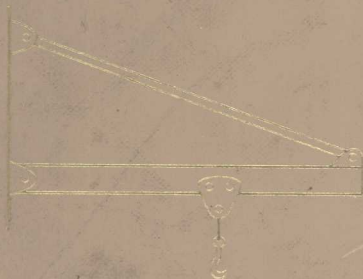


R. C. Hibbeler

FIFTH EDITION

Engineering Mechanics

STATICS AND DYNAMICS



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STATICS AND DYNAMICS

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Preface

The purpose of this book is to provide the student with a clear and thorough presentation of the theory and applications of engineering mechanics. To achieve this objective the author has by no means worked alone on this task, for to a large extent this book has been shaped by the comments and suggestions of more than a hundred reviewers in the teaching profession and many of the author's students who have used its previous editions.

Several changes have been made in preparing this new edition. Overall, particular attention has been given to developing accurate definitions and explanations of the concepts. A greater emphasis has been placed on drawing free-body diagrams by adding specific sections and examples throughout the text on this topic. Also, the importance of selecting an appropriate coordinate system and associated sign convention for vector components is stressed when the equations of kinematics, the equations of equilibrium or the equations of motion are applied. Some rewriting and rearrangement of material has occurred. In Statics, the reduction of a simple distributed loading is covered at the end of Chapter 4, and the last chapters of this volume have been renumbered. In Dynamics, relative motion analysis using Cartesian vectors and scalar components has been combined into a single section. Lastly, most of the problems in this edition are new, and there are more of them than in the previous edition. In this regard, extra care has been taken in the presentation and solution of the problems, and all the problem sets have been reviewed and the solutions checked by the author's colleagues to ensure both their clarity and numerical accuracy.

Organization and Approach. The contents of each chapter are organized into well-defined sections. Selected groups of sections contain an explanation of specific topics, illustrative example problems, and a set of homework problems. The topics within each section are placed into subgroups defined by boldface titles. The purpose of this is to present a structured method for introducing each new definition or concept, and to make the book convenient for later reference and review.

A “procedure for analysis” is given at the end of many sections of the book in order to provide the student with a review or summary of the material and a logical and orderly method to follow when applying the theory. As in the previous editions, the example problems are solved using this outlined method in order to clarify its numerical application. It is to be understood, however, that once the relevant principles have been mastered and enough confidence and judgment have been obtained, the student can then develop his or her own procedures for solving problems. In most cases, it is felt that the first step in any procedure should require drawing a diagram. By doing so, the student forms the habit of tabulating the necessary data while focusing on the physical aspects of the problem and its associated geometry. If this step is correctly performed, applying the relevant equations of mechanics becomes somewhat methodical, since the data can be taken directly from the diagram. This step is particularly important when solving problems involving equilibrium or kinetics, and for this reason, drawing a free-body diagram is strongly emphasized throughout the book.

Since mathematics provides a systematic means of applying the principles of mechanics, the student is expected to have prior knowledge of algebra, geometry, trigonometry, and, for complete coverage, some calculus. Vector analysis is introduced at points where it is most applicable. Its use often provides a convenient means for presenting concise derivations of the theory, and it makes possible a simple and systematic solution of many complicated three-dimensional problems. Occasionally, the example problems are solved using more than one method of analysis so that the student develops the ability to use mathematics as a tool whereby the solution of any problem may be carried out in the most direct and effective manner.

Problems. Numerous problems in the book depict realistic situations encountered in engineering practice. It is hoped that this realism will both stimulate the student’s interest in engineering mechanics and provide a means for developing the skill to reduce any such problem from its physical description to a model or symbolic representation to which the principles of mechanics may be applied. As in the previous edition, an effort has been made to include some problems which may be solved using a numerical procedure executed on either a desk-top computer or a programmable pocket calculator. Suitable numerical techniques along with associated computer programs are given in Appendix B. The intent here is to broaden the student’s capacity for using other forms of mathematical analysis *without* sacrificing the time needed to focus on the application of the principles of mechanics. Problems of this type

which either can or must be solved using numerical procedures are identified by a “square” symbol (■) preceding the problem number.

Throughout the text there is an approximate balance of problems using either SI or FPS units. Furthermore, in any set, an attempt has been made to arrange the problems in order of increasing difficulty.* The answers to all but every fourth problem are listed in the back of the book. To alert the user to a problem without a reported answer, an asterisk (*) is placed before the problem number.

Contents: Statics. The subject of statics is presented in 11 chapters, in which the principles introduced are first applied to simple situations. Most often, each principle is applied first to a particle, then to a rigid body subjected to a coplanar system of forces, and finally to the general case of three-dimensional force systems acting on a rigid body.

