RELATIVISTIC ASTROPHYSICS

VOLUME 2

The Structure and Evolution of the Universe

YA. B. ZEL'DOVICH AND I. D. NOVIKOV

Revised and Enlarged from the Original Russian Edition by the Authors and the Editor

Translated by LESLIE FISHBONE

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THE UNIVERSITY OF CHICAGO PRESS
Chicago and London

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Relativistic Astrophysics, Volume 2, has been translated from the original Russian *Relyativistskaya Astrofizika: Stroeniye i Evolyutsiya Vselennoi* (Moscow: Izdatel'stvo "Nauka," 1975) as revised and enlarged by the authors.

The University of Chicago Press, Chicago 60637
The University of Chicago Press, Ltd., London

Published 1983
Printed in the United States of America
90 89 88 87 86 85 84 83   5 4 3 2 1

Library of Congress Cataloging in Publication Data

Zel'dovich, ÎAkov Borisovich.
Relativistic astrophysics.

Translation of Relîativistskaià astrofizika.
Vol. 2: Translated by Leslie Fishbone; edited by Gary Steigman.
Includes bibliographies and indexes.
CONTENTS: v. 1. Stars and relativity.—v. 2. The structure and evolution of the universe.
1. Astrophysics. 2. Relativity (Physics). 3. Cosmology. 4. Gravitation. I. Novikov, Igor' Dmitrievich. II. Title.
QB461.Z4413   523.01   77-128549

ISBN 0-226-97955-5 (v. 1)   AACR1
ISBN 0-226-97957-1 (v. 2)

EDITOR'S FOREWORD

In less than two decades, modern cosmology has emerged as one of the most exciting, vibrant, and productive fields of scientific endeavor. As midwives in its birth and as shepherds of its more recent growth to maturity, Ya. B. Zel'dovich and I. D. Novikov have played a role of incalculable importance. It is no hyperbole to say that they have conceived of, and carried through, most of the developments which have led to "modern" or "physical" cosmology. This book, the second volume of *Relativistic Astrophysics*, has been long awaited. The wait has been worth it.

In its approach to modern cosmology, this book is encyclopedic. I can think of no important developments which are omitted. It should be an invaluable addition to every physicist's bookshelf. It will serve well those who wish to learn as well as those who wish to review. The approach is a pedagogic one in which no details are omitted and the physics (rather than the mathematical formalism) is always at the forefront.

The publication of this eagerly anticipated book has been much delayed. Virtually all the blame in this respect must fall on me as editor; some fault, however, belongs to the authors. I began my task in 1972 with quite a different manuscript. The subject of cosmology was then in rapid growth, and Zel'dovich and Novikov would provide new chapters as rapidly as I edited the old ones. The process was not converging. A halt was called, and Zel'dovich and Novikov produced an entirely new book, *Cosmology* (Nauka), in Russian which had to be translated before the editing could begin anew.

As editor, I have made a real attempt to verify the correctness of the text and to ensure its clarity. This, most assuredly, is the major cause of the delay in publication. Although I hope the time has been well spent, I urge the serious reader to approach the text with care; there may still be minor errors which have escaped the diligence of the authors and the editor.

As editor, I am indebted to Dr. Leslie Fishbone who provided a superior translation, greatly simplifying my task. Dr. Fishbone worked on the manuscript while at the Department of Physics and Astronomy of the University of Maryland and at the Department of Physics of the University of Utah. He wishes to express his appreciation to the faculty and staff of both departments for their support and assistance. He particularly wishes to thank Anita Wellin for translating the references.

Phyllis Jackson typed most of the manuscript, and I am grateful to her for her patience and her competence. Thanks too are due Bernard Carr, Doug Macdonald, Ian Redmount, and Mike Wilson for editorial assistance. Finally, I must thank Kip S. Thorne for always being available to help when I needed it.

GARY STEIGMAN

PREFACE TO THE ENGLISH EDITION

This book constitutes the second volume of our monograph *Relativistic Astrophysics*. It is devoted to cosmology and can be read independently of the first volume, *Stars and Relativity* (Zel'dovich and Novikov 1971).

