

Magnetic Techniques for the Treatment of Materials

Jan Svoboda

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by

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Preface

In the years following the publication of *Magnetic Methods for the Treatment of Minerals* by Elsevier in 1987, many changes have taken place in magnetic technology. While fundamental and thorough, the above treatise reflected preferences and philosophy of research and the development and application of magnetic methods as they were practiced in the second half of the last century. Although demand for metals and minerals has not diminished, the dominant drivers of the early 21st century differ significantly from those of the late 20th. The priorities of nuclear power, defence, and energy-demanding and waste-generating beneficiation of mineral resources are being replaced by those associated with technology sustainability, environmental and knowledge management, recycling, and health care. Research priorities and product development of the last century cannot, therefore, satisfactorily meet criteria of the 21st century.

Considerable technological progress has been achieved in areas such as automation, computerization, sustainable material science, laboratory and plant practices and separation equipment. New permanent magnetic materials, advances in practical applications of superconductivity and availability of sophisticated modelling tools have changed the technological landscape. As a result, innovation and technology transfer in magnetic technology have been remarkably successful during the last two decades.

The title of this monograph reflects the fact that the book covers not only the application of magnetic techniques in the minerals industry, but also in recycling, environmental engineering and biomedical sciences. The goal of the book is to reflect the current technological trends and to re-position the research, development and practice of magnetic methods of material manipulation, according to current needs and strategic thinking. The book is intended for researchers, students, development workers and consulting metallurgists, as well as plant practitioners and equipment manufacturers who wish to acquire comprehensive knowledge of magnetic separation technology.

The book is based on work of numerous scientists and engineers. I am grateful to many of my colleagues and friends at universities, research organizations, and mining and manufacturing companies around the world, for their contribution in advancing magnetic separation technology. Interaction and collaboration with them over the years has been enriching and rewarding and they will be identified in this monograph by citation of their published work. I am

also indebted to many individuals and companies who have given me invaluable help by providing data and photographs for reproduction. Their important contributions have been acknowledged directly in the text.

I am grateful to Mr. E. H. Roux and Drs. D. Slatter, G. Hill and N. Allen for their kindness in reading portions of the manuscript and suggesting improvements. I also wish to express my appreciation to my colleagues Drs. V. Murariu and L. Krüger and Mr. A. Sarelis for assistance with various aspects of this book.

Jan Svoboda
Johannesburg, South Africa
February 2004

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Chapter 1

Principles of Material Treatment by Magnetic Means

1.1 Magnetism and the innovation milestones in magnetic separation

Magnetic phenomena have been known and exploited for many centuries. The earliest experiences with magnetism involved magnetite, the only material that occurs naturally in a magnetic state. Thales of Miletus stated that the magnetic interaction between lodestone, or magnetite, and iron was known for at least as long ago as 600 B.C. That magnetite can induce iron to acquire attractive powers, or to become magnetic, was mentioned by Socrates. Permanent and induced magnetism, therefore, represents one of man's earliest scientific discoveries.

Practical significance of magnetic attraction as a precursory form of magnetic separation was recognized in 1792, when W. Fullarton obtained an English patent for separating iron ore by magnetic attraction [D1]. Since that time the science and engineering of magnetism and of magnetic separation have advanced rapidly and a large number of patents have been issued. While separation of inherently magnetic constituents was a natural early application of magnetism, Wetherill's separator, devised in 1895, was an innovation of significant proportions. It demonstrated that it was possible to separate two components, both of which were commonly considered to be non-magnetic. In the ensuing time various types of disk, drum and roll dry magnetic separators were developed although the spectrum of minerals treatable by these machines was limited to rather coarse and moderately strongly magnetic materials. Since the end of the nineteenth century there has been a steady expansion of both the equipment available and the range of ores to which magnetic separation is applicable.

