of passive, reactive, and active elements such as resistors, capacitors, inductors, and amplifiers, or combinations thereof.

There are basically five types of filters used to pass or reject such signals and they are defined as follows:

1. A *lowpass* filter allows a specific band of frequencies to pass. This band of frequencies, called the *passband*, ranges from zero frequency or dc to a certain *cutoff frequency* ω_c*, and has a maximum attenuation or ripple level of A_{max} within the passband (see Fig. 3–1). Frequencies beyond the ω_c may have an attenuation greater than A_{max}, but at the specific frequency ω_s, defined as the *stopband frequency*, a minimum attenuation (A_{min}) must prevail. The band of frequencies higher than ω_s and maintaining an attenuation greater or equal to A_{min} is called the *stopband*. The transition region or *transition band* is the band of frequencies between ω_c and ω_s.
2. A *highpass* filter allows frequencies above the passband frequency ω_c to pass and rejects frequencies below this point. A_{max} must be maintained in the passband, and frequencies equal to and below the stopband frequency ω_s must have a minimum attenuation (A_{min}) (see Fig. 3–2).

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* Recall that the radian frequency $\omega = 2\pi f$

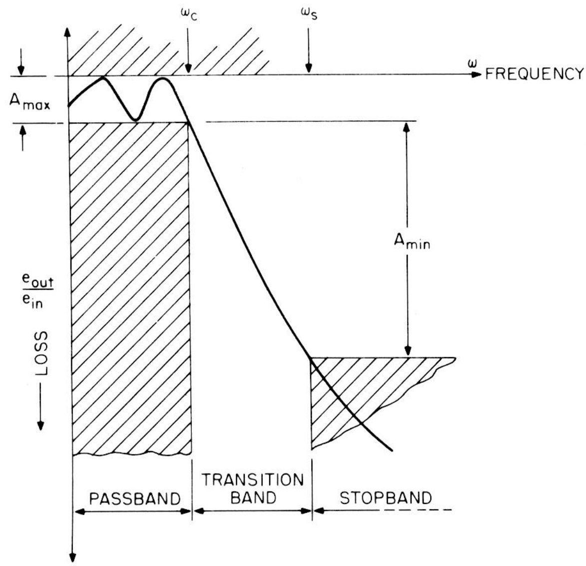


Fig. 3–1. Common lowpass filter response.

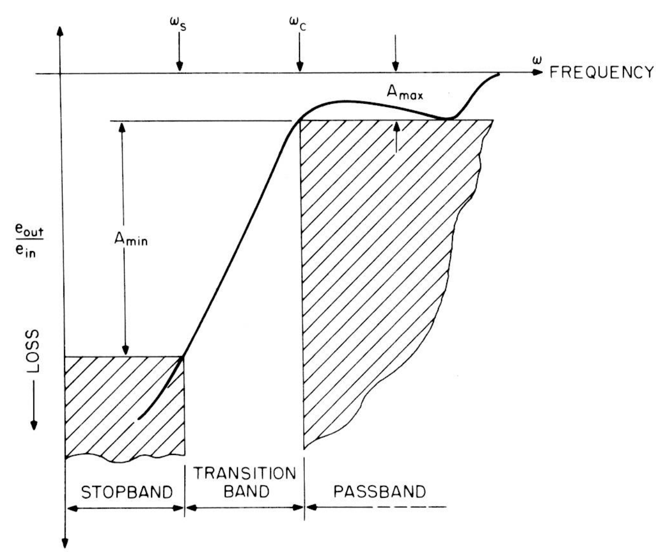


Fig. 3–2. Common highpass filter response.

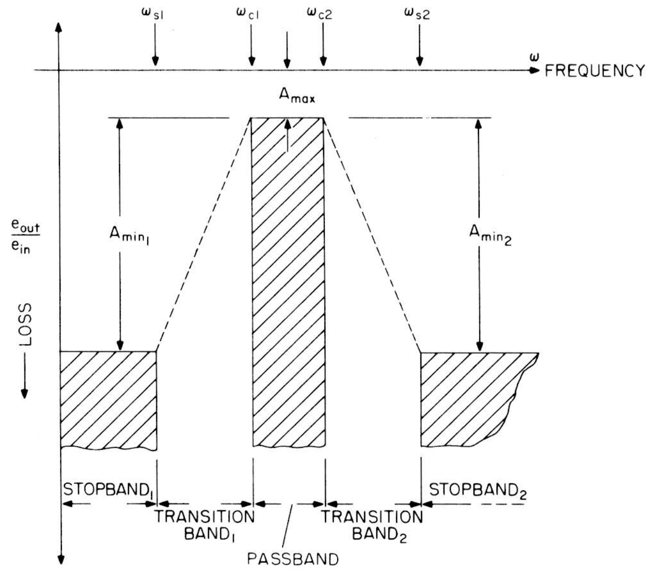


Fig. 3–3. Common bandpass filter response.