Modern cosmology is a vast, rapidly developing field of knowledge. Its theoretical basis is the set of cosmological models devised by the Soviet mathematician Alexander Friedmann. Its observational basis is the work that led to the discovery by the American astrophysicist Edwin Hubble of the law describing the redshift in the spectra of galaxies.

Though the kinematics of the evolving Universe became known decades ago, only in more recent years did research into the physics of processes occurring in the expanding Universe receive a reliable observational and theoretical basis. Indeed, confirmation of the so-called hot-Universe theory has come only during the past decade. Now, with respect to its overall features, one can consider as known the present state of the Universe and that of its most recent past. The results achieved are of permanent significance, forever to be counted in the golden stock of science.

However, these achievements have led to the emergence of new problems, on which an unusually active assault has begun. Diverse attempts have been undertaken to work out the deepest and most difficult questions, those relating to the singular state in the very distant past of the Universe, to the origin of galaxies, and to other matters.

Research concerned with the cosmological singularity is of particular theoretical significance because, though the singular state is distant from us by a time greater than 10^{10} years, every particle (or its predecessor) has emerged from the crucible of the singularity. Said otherwise, the present and future states of the Universe depend upon its past, and consequently on the singular state.

Modern cosmology makes use of the entire arsenal of knowledge in both physics and astronomy. The aforementioned classical hot-Universe theory utilizes the general theory of relativity, thermodynamics, hydrodynamics, and plasma theory. In studies of the singularity, wide use is made of such younger branches of physics as quantum-field theory. It is therefore natural to find as neighbors in this book respected and established theories more than 50 years old and hypotheses which have hardly emerged from infancy. Sometimes they contradict each other!

The parts of this monograph in which are described the basic observational facts and classical theories should form part of the general-education background of every astronomer—and even, in our opinion, of every physicist. On the other hand, knowledge of the entire content of the book is necessary only for those who are working or intending to work in the field of cosmology. Though not a substitute for the original literature, this monograph gives a sufficiently complete picture of the present state of cosmology, especially theoretical cosmology, to satisfy most.

Combining the heterogeneous problems of cosmology in a single volume seemed to us unusually difficult. What seemed utterly unrealistic was our original intention to redo for this edition the cosmology section of our Russian-language book *Relyativistskaya Astrofizika* (Zel'dovich and Novikov 1967*b*). A monograph had to be written all over again. In spite of the large size of the resulting book, it was only with great difficulty that we succeeded in covering all parts of modern cosmology. As in our previous books, along with the mathematical theory of processes, we have tended to give a clear interpretation of the theory, showing especially how the formalism of the theory is applied to the solution of concrete problems.

The Russian version of this book was published in 1975. The volume in hand represents a translation of the 1975 Russian edition. In the course of the work on the translation, however, we once again had to redo several sections in order to describe the achievements of the last few years.

To conclude our excuses regarding the difficulties involved in writing a monograph of this sort, we present the following topical citation from the introduction to Kepler's book *The New Astronomy Based upon Causes, or The Physics of the Heavens:*

"At the present time, the gravest fate is reserved for those who write mathematics or, especially, astronomy books. If the necessary strictness in terminology, explanations, proofs, and conclusions is not maintained, then the book will not be mathematical. If this strictness is maintained, then it will be very tiring to read the book, especially in Latin, which is devoid of the charm peculiar to text written in Greek. Therefore, it is now very rare that one encounters suitable readers; the majority prefers to avoid reading altogether."

While working on this monograph, we enjoyed the unfailing support of our comrades in the theoretical astrophysics section of the Institute of Applied Mathematics of the Academy of Sciences of the U.S.S.R. and of the Institute for Cosmic Research of the Academy of Sciences of the U.S.S.R. We mention particularly A. G. Doroshkevich and R. A. Sunyaev, together with whom Chapter 8 was written. Highly useful too were discussions and debates with workers at the Institute for Cosmic Research, the Shternberg State Astronomical Institute, the Physical Institute of the Academy of Sciences, the Institute for Theoretical Physics, the Institute for Physical Problems, and other institutes, and also with foreign colleagues.

We want to thank especially our foreign colleagues Dr. Leslie Fishbone,

the translator, and Dr. Gary Steigman, the editor of the translation, for having made our book accessible to English-language readers. Dr. Steigman made particularly important contributions by commenting on the substance of the text and by correcting several misprints and errors in the Russian edition. Finally, we hope that the joint work by the editor and us on the translation itself, both through correspondence and during Dr. Steigman's visit to us in Moscow, has improved the English edition.

YA. B. ZEL'DOVICH
I. D. NOVIKOV

INTRODUCTION

Scientific cosmology arose significantly earlier in history than did other astronomical disciplines. In spite of this earlier flowering, and indeed, perhaps as a consequence of it, the subject is presently being developed particularly intensively. Discoveries of enormous significance have resulted.

In the following short review, we shall try to display the structure and logic of modern cosmology. In so doing, it is not our intention to give a historical account. As a result we shall refrain from giving references to the literature; these will be presented when a detailed account of the question at issue is presented. Our philosophy is that the history of the Universe is infinitely more interesting than the history of the study of the Universe.

The most important fact of cosmology is the observed isotropy of the Universe. In other words, the visible picture of the Universe is independent of the direction in which an observer looks. This applies to the radio emission penetrating the entire Universe (the "relic radiation") to which we shall return for a more detailed discussion; to the longer wavelength radio emission from discrete sources, when averaged with respect to an area on the sky containing many sources; and to the X-ray emission, the origin of which is not entirely clear. These forms of radiation reach us from great distances, yet the deviation from isotropy in their intensity does not exceed 0.1–1.0%.

The next step in understanding the structure of the Universe consists of a bold extrapolation. We know that we are located in a spiral galaxy, which in turn is part of a cluster of galaxies. We see other clusters of galaxies scattered in space. However, the isotropy of the relic radiation (RR) and of the other forms of radiation incident from afar is more fundamental than the local inhomogeneity of our immediate surroundings. Indeed, the distribution on the sky of galaxy clusters also presents on the average an isotropic picture. It is therefore natural to assume that the Universe is homogeneous on large scales, although inhomogeneous on small scales. The observed isotropy of the radiation gives a quantitative estimate of the degree of homogeneity. The deviations from homogeneity turn out to be less than 0.1–1.0% on a scale of $\sim 10^{10}$ light-years.

It is necessary to emphasize that, at first glance, the observational data do not by themselves contradict an assumption of spherical symmetry for the Universe. It is possible to assume that the Earth (the solar system or the Galaxy) is located at the center and that the average density of the matter in the Universe depends on the distance from the center, but not on the direction.

The selection of a homogeneous model instead of a model with a center in the Galaxy is based on general philosophical reasoning: In the Middle Ages it seemed natural simply to assume that the Earth is in the central position. Since that time, the prevailing philosophical views have changed; we have become more modest. Finally, an in-depth analysis of all of the observational data leads to the conclusion that the Universe is homogeneous.

Accepting the homogeneous model of the Universe, we are led to several conclusions which follow from well-known physical laws. But it is also appropriate to ask: How justifiable is the use of known physical laws established in the laboratory? Should one expect, upon passing to the grand scale of the Universe, that these laws must be changed? Such a change in the laws of physics over large scales did occur when the general theory of relativity (GTR), with its notion of spacetime curvature, was developed. Curvature clearly must be manifested over large cosmological scales. Now GTR is so designed that the correspondence principle is satisfied: it gives the known laws of physics identically over small scales. Of course, these known laws have been verified with great accuracy over laboratory and even solar-system scales. However, even within the limits of the solar system (and in the most refined, accurate experiments in the laboratory), GTR predicts specific corrections whose existence has been experimentally confirmed. Finally, besides the demonstrated existence of spacetime curvature, there is still the unanswered question that applies specifically to large scales—the question of the cosmological constant.

The cosmological constant describes the (positive or negative) energy density and corresponding pressure attributed to the vacuum. This leads to the appearance of an additional relative acceleration (positive or negative) between any two particles, an acceleration depending only on the distance between the particles and not on their mass. This is a very small addition, however. The cosmological constant describes a universal force of repulsion or attraction proportional to distance. This change in the law of gravitation cannot be measured in laboratory experiments. However, for the dynamics of the Universe as a whole, because of the enormous distances involved, the effects of the cosmological constant can be substantial.