The development of permanent magnetic materials and improvement in their

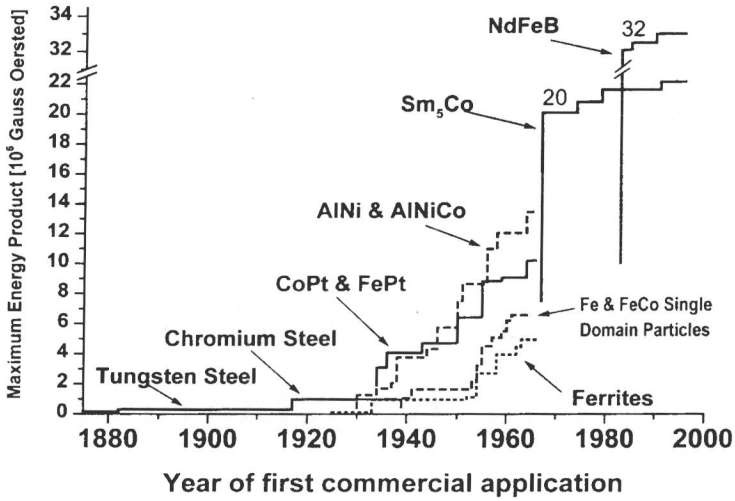


Figure 1.1: Development of permanent magnet materials.

magnetic properties have been main drivers of innovation in magnetic separation. Figure 1.1 illustrates the history of improvement of the maximum energy product $(BH)_{\max}$ of permanent magnets. Three innovation milestones can be identified in the graph. At the end of the nineteenth century very feeble steel-based magnets were employed, while in the 1940s permanent magnets that were able to compete with electromagnets, were developed. Probably the most important innovation step was made in the late seventies of the last century when rare-earth magnets became available. These magnets allowed new solutions for challenges that were not possible or feasible with electromagnets.

Another significant driver of innovation in magnetic separation was the introduction of ferromagnetic bodies (such as balls, grooved plates or mesh) into the magnetic field of a separator. In 1937 Frantz developed a magnetic separator consisting of an iron-bound solenoid packed with ferromagnetic steel ribbons [F1] and this proved to be an important milestone in the development of the present high-intensity and high-gradient magnetic separators. This innovation extended the range of applicability of magnetic separation to many weakly magnetic and even to diamagnetic minerals of the micrometer size.

Although the significance of the discovery of superconductivity has been equated with the invention of the wheel [K1], its importance for magnetic separation does not seem to represent a major breakthrough. The need for magnetic induction greater than 2 T has never been convincingly demonstrated in matrix separators [S1, S2] and the main advantage of superconducting magnets is, therefore, the reduced energy consumption and the possibility of generating a high magnetic force in large volumes, even without using matrices.

The concentration of various ferrous and non-ferrous minerals has been an

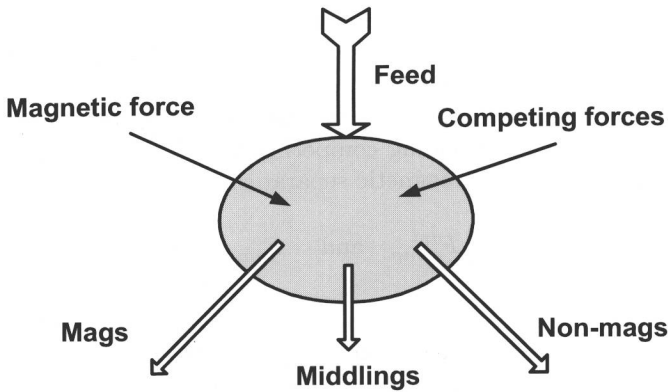


Figure 1.2: Schematic representation of the process of magnetic separation.

important application of magnetic separation, as has the removal of low concentrations of magnetizable impurities from industrial minerals. In recent years, as a result of numerous economic, environmental and social challenges, the recycling of metals from industrial wastes and the concentration or removal of biological objects in medicine and biosciences have become important areas of application of magnetic technology. Recent research and development of eddy-current separators, magnetic fluids and magnetic carriers illustrate the enormous effort that has been expended over the last twenty years in order to convert a wealth of novel ideas into workable techniques and introduce them into material manipulation operations.