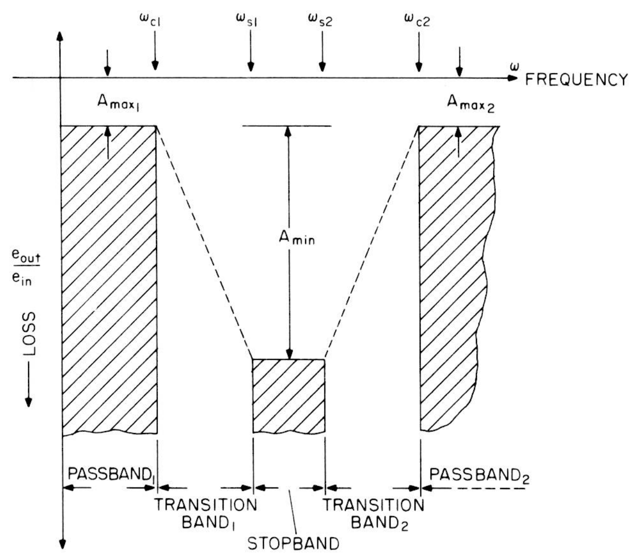


Fig. 3–4. Common bandreject filter response.

3. A *bandpass* filter performs the function of passing a specific band of frequencies while rejecting those frequencies above and below the band's upper ω_{c2} and lower ω_{c1} cutoff frequency limits (see Fig. 3–3). As in the previous two cases the passband is required to sustain an attenuation of A_{max} and the stopband of frequencies above ω_{s2} and below ω_{s1} must have a minimum attenuation of A_{min_1} and A_{min_2}.
4. A *bandreject* filter or *notch* filter allows all but a specific band of frequencies to pass. As shown in Fig. 3–4, the frequencies between ω_{s1} and ω_{s2} are filtered out, and the frequencies above ω_{c2} and below ω_{c1} are passed. The attenuation requirements of the stopband A_{min} and passband A_{max} must still hold.
5. An *allpass* (or *phase-shift*) filter allows all frequencies to pass without any appreciable attenuation. It also introduces a predictable phase shift to all frequencies passed, though not restricting the entire range of frequencies to a specific phase shift (i.e., a phase shift may be imposed upon a selected band of frequencies and appear invisible to all others).

IV

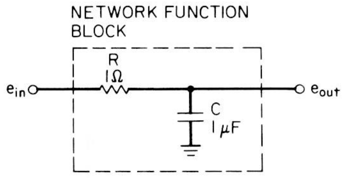
Transfer Functions: Decibels and the Quality Factor

In network synthesis, or in this case active filter design, it is usually desired to describe the network function block in terms of *gain* (what you get out of a filter compared to what you put in). Gain is usually expressed as the ratio e_{out}/e_{in}, which is typically called the network transfer function. Measurement of gain or loss is usually expressed logarithmically in terms of decibels (dB).

A voltage ratio expressed in decibels is defined as 20 $\log_{10}$ (e_{out}/e_{in}). Figure 4–1 summarizes the points made above and Fig. 4–2 summarizes a few useful decibels relationships.

In Fig. 4–1 the points are collectively illustrated by: solving for the transfer function e_{out}/e_{in} of the ***RC*** network in (a); determining the voltage magnitude (to be covered in the next chapter) gain in (b); expressing the gain in terms of decibels in (c). This simple procedure is the same as is used for complex filter networks with the equations becoming somewhat more cumbersome to work with in the latter case.

In a filter network the rolloff rate beyond the cutoff frequency is normally expressed as a slope having a gain (attenuation) of X dB per Y amount of frequency change. This slope is defined as $n20$



(a)

$$e_{out}(s)/e_{in}(s) = \frac{1}{sRC + 1}$$

or

$$e_{out}(j\omega)/e_{in}(j\omega) = \frac{1}{j\omega RC + 1}$$

(b)

$$|e_{out}/e_{in}| = \text{Voltage gain magnitude}$$

$$20 \log_{10}|e_{out}/e_{in}| = \begin{matrix}\text{Voltage gain magnitude} \\ \text{in terms of decibels}\end{matrix} = 20 \log_{10} \left| \frac{1}{sRC + 1} \right|$$

(c)

Fig. 4–1. (a)A passive first-order lowpass filter. (b)Its transfer function. (c)Its gain in terms of decibels.