The appearance in physics of new laws and new formulations has always been connected with old laws falling into irreconcilable contradiction with experiment or proving to be logically inconsistent when applied to a new region of experience. So it was, for example, when prequantum physics collided with the ultraviolet catastrophe in the radiation law and with the unexplained stability of atoms. So it was, also, when a relativistic generalization of Schrödinger's equation was required for a description of high-velocity quantum phenomena. Let us now approach GTR in this spirit. Applying GTR to the unbounded Universe, we confront neither logical contradictions in the theory itself nor any glaring contradictions between theory and observations. In particular, there are no observational data suggesting a limitation on the applicability of GTR to the scales of the Universe.[1] Therefore, the assumption that a change in GTR is needed in applications to cosmology is unfounded.

Thus, the aggregate of theoretical, experimental, and observational facts stand in favor of the applicability of the physical laws and GTR to a description of the evolution of the Universe from *almost* the very beginning of the expansion. They apply from times when the matter density is much greater than the density of nuclear matter, ($\rho > 10^{14}$ g cm^{-3}), up to the present time.

The authors want to emphasize the "orthogonality" of their point of view from that of some others. We do not agree with the "theories" appearing from time to time with features that violate the fundamental laws of physics. Such theories are, for example, those in which there is constant creation of matter "out of nothing" far from the singularity (the theory of the steady-state universe), or those which involve a decrease of the gravitational constant with time.[2] Concrete reasons for our disagreement and a criticism of such theories are given in Part V of the book. In short, we adopt the viewpoint that the homogeneous and isotropic Universe can be examined within the realm of GTR.

As is well known, there exists an extensive family of solutions to the Einstein equations for the evolution of the Universe which preserves the homogeneous and isotropic properties in the course of time. But what, specifically, are these properties?

Observations show that we live in an expanding, indeed an evolving, Universe. The redshift of the spectral lines of light from distant galaxies is a consequence of the recession from us of these galaxies. The latest measurements confirm the proportionality between the velocity and the distance: $v = H_0 r$, where $H_0 \approx 50$ km s^{-1} Mpc^{-1} $= (6 \times 10^{17}\text{ s})^{-1} = (2 \times 10^{10}\text{ yr})^{-1}$. The quantity with the dimensions of time inside the parentheses corresponds roughly to the time for expansion from the high-density state. The quantity H_0—"Hubble's constant" (the present value of the "Hubble parameter")—is known from observations to an accuracy of at least 50%.

Many solutions of the equations of GTR agree with the linear relation $v = H_0 r$. Indeed, such an expansion law is necessary in order that the most general properties of such a solution—homogeneity and isotropy—be preserved for the duration of the evolution. At the same time one also assumes

1. As we shall see, relativistic cosmology leads to the necessity for a state in the past with enormous density and with enormous spacetime curvature. This is the so-called singular state. In such a situation, GTR in its existing form is not applicable. A more detailed discussion of this will appear later.

2. We do not exclude unexpected properties of matter at high temperature and density, subject to energetic and thermodynamic limitations.

that the density and pressure of the matter are everywhere the same. That is, they do not depend on the spatial coordinates, though they do depend on time. For a complete solution to the problem of specifying a solution of the equations, however, it is not sufficient to give these properties and the value of H_0 alone. It is also necessary to know the numerical values of the matter density and the pressure today, and the value of the cosmological constant Λ.

Very likely, $\Lambda \approx 0$. In this case one can show that for any matter density, the duration of the past evolution is less than H_0^{-1}. The future evolution then depends essentially on ρ_0, the present-day value of the matter density only. There is a critical value of the density, ρ_c, depending on H_0 (for $H_0 = 50$ km s^{-1} Mpc^{-1}, this critical value is $\rho_c \approx 0.5 \times 10^{-29}$ g cm^{-3}) such that for $\rho_0 < \rho_c$ the expansion will continue without bound, while for $\rho_0 > \rho_c$ the expansion will change to contraction.

The most important property of the Universe—its geometrical structure, whether it is finite or infinite—also depends on the density alone (if $\Lambda \approx 0$). For $\rho_0 > \rho_c$, the Universe is finite although unbounded. This is similar, for example, to the surface of a sphere, which is finite but has no boundaries. For $\rho_0 < \rho_c$, the Universe is infinite and, in this sense, does not differ from classical, three-dimensional, Euclidean space. Thus the density, the future, and the geometrical structure of the Universe are related to one another.

The case of a nonzero cosmological constant is more complicated. For example, given a definite relation between Λ and ρ_0, the finite-Universe case is possible, though in the future, unlimited expansion is in prospect. Here, in a short review, it is inappropriate to present a complete classification of such solutions.

As we shall see later, the direct determination of the average density in the Universe from observations of diverse objects is a very difficult problem since many forms of matter are difficult to observe. This is, however, a major aspect of cosmological research.

In principle, one can determine the structure of the Universe from the curvature of space by observing how the brightness of sources with known absolute luminosity (e.g., galaxies) changes in relation to their distance or to their redshift. One can use the redshift because it is directly connected to distance. However, this redshift-distance dependence varies from model to model.

The theory of light propagation and the connections between the properties of distant objects and observable quantities are those aspects of cosmology which have been worked out most extensively. It would, in principle, be sufficient to measure the redshift and the light flux of two distant objects with known absolute luminosity in order to determine H_0 and the density ρ_0. This is true only when $\Lambda = 0$; however, if $\Lambda \neq 0$, an additional observation is necessary. In order to establish the structure of the Universe, one can measure in place of the luminosity the number of objects of a given type in a given element of solid angle and in a specified interval of redshift. Again though, this must be for objects whose density in space is known from independent considerations. However, this simple and clear program has to this day not been carried out—and not only because of practical, technical difficulties. A deep problem of principle arises because celestial bodies evolve. In observing distant objects, we are also observing them in the distant past. Even for objects of known type (galaxies and quasars), we therefore cannot know with certainty their absolute luminosity and number per unit volume at the instant when the radiation, received by us today, is emitted.

Therefore, a comparison of the theory with observations has not led to the establishment of the structure of the Universe. Rather, proof has been obtained that the specific number density[3] (or the intrinsic luminosity) of the most powerful sources of energy in the Universe—quasars and radio sources—was substantially higher in the past than it is today. For ordinary galaxies, one expects much greater stability and smaller evolutionary effects. But the distances over which galaxies can be observed are small compared to cosmological scales. To this day, therefore, the structure of the Universe has not been established through observations of ordinary galaxies either.

For a determination of the structure of the Universe, it thus became necessary to return to the difficult problem of finding the average density by means of direct observations of the various celestial bodies and of the radiation in the Universe. Indeed, these observations are necessary for one additional reason.

The development of cosmology is inseparable from the question of the material content of the Universe. For the dynamics of the Universe, it is sufficient to know only one quantity, the average density. But, for an understanding of the physical processes, it is necessary to know which particles fill the Universe. We are interested in both questions.

The average density of visible matter in galaxies today is known approximately. If one "spreads" it over all of space, then the resulting density is about 10^{-31} g cm^{-3}, i.e., about 50 times less than the critical density. Very recently it has been shown that massive elliptical galaxies are possibly surrounded by halos consisting of stars of low luminosity. Taking account of this, the average density can be increased several-fold. Now the mass of galaxies is determined from an analysis of the motions of stars and gas clouds under the natural assumption that the age of the galaxies is many times larger than the period of a star in a galactic orbit. Such a method permits one to determine the total mass of a galaxy, a mass including that of invisible forms of material such as extinct stars.[4] Since the mass of a galaxy

3. That is, the number of sources taken o . . a fixed mass of matter in the Universe.

4. Notice, however, that this applies to the mass within the galaxy. Mass in the hypothetical halo surrounding the galaxy does not affect motion in the inner part.

is approximately equal to the sum of the rest masses of the baryons (neutrons and protons) of which the galaxy is composed, one can thereby obtain the average number density of baryons in the Universe today—a density whose order of magnitude is $\sim 6 \times 10^{-8}\ \mathrm{cm}^{-3}$.