The text begins in Chapter 1 with an introduction to mechanics and a discussion of units. The notion of a vector and the properties of a concurrent force system are introduced in Chapter 2. This theory is then applied to the equilibrium of particles in Chapter 3. Chapter 4 contains a general discussion of both concentrated and distributed force systems and the methods used to simplify them. The principles of rigid-body equilibrium are developed in Chapter 5 and then applied to specific problems involving the equilibrium of trusses, frames, and machines in Chapter 6, and to the analysis of internal forces in beams and cables in Chapter 7. Applications to problems involving frictional forces are discussed in Chapter 8, and topics related to the center of gravity and centroid are treated in Chapter 9. If time permits, sections concerning more advanced topics, indicated by stars, (★) may be covered. Most of these topics are included in Chapter 10 (area and mass moments of inertia) and Chapter 11 (virtual work and potential energy). Note that this material also provides a suitable reference for basic principles when it is discussed in more advanced courses.

At the discretion of the instructor, some of the material may be presented in a different sequence with no loss in continuity. For example, it is possible to introduce the concept of a force and all the necessary methods of vector analysis by first covering Chapter 2 and Sec. 4.1. Then, after covering the rest of Chapter 4 (force and moment systems), the equilibrium methods in Chapters 3 and 5 can be discussed.

Contents: Dynamics. The subject of dynamics is presented in the last 11 chapters. In particular, the kinematics of a particle is discussed in Chapter 12, followed by a discussion of particle kinetics in Chapter 13 (equation of motion), Chapter 14 (work and energy), and Chapter 15 (impulse and momentum). The concepts of particle dynamics contained in these four chapters are then summarized in a “review” section and the student is given the chance to identify and solve a variety of different types of problems. A similar sequence

*Review problems, wherever they appear, are presented in random order.

of presentation is given for the planar motion of a rigid body: Chapter 16 (planar kinematics), Chapter 17 (equations of motion), Chapter 18 (work and energy), and Chapter 19 (impulse and momentum), followed by a summary and review set of problems for these chapters. If desired, it is possible to cover Chapters 12 through 19 in the following order with no loss in continuity: Chapters 12 and 16 (kinematics), Chapters 13 and 17 (equations of motion), Chapters 14 and 18 (work and energy), and Chapters 15 and 19 (impulse and momentum).

Time permitting, some of the material involving three-dimensional rigid-body motion may be included in the course. The kinematics and kinetics of this motion are discussed in Chapters 20 and 21, respectively. Chapter 22 (vibrations) may be included if the student has the necessary mathematical background. Sections of the book which are considered to be beyond the scope of the basic dynamics course are indicated by a star and may be omitted. As in *Statics*, however, this more advanced material provides a suitable reference for basic principles when it is covered in other courses.

Acknowledgments. I have endeavored to write this book so that it will appeal to both the student and instructor. Through the years many people have helped in its development and I should like to acknowledge their valued suggestions and comments. Specifically, I wish to personally thank the following individuals who have contributed to this edition, namely, Professors Robert D. Celmer, University of Hartford; Major William Conner, U.S. Military Academy, West Point; Mitsunori Denda, Rutgers University; Robert W. Fuessle, Bradley University; John E. Griffith, University of Southern Florida; William Q. Gurley, University of Tennessee, Chattanooga; Edward E. Hornsey, University of Missouri, Rolla; Jim Kauzlarich, University of Virginia; Mohammad Khosrowjerdi, Western New England College; William M. Lee, U.S. Naval Academy, Annapolis; Captain Al Lewis, U.S. Military Academy, West Point; Will Liddell, Jr., Auburn University at Montgomery; John C. McWhorter, Mississippi State University; Nasser Moshrefi, Michigan Technical University; John Peddieson, Tennessee Technical University; Robert Schmidt, University of Detroit; Robert Snell, Kansas State University; Henry L. Sundberg, Jr., Western New England College; Leang-Neng Tao, Illinois Institute of Technology; Theodore Tauchert, University of Kentucky; Donald M. Wallace, Norwich University; Han-Chin Wu, University of Utah.

Many thanks are also extended to all my students and to those in the teaching profession who have freely taken the time to send me their suggestions and comments. Since the list is too long to mention, it is hoped that those who have given help in this manner will accept this anonymous recognition. Furthermore, I greatly appreciate the personal support and attention given to me by my editors and the staff at Macmillan. Lastly, I should like to acknowledge the encouragement and assistance of my wife, Conny, during the time it has taken to prepare the manuscript for publication.

Russell Charles Hibbeler

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1

General Principles

1.1 Mechanics

Mechanics can be defined as that branch of the physical sciences concerned with the state of rest or motion of bodies that are subjected to the action of forces. A thorough understanding of this subject is required for the study of structural engineering, machine design, fluid flow, electrical devices, and even the molecular and atomic behavior of elements.

In general, mechanics is subdivided into three branches: *rigid-body mechanics*, *deformable-body mechanics*, and *fluid mechanics*. This book treats only rigid-body mechanics, since this subject forms a suitable basis for the design and analysis of many engineering problems, and it provides part of the necessary background for the study of the mechanics of deformable bodies and the mechanics of fluids.

Rigid-body mechanics is divided into two areas: statics and dynamics. *Statics* deals with the equilibrium of bodies, that is, those that are either at rest or move with a constant velocity; whereas *dynamics* is concerned with the accelerated motion of bodies. Although statics can be considered as a special case of dynamics, in which the acceleration is zero, statics deserves separate treatment in engineering education, since most structures or frameworks are designed with the intention that they remain in equilibrium.

Historical Development

The subject of statics developed very early in history, because the principles involved could be formulated simply from measurements of geometry and force. For example, the writings of Archimedes (287–212 B.C.) deal with the principle of the lever. Studies of the pulley, inclined plane, and wrench are also recorded in ancient writings—at times when the requirements of engineering were limited primarily to building construction.

Since the principles of dynamics depend upon an accurate measurement of time, this subject developed much later. Galileo Galilei (1564–1642) was one of the first major contributors to this field. His work consisted of experiments using pendulums and falling bodies. The most significant contributions in dynamics, however, were made by Isaac Newton (1642–1727), who is noted for his formulation of the three fundamental laws of motion and the law of universal gravitational attraction. Shortly after these laws were postulated, important techniques for their application were developed by Euler, D'Alembert, Lagrange, and others.

1.2 Fundamental Concepts

Before beginning our study of rigid-body mechanics, it is important to understand the meaning of certain fundamental concepts and principles.

Basic Quantities

The following four quantities are used throughout rigid-body mechanics.

Length. Length is needed to locate the position of a point in space and thereby describe the size of a physical system. Once a standard unit of length is defined, one can then quantitatively define distances and geometrical properties of a body as multiples of the unit length.

Time. Time is conceived as a succession of events. Although the principles of statics are time independent, this quantity does play an important role in the study of dynamics.

Mass. Mass is a property of matter by which we can compare the action of one body with that of another. This property manifests itself as a gravitational attraction between two bodies and provides a quantitative measure of the resistance of matter to a change in velocity.

Force. In general, force is considered as a “push” or “pull” exerted by one body on another. This interaction can occur when there is either direct contact between the bodies, such as a person pushing on a wall, or it can occur through a distance when the bodies are physically separated. Examples of the latter type include gravitational, electrical, and magnetic forces. In any case, a force is completely characterized by its magnitude, direction, and point of application.

Idealizations

In mechanics models or idealizations are used in order to simplify application of the theory. A few of the more important idealizations will now be defined. Others that are of importance will be discussed at points where they are needed.

Particle. A *particle* has a mass but a size that can be neglected. For example, the size of the earth is insignificant compared to the size of its orbit, and therefore the earth can be modeled as a particle when studying its orbital motion. When a body is idealized as a particle, the principles of mechanics reduce to a rather simplified form since the geometry of the body will not be involved in the analysis of the problem.

Rigid Body. A *rigid body* can be considered as a combination of a large number of particles in which all the particles remain at a fixed distance from one another both before and after applying a load. As a result, the material properties of any body that is assumed to be rigid will not have to be considered when analyzing the forces acting on the body. In most cases the actual deformations occurring in structures, machines, mechanisms, and the like are relatively small, and the rigid-body assumption is suitable for analysis.

Concentrated Force. A *concentrated force* represents the effect of a loading which is assumed to act at a *point* on a body. We can represent the effect of the loading by a concentrated force, provided the area over which the load is applied is *small* compared to the overall size of the body.

Newton's Three Laws of Motion

The entire subject of rigid-body mechanics is formulated on the basis of Newton's three laws of motion. These laws, which apply to the motion of a particle as measured from a nonaccelerating reference frame, may be briefly stated as follows:

First Law. A particle originally at rest, or moving in a straight line with constant velocity, will remain in this state provided the particle is *not* subjected to an unbalanced force.

Second Law. A particle acted upon by an *unbalanced force* \mathbf{F} experiences an acceleration \mathbf{a} that has the same direction as the force and a magnitude that is directly proportional to the force.* If \mathbf{F} is applied to a particle of mass m , this law may be expressed mathematically as

$$\mathbf{F} = m\mathbf{a} \quad (1-1)$$

Third Law. The mutual forces of action and reaction between two particles are equal, opposite, and collinear.

* Stated another way, the unbalanced force acting on the particle is proportional to the time rate of change of the particle's linear momentum.

Newton's Law of Gravitational Attraction

Shortly after formulating his three laws of motion, Newton postulated a law governing the gravitational attraction between any two particles. This law can be expressed mathematically as

$$F = G \frac{m_1 m_2}{r^2} \quad (1-2)$$

where F = force of gravitation between the two particles
 G = universal constant of gravitation; according to experimental evidence, $G = 66.73(10^{-12}) \text{ m}^3/(\text{kg} \cdot \text{s}^2)$
 m_1, m_2 = mass of each of the two particles
 r = distance between the two particles

Weight

Any two particles or bodies have a mutual attractive (gravitational) force acting between them. In the case of a particle located at or near the surface of the earth, however, the only gravitational force having any sizable magnitude is that between the earth and the particle. Consequently, this force, termed the *weight*, will be the only gravitational force considered in our study of mechanics.

From Eq. 1-2, we can develop an approximate expression for finding the weight W of a particle having a mass $m_1 = m$. If we assume the earth to be a nonrotating sphere of constant density and having a mass m_2 , then if r is the distance between the earth's center and the particle, we have

$$W = G \frac{m m_2}{r^2}$$

Letting $g = G m_2 / r^2$ yields

$$W = mg \quad (1-3)$$

By comparison with Eq. 1-1, we term g the acceleration due to gravity. Since it depends upon r , it can be seen that the weight of a body is *not* an absolute quantity. Instead, its magnitude depends upon where the measurement was made. For most engineering calculations, however, g is determined at sea level and at a latitude of 45° , which is considered the "standard location."

1.3 Units of Measurement

The four quantities—length, time, mass, and force—are not all independent from one another; in fact, they are *related* by Newton's second law of motion, $\mathbf{F} = m\mathbf{a}$. Hence, the *units* used to define force, mass, length, and time cannot *all* be selected arbitrarily. The equality $\mathbf{F} = m\mathbf{a}$ is maintained only if three of the four units, called *base units*, are *arbitrarily defined* and the fourth unit is *derived* from the equation.

SI Units

The International System of units, abbreviated SI after the French “Système International d’Unités,” is a modern version of the metric system which has received worldwide recognition. As shown in Table 1–1, the SI system specifies length in meters (m), time in seconds (s), and mass in kilograms (kg). The unit of force, called a newton (N), is *derived* from $F = ma$. Thus, 1 newton is equal to a force required to give 1 kilogram of mass an acceleration of 1 m/s^2 ($N = \text{kg} \cdot \text{m/s}^2$).

If the weight of a body located at the “standard location” is to be determined in newtons, then Eq. 1–3 must be applied. Here $g = 9.806 \text{ 65 m/s}^2$; however, for calculations, the value $g = 9.81 \text{ m/s}^2$ will be used. Thus,

$$W = mg \quad (g = 9.81 \text{ m/s}^2) \quad (1-4)$$

Therefore, a body of mass 1 kg has a weight of 9.81 N, a 2-kg body weighs 19.62 N, and so on.

U.S. Customary

In the U.S. Customary system of units (FPS) length is measured in feet (ft), time in seconds (s), and force in pounds (lb), Table 1–1. The unit of mass, called a *slug*, is *derived* from $F = ma$. Hence, 1 slug is equal to the amount of matter accelerated at 1 ft/s^2 when acted upon by a force of 1 lb ($\text{slug} = \text{lb} \cdot \text{s}^2/\text{ft}$).

In order to determine the mass of a body having a weight measured in pounds, we must apply Eq. 1–3. If the measurements are made at the “standard location,” then $g = 32.2 \text{ ft/s}^2$ will be used for calculations. Therefore,

$$m = \frac{W}{g} \quad (g = 32.2 \text{ ft/s}^2) \quad (1-5)$$

so that a body weighing 32.2 lb has a mass of 1 slug, a 64.4-lb body has a mass of 2 slugs, and so on.

Table 1–1 Systems of Units

Name	Length	Time	Mass	Force
International System of Units (SI)	meter (m)	second (s)	kilogram (kg)	newton* (N) $\left(\frac{\text{kg} \cdot \text{m}}{\text{s}^2}\right)$
U.S. Customary (FPS)	foot (ft)	second (s)	slug* $\left(\frac{\text{lb} \cdot \text{s}^2}{\text{ft}}\right)$	pound (lb)

*Derived unit.