Voltage decibels $= 20 \log_{10} e_{out}/e_{in}$

A 1-dB change is equivalent to approximately 10 percent change
A 3-dB change drops to approximately 70 percent amplitude
A 6-dB change is equivalent to a 2:1 ratio
A 10-dB change is equivalent to approximately a 3:1 ratio
A 12-dB change is equivalent to a 4:1 ratio
A 20-dB change is a 10:1 ratio
A 30-dB change is approximately a 30:1 ratio
A 40-dB change is a 100:1 ratio
A 50-dB change is approximately a 300:1 ratio
A 60-dB change is a 1,000:1 ratio
A 70-dB change is approximately a 3,000:1 ratio
An 80-dB change is a 10,000:1 ratio

$$X = 10 \log_{10} P_a/P_b = 20 \log_{10} I_a/I_b = 20 \log_{10} V_a/V_b$$

$$P = IE = E^2/R = I^2R$$

Fig. 4–2. Decibel ratios (logarithmic) versus linear ratios.

dB/decade or $n6$ dB/octave where n is the order (degree) of the filter. A decade is a 10:1 frequency interval and an octave is a 2:1 frequency interval. (These notations represent equal slope and are used interchangeably in the literature.) This is more clearly illustrated in Fig. 4–3.

Example 4–1

If the output response of a filter network to a 20 mV, 1 kHz input signal is 3 V, determine the voltage gain in decibels that is associated with the filter at 1 kHz.

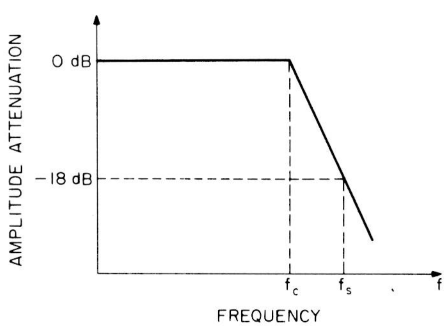
Solution

$$X \text{ dB} = 20 \log_{10} (e_{out}/e_{in})$$
$$= 20 \log_{10} (3 \text{ V}/20 \text{ mV}) \quad (4.1\text{–}1)$$
$$= 43.52 \text{ dB}$$

The filter network has a voltage gain of 43.52 dB at a frequency of 1 kHz.

Example 4–2

What order of lowpass filter is required to meet the specifications shown in the figure below?



Solution

$$n\,(-6 \text{ dB/octave}) = (-18 \text{ dB/octave}) \quad (4.2\text{–}1)$$
$$n = 3$$

In order to satisfy the lowpass filter stopband specifications a third-order filter is required

$$f_c = 1 \text{ kHz}$$
$$f_s = 2 \text{ kHz} \qquad \textbf{(4.2–2)}$$

The passband attenuation is 0 dB for $f \leq f_c$, and the stopband attenuation is more negative than or equal to -18 dB for $f \geq f_s$.

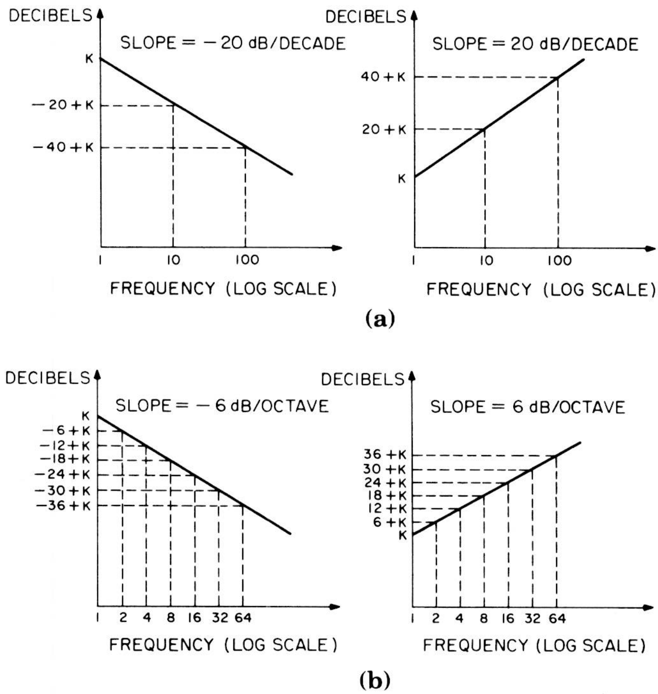


Fig. 4–3. (a)Plots of ±20 dB/decade slopes. (b)Plots of ±6 dB/octave slopes.

Another term to be discussed is the quality factor (Q). Perhaps it will be easier to understand Q if a parallel is made to a term called the damping factor (ζ). The Q of a circuit may be thought of as having the inverse effect of damping.* The following will clarify this statement.

Recall from basic circuit analysis that the response of a resonant circuit excited by an impulse voltage may be characterized as

* Actually $Q = \dfrac{1}{2\zeta}$ where $\zeta = \cos\theta$