1.2 Principles of magnetic separation

Magnetic separation is used for the concentration of magnetic materials and for the removal of magnetizable particles from fluid streams. The separation is achieved by passing the suspensions or the mixtures of particles through a non-homogeneous magnetic field. This process leads to preferential retention or deflection of the magnetizable particles. The same objective is often achieved in a very different fashion, the common features being a competition between a wide spectrum of forces of various magnitudes and ranges.

In magnetic separation the separating (differentiating) external force is the magnetic force. The separation of one material from another or the removal of magnetizable particles from streams depend upon their motion in response to the magnetic force and to other competing external forces, namely gravitational, inertial, hydrodynamic and centrifugal forces. Interparticle forces of electromagnetic and electrostatic origin contribute to the overall scenario, which is depicted in Fig. 1.2.

It is thus clear that a necessary (but not sufficient) condition for a successful

separation of more strongly magnetic from less strongly magnetic particles in a magnetic field is that the magnetic force F_{mag}^m acting on more magnetic particles must be greater than the sum of all the competing forces F_{comp}^{im} . Simultaneously, a magnetic force acting on less strongly magnetic particles, F_{mag}^n must be smaller than the sum of the corresponding competing forces. Therefore, the following conditions must be met in a magnetic separator:

$$F_{mag}^m \geq \sum_i F_{comp}^{im} \quad \text{and} \quad F_{mag}^n \leq \sum_i F_{comp}^{in} \quad (1.1)$$

For different objectives eqs. (1.1) will have specific forms. For instance, in order to achieve high recovery of magnetic particles, the magnetic separating force must be greater than the sum of the competing forces as shown in eqs. (1.1). If, however, the magnetic force is much greater than the competing forces, i.e.

$$F_{mag} \gg F_{comp} \quad (1.2)$$

selectivity of separation will be poor, as no distinction will be made between magnetizable species of different values of magnetic susceptibilities. High selectivity of the separation process can thus be obtained when the magnitudes of the magnetic and competing forces are of comparable magnitudes, compatible with conditions given by eqs. (1.1). The selectivity of the process will be, therefore, critically determined by the relative values of the magnetic and competing forces. And these are affected by the correct choice of a separator and its operating conditions. For instance, selective separation of a magnetizable material (1) from magnetizable material (2) will be achieved when the following relationship is met:

$$F_{mag}^{(1)} > F_{comp} > F_{mag}^{(2)} \quad (1.3)$$

If the non-magnetic fraction is the valuable product, the "brute force" approach to separation, expressed by relationship (1.2), will result in poor yield of the non-magnetic product, as a result of mechanical and magnetic entrainment of the non-magnetic material in the magnetic tailings.

In general, a mixture of particles introduced into the magnetic separator will be split into two or more components. However, in any real separation, both magnetic and non-magnetic particles can be found in the magnetic fraction, non-magnetic fraction and the middling fractions. The efficiency of separation is usually expressed by the recovery of the magnetic component, and by the grade of the magnetic product. However, these criteria are not unique and they must be selected on the basis whether the useful product is the magnetic or non-magnetic fractions.

Since all materials are magnetic to some extent, methods of separation that use magnetic force offer a unique approach to material manipulation in a wide array of industries. One of the main advantages of material treatment in a magnetic field is that the magnetic force can be applied in a controlled manner, in a wide range of values. Moreover, this force can be superimposed on other physical forces and several physical properties of materials can thus be exploited

simultaneously. In addition, magnetic separation can treat a wide range of types of materials, ranging from colloidal to large sizes and from "non-magnetic" to strongly magnetic. Magnetic separation is an environmentally friendly technique and it can operate in wet and dry modes, making it a technique of choice for arid or arctic regions.