One common assumption concerns the charge symmetry of the Universe, i.e., the existence of equal amounts of matter and antimatter on the average. It is necessary to emphasize that, from the point of view of the theory of elementary particles, such an assumption is *a priori* possible but certainly not required. Special searches have been undertaken for the effects of the annihilation of particles and antiparticles between matter galaxies and antimatter galaxies. These searches have yielded a negative result. Therefore, at the present time the assumption of charge asymmetry in the Universe is more probable. This assumption means, for example, that there is only an insignificant number of antibaryons among the cosmic rays.

Observations show the presence of intense radio emission in the centimeter and millimeter bands which cannot be ascribed to discrete sources. It has been assumed, therefore, that this radiation is primordial and arises in the very early stages of the Universe.

Using as a basis the observational data and a minor theoretical extrapolation, the number density of the primordial (relic) photons has been calculated to be about $400\ \mathrm{cm}^{-3}$, which is 10^{8}–10^{10} times greater than the average number density of baryons in the Universe. This RR corresponds to black-body radiation at a temperature of 2.7 K. The average energy of the photons is about 0.0007 eV, while the energy density of the radiation is $\epsilon = 4 \times 10^{-13}$ ergs cm^{-3}. The corresponding mass density is $\rho_{\mathrm{rad}} = \epsilon/c^2 = 5 \times 10^{-34}\ \mathrm{g\ cm}^{-3}$. Thus the contribution of the radiation to the total density is quite small at the present time.

The numerous theoretical and observational studies of the intergalactic gas are still incomplete. The neutral hydrogen content of this gas is quite low, for any such gas must be almost entirely ionized. Its temperature presumably lies within the limits $2 \times 10^4\ \mathrm{K} < T < 3 \times 10^8\ \mathrm{K}$. Such gas is practically transparent at all wavelengths; indeed, the gas itself would emit somewhere between the ultraviolet and X-ray part of the spectrum. The best estimates give the inequality $\rho_{\mathrm{gas}} \lesssim \rho_c$.

A direct determination of the density of neutrinos and gravitational waves is quite difficult. Their density could exceed by many times the density of ordinary matter (baryons). Even so, direct physical methods would still not have sufficient sensitivity to detect these neutrinos and gravitons. Indirect estimates suggest that the density of such particles is substantially less than the density of ordinary matter.

At present, there are no reliable estimates of the possible numbers of extinct quasars, stars, and other low-luminosity or nonluminous objects between galaxies. Concerning galaxies though, the average density of matter in the Universe contributed by them has been fairly reliably established.

This value is $\rho_{gal} \approx 10^{-31}$ g cm$^{-3} \approx 0.02\ \rho_c$. Concerning other forms of matter, one can only say that well-studied forms (for example, RR) have a density substantially less than ρ_c. The density of the remaining forms is very poorly known.

Thus, there is as yet no answer to the question of whether or not ρ_0 is larger than ρ_c, and, consequently, whether or not the Universe is finite or infinite.

Knowing the present content of the Universe, one can trace the earlier stages of evolution. The most important fact is that of the expansion of the Universe. As the expansion proceeds, the RR temperature falls. The value of this temperature today is 2.7 K; in the past it was much higher. Early in the expansion, the dense matter is opaque to radiation. The matter and radiation are in thermodynamic equilibrium and have a very high temperature. Hence the name, the "theory of the hot Universe."

One can consider as firmly established the general picture of evolution that is encompassed by the name, the "theory of the hot Universe." This picture entails first of all the homogeneous (the same at all points in space) and isotropic (the same in all directions) expansion of the Universe. Second, it entails a Universe filled with material that is numerically dominated by photons. Thus, knowing the expansion law and making use of the laws of physics, one can compute the past history of the matter and determine the ongoing physical processes.

The mass density of ordinary matter (baryons) falls with the expansion inversely as the volume, $\rho_b \sim V^{-1}$. The RR energy density falls faster than ρ_b: $\rho_{rad} \sim V^{-4/3}$. Consequently, in the past, the photons predominate at an early stage, not only with respect to number but also with respect to mass. At this stage, called the radiation-dominated (RD) era, the ordinary matter consists of completely ionized helium and hydrogen.

From the equations of mechanics and from the known interactions between photons and atoms, one can find the temperature and composition as a function of time (where we will reckon the time t from the moment when $\rho = \infty$ in the Friedmann solution). For example, at the time $t = 1$ s, the temperature is about 1 megavolt or 10^{10} K, while the density is 10^6 g cm^{-3}. Besides photons, there are at this time almost as many pairs of electrons and positrons present. However, complex nuclei cannot exist at this time. Protons and neutrons exist in almost equal numbers due to the collisions with electrons and positrons, collisions which lead to mutual transformations of the protons and neutrons into each other.

As the expansion proceeds, the positrons disappear. A portion of the neutrons decay, while the remaining neutrons combine with the protons, yielding finally 70% hydrogen and 30% helium by mass. Traces of deuterium and of helium-3 may result, but there is almost a complete absence of heavier elements. This prediction of the theory does not contradict the meager evidence on the possible content of primordial matter (matter not processed by stellar nucleosynthesis). Neutrinos and antineutrinos from this period must also remain. Moreover, those remaining must be roughly equal in number to the number of photons and must have an average energy today corresponding to a temperature of several degrees.

During the expansion following the RD era, i.e., in an epoch close to ours, the mass density of ordinary matter exceeds the mass density of photons. Moreover, this matter is in the form of neutral atoms.

In this general picture many details are lacking. These include a treatment of deviations from homogeneity and a treatment of the earliest stages, when, along with the photons, diverse particles and antiparticles are present and quantum phenomena are important.[5] As we shall see later, there is a definite connection between the question of inhomogeneity and that of the evolution during the earliest stages.

From the observational point of view, the possibility of studying the early stages is limited by the scattering of electromagnetic waves by electrons. All RR received today is repeatedly scattered by electrons when the scale of the Universe is roughly 1000 times smaller than that now, i.e., at a redshift $z \equiv \Delta\lambda/\lambda \approx 1000$. At that time the matter is in the form of an ionized plasma at a temperature of 3000 K. Only in the epoch nearest ours does the temperature fall sufficiently so that the recombination of hydrogen occurs. When the plasma is neutral, it becomes transparent to radiation. One cannot "see" the more distant past with the help of electromagnetic waves. Any contention concerning the homogeneity and isotropy of the Universe which follows from the absence of visible fluctuations in the RR temperature on the celestial sphere applies specifically to the epoch of recombination, when $z \approx 1000$. This is the same as $\sim 10^6$ yr from the beginning of the expansion of the Universe (recall that our epoch corresponds to the time 10^{10} years).

We call "observable" that part of the Universe from which electromagnetic waves can reach the observer without being absorbed or scattered en route. In principle, neutrinos, which are very penetrating particles, could enable one to observe an even larger part of the Universe. Particles emitted at the singularity, moving with the speed of light, could reach the observer; the corresponding distance is called the "horizon."

The "observable" radius is large—about 97% of the "horizon" radius. The consequent volume accessible to observations with the help of RR is 90% of the maximum possible volume. Homogeneity on a large scale is thereby established. However, not excluded is the possibility of inhomogeneity in the Universe and of anisotropy of the expansion before the recombination time of $z \approx 1000$, i.e., for times $t < 10^6$ yr from the start of

5. Note too the still unsolved problems of the energy density of gravitational waves of various wavelengths in the Universe and of whether the densities of neutrinos and antineutrinos can be considered equal.

the expansion. What is more, the present-day structure of the Universe, with its separate galaxies and clusters of galaxies, demonstrates the existence of and the requirement for deviations from the ideal picture of homogeneity and isotropy on all scales.

The general picture of an expanding "hot Universe" set forth above is reliably established and is one of the most important achievements of twentieth-century science. Now we shall pass to an account of the searches for answers to more subtle questions, such as the problem of the origin of galaxies and the problem of the beginning of the cosmological expansion. Here we confront questions at the forefront of today's research. Not surprisingly, we will speak in a less definite tone and will enumerate various hypotheses.

Producing a thorough analysis of the deviations from the ideal picture of a homogeneous, isotropic Universe is the most important task confronting modern cosmology. In the analysis it is necessary to combine observations and theory. The theory of small perturbations provides a solid foundation for the research. The most general property of a Universe close to the ideal is that one can classify the deviations from the ideal (perturbations) into separate forms ("modes") that evolve independently of one another. Such a theory is of great value in an analysis of the observations.

One can successfully carry out a complete examination of the ideal model by virtue of its utterly exceptional properties which significantly simplify the mathematics. At each instant the ideal model is characterized by several numbers in all (the radius of the Universe, the expansion rate, the density, the temperature, and so forth). The complete evolutionary picture in this approximation is then described by the dependence of these several numbers on time. Consequently, the problem is no more complicated than the problem of the motion of a single particle. This simplification is achieved by limiting the problem to the homogeneous case, or, as is said in mathematics, by considering a "degenerate" case. In distinction, the most general formulation of the problem requires the consideration of functions of at least four variables—time and the three spatial coordinates. In such a formulation the problem is exceptionally complicated. The difficulty is further aggravated by our lack of knowledge of the initial conditions—the conditions at the beginning of the expansion. It is necessary either to investigate the large number of variants of the initial state or, having an (incomplete!) idea of the present-day state, to solve the equations for the behavior in the past. Both approaches encounter great difficulties.

The theory of small perturbations is remarkable in that it combines generality of formulation with mathematical simplicity. On the other hand, by virtue of the small amplitude of the perturbations or, as is said in mathematics, by virtue of the linearity of the problem, an examination of each mode reduces simply to finding several functions of time. In other words, the problem is of the same class of difficulty as the problem of the motion of a point. This remarkable combination of generality and simplicity explains why a significant part of the book is devoted to a study of the theory of small perturbations of the homogeneous, isotropic Universe.

In the theory of small perturbations, one considers the evolution of separate modes. This is, however, incomplete in the sense that a determination of the absolute value of the amplitude of the various modes is beyond the theory. One must take this quantity from outside the theory—from observations or from other theoretical considerations. The theory tells us, for example, that a density perturbation encompassing a mass greater than $10^{15} M_\odot$ grows by a factor of 100 in the time that the scale of the Universe also grows by a factor of 100 (for example, from $t = 10^6$ yr to $t = 10^9$ yr). However, whether or not this kind of perturbation occurs in nature, its amplitude remains unknown in terms of this theory alone. But what do observations tell us? Direct observations of the matter distribution tell us that on scales of the order of 10 Mpc and less, the inhomogeneity is large. This maximum scale is roughly the average distance between neighboring clusters of galaxies. For ρ_0 between 10^{-31} g cm^{-3} and $\rho_c \approx \frac{1}{2} \times 10^{-29}$ g cm^{-3}, the mass in a sphere whose diameter is 10 Mpc lies between $7 \times 10^{11} M_\odot$ and $3 \times 10^{13} M_\odot$. The density perturbations on this scale today satisfy $(\delta\rho/\rho)_0 \approx 1$. When do such large perturbations form? Apparently in the recent past, at a redshift $z \approx 5$.[6]

Indeed, one can compute, though only very roughly, that the matter density in clusters is of the order of the average density at just that epoch when the clusters-to-be separate from the expanding background and transform into bound configurations. One can clearly imagine that before some definite redshift z_1 (i.e., for $z > z_1$), the matter is expanding, while thereafter (for $z < z_1$), it divides into separate parts—the clusters of galaxies. The distance between these parts continues to increase, but the parts themselves are "preserved." They are maintained in a state of dynamic equilibrium without a change in their average density. Therefore, the present average density of the matter in clusters is roughly characteristic of the average matter density at the epoch of formation. It is this fact which permits one to estimate the formation time.

Using perturbation theory, one can draw from this a conclusion concerning the amplitude of density perturbations and of other quantities in the distant past. For the plasma state in the RD stage, the amplitude $\delta\rho/\rho \approx 10^{-3}$ is obtained for density oscillations. Correspondingly, both the velocity of the separate elements of matter (expressed in units of the velocity of light) with respect to the background of the general expansion and the dimensionless

6. To each value of the redshift z, there corresponds a definite time in the past. This time is at an epoch in the past separated from our epoch by the time interval necessary for the light ray to undergo the corresponding redshift. As an aid to the reader, it is convenient to remember that the quantity $(1 + z)$ shows how many times the scale of the expanding Universe increases from the given epoch to ours.