1.2.1 Magnetic force on a magnetizable particle

The density of the magnetic energy w_m in a linear isotropic medium is given by:

$$w_m = \frac{1}{2} \vec{H} \cdot \vec{B} \quad (1.4)$$

where H and B are, respectively, the magnitudes of the magnetic field and the magnetic flux density (magnetic induction).

It transpires from eq. (1.4) that the magnetic energy U_{mp} of a magnetizable particle of volume V_p placed in the magnetic field is:

$$U_{mp} = \frac{1}{2} \mu_p V_p H^2 \quad (1.5)$$

while the magnetic energy of a fluid of the same volume is given by

$$U_{mf} = \frac{1}{2} \mu_f V_p H^2 \quad (1.6)$$

In eqs. (1.5) and (1.6), μ_p and μ_f are magnetic permeabilities of the particle and the fluid, respectively. The energy increment U of the system (particle + fluid) is given, to first order, as the difference between the energies given by eqs. (1.5) and (1.6). For weakly magnetic particles this is a good approximation [G1]. Thus:

$$U = \frac{1}{2} (\mu_f - \mu_p) V_p H^2 \quad (1.7)$$

In general, a force can be expressed as $\vec{F} = -\nabla U$, where ∇ is the operator of the gradient. Taking into account that $\mu_j = \mu_0(1 + \kappa_j)$, where κ_j is the volume magnetic susceptibility of material j and μ_0 is the magnetic permeability of vacuum, the magnetic force can be written (in SI units):

$$\vec{F}_m = \frac{1}{2} \mu_0 (\kappa_p - \kappa_f) V_p \nabla H^2 \quad (1.8)$$

In practical situations the magnetic flux density B is frequently used, rather than the magnetic field strength H , and eq. (1.8) can then be expressed as:

$$\vec{F}_m = \frac{1}{\mu_0} (\kappa_p - \kappa_f) V_p B \nabla B \quad (1.9)$$

For sufficiently strongly magnetic particles ($\kappa_p \gg \kappa_f$) it is advantageous to re-write eq. (1.8) as:

$$\vec{F}_m = \mu_0 V_p M \nabla H \quad (1.10)$$

where M is the magnetization of the particle.

Researchers as well as practitioners of magnetic separation sometimes use old *cgs* system of units, in which eq. (1.9) can be written

$$\vec{F}_m = (\kappa_p - \kappa_f) V_p B \nabla B \quad (1.11)$$

In practice it is easier to measure specific (mass) magnetic susceptibility χ rather than volume susceptibility and eq. (1.9) can be re-written, assuming that $\chi_f \ll \chi_p$:

$$\vec{F}_m = \frac{1}{\mu_0} (\chi_p - \chi_f) m_p B \nabla B \quad (1.12)$$

where m_p is the mass of the particle.

It can be seen from eqs. (1.8) to (1.12) that the magnitude of the magnetic force is proportional to the product of the magnetic flux density and its gradient. This force is in the direction of the field gradient, not of the magnetic field. It is also clear that in a homogeneous magnetic field, i.e. when $\nabla B = 0$, the force on a particle is zero. It can also be seen from the above equations that

$$F_m \approx b^3 \quad (1.13)$$

where b the particle radius.

1.2.2 Competing forces in a magnetic separator

As has been mentioned above, magnetic force in a magnetic separator competes with various external forces such as forces of gravity and inertia, centrifugal force and hydrodynamic drag. The importance of each force is a function of the type of the separator and of the mode in which it operates.

For a spherical particle of density ρ_p the gravitational force is given by:

$$\vec{F}_g = (\rho_p - \rho_f) V_p \vec{g} \quad (1.14)$$

where ρ_f and g are the density of the fluid medium and the acceleration by gravity, respectively.

The centrifugal force can be expressed as:

$$\vec{F}_c = (\rho_p - \rho_f) \omega V_p \vec{r} \quad (1.15)$$

where r is the radial position of the particle and ω is the angular velocity.

The hydrodynamic drag force can be obtained from Stokes's equation: