

CONTROL AND DYNAMIC SYSTEMS

ADVANCES IN THEORY
AND APPLICATIONS

Edited by
C. T. LEONDES

VOLUME 27: SYSTEM IDENTIFICATION
AND ADAPTIVE CONTROL
Part 3 of 3

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C. T. LEONDES

School of Engineering and Applied Science
University of California, Los Angeles
Los Angeles, California

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AND ADAPTIVE CONTROL**
Part 3 of 3



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DYNAMIC SYSTEMS

ADVANCES IN THEORY
AND APPLICATIONS

ACADEMIC PRESS RAPID MANUSCRIPT REPRODUCTION

Edited by
C. T. LEONDES

School of Engineering and Applied Science
University of California, Los Angeles
Los Angeles, California

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PREFACE

Volume 27 of *Control and Dynamic Systems* is the third volume in a trilogy whose theme is advances in the theory and application of system parameter identification and adaptive control. System parameter identification and adaptive control techniques have now matured to the point where such a trilogy is useful for practitioners in the field who need a comprehensive reference source of techniques having significant applied implications.

The first contribution in this volume, "A New Approach to Adaptive Control," by C. D. Johnson, presents a powerful new approach to multivariable model reference adaptive control based on the ideas and techniques of disturbance-accommodating control theory, which Professor Johnson also originated. The remarkable degree of adaptive performance that these new controllers can achieve is demonstrated by numerous computer simulation results. The chapter begins with a significant discussion noting the distinction between "robust controllers" and "adaptive controllers."

Certainly one of the most important areas of research over the past two or three decades is the modeling of biological systems, and among the most important researchers on the international scene are G. G. Jaros and his colleagues at the University of Cape Town. In the second chapter, "Biological Systems: A General Approach," Jaros, Belonje, and Breuer present a generalized methodology and terminology for modeling biological systems based on a top-down systems approach. The powerful results presented make this an important reference source for research workers in this significant field.

Among the large-scale complex systems applications benefiting from the large knowledge base developed in system identification and adaptive control is that of optimal environmental control of large buildings. This is becoming increasingly more important with the clear trend toward "megastructures" on the international scene. The third contribution, "Optimal Control for Air Conditioning Systems: Large-Scale Systems Hierarchy," by C. E. Janeke, one of the major international contributors in this area, is a comprehensive treatment of the essential techniques in this extremely important and complex area. The next contribution, "A Linear Programming Approach to Constrained Multivariable Process Control," by C. Brosilow and G. Q. Zhao, presents significant new results for the efficient operation of a process operating at or near constraints on the control efforts and/or process output variables in order to prevent an often inevitable, and possibly severe, degradation in performance. Starting with the control structure used by inferential and internal model controllers, design methods which present signifi-

cantly less computational burden on the controller (and, therefore, are capable of being implemented with microprocessor-based hardware) are developed and illustrated by computer simulations.

The next contribution, "Techniques for the Identification of Distributed Systems Using the Finite Element Approximation," by K. Y. Lee, is a remarkably comprehensive treatment of this subject of broad applied significance. Many engineering physical systems as well as environmental and ecological systems are distributed systems and, therefore, are described by partial differential equations. Their modeling and parameter identification, for the purposes of an implementable control, present formidable problems. Two rather powerful approaches to this problem are presented in this contribution: one is to approximate the distributed system as a finite-dimensional (lumped) system and then to develop a parameter identification scheme; the other is first to develop an infinite-dimensional parameter estimation scheme and then to approximate the solution algorithm using the finite-element method. Both approaches for distributed system modeling and parameter identification are demonstrated numerically and, by numerical example, shown to be highly efficient and effective. In the next chapter, "An Identification Scheme for Linear Control Systems with Waveform-Type Disturbances," by J. Chen, a powerful method is presented for system parameter identification in the presence of system environmental disturbances which cannot be measured. An identification technique is developed that utilizes the disturbance-accommodation control technique of C. D. Johnson to counteract waveform disturbances and applies the maximum likelihood method to identify unknown parameters. Noiselike disturbances are shown to be included as a special case. Numerical examples included in this contribution illustrate the satisfactory convergence of this technique.

"Realizations for Generalized State Space Singular Systems," by M. A. Christodoulou, deals with the fundamentally significant problem of developing irreducible state space realizations or equations for systems from their transfer function representation. The requirement for state space representations of multivariable systems in irreducible or lowest order state vector form deals exactly with the issue of system models of a most efficient form or description. This contribution, which includes significant extensions to the current literature, presents a comprehensive treatment of this fundamentally important subject. The final chapter, "Discrete Systems with Multiple Time Scales," by M. S. Mahmoud, notes that many physical and engineering problems are described by large-scale dynamic models that, as a result, require computational efforts for control analysis and optimization which can be quite excessive. Fundamental techniques for the use of multiple time scales to develop reduced order models which approximate the dynamic behavior of large-scale systems are developed. Interesting adaptive control problems for dominant and nondominant time scale elements of large-scale systems are also developed.

When the theme for this trilogy of volumes was decided upon, there seemed little doubt that it was most timely. The field has been quite active for nearly three decades and has now reached a level of maturity calling for such a trilogy. Because of the substantially important contributions of the authors of this volume and the two previous volumes, however, all three volumes promise to be not only timely but also of lasting fundamental value.

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A NEW APPROACH TO ADAPTIVE CONTROL

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I. INTRODUCTION

In practical applications of control engineering it is common to find that the physical plant (system) one is attempting to control is subject to a range P of plant uncertainties. These plant uncertainties can take the form of uncontrollable changes in plant parameters that occur during operation of the system and/or can arise from the inevitable modeling errors and modeling approximations associated with controller designs for complex systems. In addition, controlled plants typically must operate over a range E of uncertain environmental conditions involving a variety of measurable and unmeasurable external input disturbances which are uncontrollable and have uncertain behavior. It follows that an effective controller must be capable of achieving and maintaining system closed-loop performance specifications in the face of all anticipated plant uncertainties P and environment uncertainties E .

If the range P of plant uncertainty is sufficiently "small," and the effects of environment uncertainty E are sufficiently "mild," a satisfactory controller can usually be designed by elementary control engineering procedures. However, as the effects of P and/or E become increasingly more severe, the design procedures of elementary control engineering become ineffective and one must then resort to a more advanced form of controller design. Controllers of this advanced type belong to a broad category generally referred to as "adaptive controllers" [1]. It should be mentioned, however, that in recent years some researchers have proposed introducing an intermediate category of advanced controllers called "robust controllers" [2], which lie somewhere between elementary controllers and adaptive controllers. There may be a useful purpose served by the concept of a robust (but nonadaptive) controller. Unfortunately, the essential scientific distinction(s) between "nonlinear robust controllers" and "nonlinear adaptive controllers" has never been made clear

in the literature and, consequently, it is difficult to distinguish between robust and adaptive controllers in general; see remarks at the end of Section V,B. The recent introduction of the new term "Robust adaptive controller" [59, 60] has not helped to clarify this issue.

A. DEFINITION OF AN ADAPTIVE CONTROLLER

The precise, universal definition of an adaptive controller is a topic which has been argued among control researchers for many years and the issue still remains unsettled. Our own researches have led us to propose the following definition of an adaptive controller.

Proposed Definition of an Adaptive Controller. Let $x_m(t; x_{m0})$ be the ideal (desired) closed-loop state-trajectory motion for a controlled plant, and let $x(t, u(t); x_0)$ denote the plant's actual state-trajectory motion under control action $u(t)$. Suppose the plant must operate in the presence of major plant uncertainties P and environment uncertainties E . Further, let the adaptation error state $e_a(t)$ be defined as

$$e_a(t) = x_m(t; x_{m0}) - x(t, u(t); x_0).$$

Then, an adaptive controller is defined as any controller that can consistently regulate $e_a(t) \rightarrow 0$ with satisfactorily small settling time, for *all* anticipated plant initial conditions x_0 , *all* anticipated plant uncertainties P , and *all* anticipated environment uncertainties E . The geometric interpretation of this definition of adaptive control is shown in Fig. 1. It should be noted in Fig. 1 that the space (domain) X_m on which x_m is defined need not be the same as that for x , in general; see Section I,B.

Remarks on the Proposed Definition. The "ideal" state-trajectory motion $x_m(t; x_{m0})$ is commonly called the "ideal model" of behavior \mathcal{M} and reflects the desired quality of plant behavior in response to plant setpoints, servo-commands, etc. The ideal model \mathcal{M} can be defined either explicitly or implicitly. In the explicit case the definition takes the form of either a given vector function $x_m(t; x_{m0}) = \mu(t)$ or a given ideal state-evolution equation $\dot{x}_m = F_m(x_m, t)$, $x_m(0) = \mu(0) = x_{m0} \in X_m$, for the plant, where X_m is invariant for all $x_m(t)$. In the implicit case, the ideal model $x_m(t; x_{m0})$ is defined as the plant trajectory which minimizes some given functional J on $x(t, u(t); x_0)$ and/or $u(t)$, subject to specified constraints and boundary conditions. When $x_m(t; x_{m0})$ is defined explicitly, the controller is called a model-reference adaptive controller. When $x_m(t; x_{m0})$ is defined implicitly, the controller is called a self-optimizing adaptive controller. Note that, in general, the definition of $x_m(t; x_{m0})$ can be either independent of the plant characteristics, dependent on only (known) nominal plant characteristics, or dependent on the actual (real-time) perturbed plant characteristics. The latter often arises, for instance, when $x_m(t; x_{m0})$ is defined implicitly.

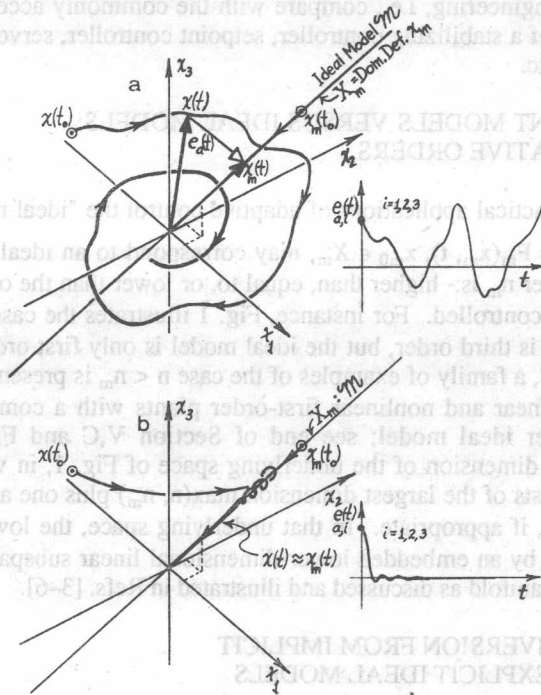


Fig. 1. Proposed definition of an adaptive controller, (a) without and (b) with adaptive control.

The two nebulous terms "major uncertainties" and "satisfactorily small settling time" are admittedly a weakness in our definition of an adaptive controller — but it appears such a weakness is unavoidable. For instance, if the definition allowed "minor" uncertainties and/or "arbitrarily large" settling times for $e_a(t) \rightarrow 0$, then almost any well-designed elementary controller would qualify as an adaptive controller. Thus, any definition of adaptive control must impose thresholds on the extent of uncertainty and limits on the $e_a(t)$ settling time to avoid such degeneracy. Otherwise, one must introduce equally nebulous qualifying terms such as weakly adaptive, strongly adaptive, etc.

Our definition of an adaptive controller is based on the time behavior of the adaptation error state $e_a(t)$ and imposes no restrictions or requirements on the mathematical, algorithmic, or physical structure of the controller itself. Indeed, adaptive controller definitions which impose controller structural requirements such as: an adaptive controller must always be "nonlinear," or an adaptive controller must always have "adjustable controller gains," appear to have no rational

scientific basis, in our opinion, and such structural requirements have no precedent in control engineering, i.e., compare with the commonly accepted structure-free definitions of a stabilizing controller, setpoint controller, servocontroller, optimal controller, etc.

B. PLANT MODELS VERSUS IDEAL MODELS: RELATIVE ORDERS

In practical applications of adaptive control the "ideal model" $x_m(t; x_{m0}) = \mu(t)$, or $\dot{x}_m = F_m(x_m, t)$, $x_{m0} \in X_m$, may correspond to an ideal system whose dynamical order n_m is: higher than, equal to, or lower than the order n of the actual plant being controlled. For instance, Fig. 1 illustrates the case $n > n_m$ where the actual plant is third order, but the ideal model is only first order (and linear). In Section V,C, a family of examples of the case $n < n_m$ is presented, consisting of a family of linear and nonlinear first-order plants with a common (fixed) linear second-order ideal model; see end of Section V,C and Fig. 11. Thus, the appropriate dimension of the underlying space of Fig. 1, in which to view such cases, consists of the largest dimension $\max(n, n_m)$ plus one additional dimension for $x_{n+1} = t$, if appropriate. In that underlying space, the lower-order system is represented by an embedded lower-dimensional linear subspace, affine space, or nonlinear manifold as discussed and illustrated in Refs. [3-6].

C. CONVERSION FROM IMPLICIT TO EXPLICIT IDEAL MODELS

Implicit ideal models are specified only in an indirect manner, as the trajectory family $x_m(t; x_{m0})$ which minimizes some given functional (optimization criterion) J on $\dot{x}(t)$ and/or $u(t)$. However, in some cases it is possible to explicitly identify the implicit-ideal model $x_m(t; x_{m0})$ from the form of the functional J . For example, it has been shown in [7, 8] that the implicit ideal model associated with the n -th-order, time-invariant plant $\dot{x} = Ax + bu$ and quadratic optimization criterion $J = \int_0^\infty [x^T(t)Qx(t) + \rho u^2] dt$, $\rho \rightarrow 0$, can be explicitly described as a certain well-defined $n_m = (m - 1)$ -th order ($m \leq n$) linear differential equation $\dot{x}_m = A_M x_m$ which manifests itself as a certain $(m - 1)$ -dimensional hyperplane (linear subspace) in the underlying n -dimensional state space of Fig. 1. Moreover, the procedures developed [7, 8] are reversible, allowing one to go from a given $(m - 1)$ -th-order ideal model $\dot{x}_m = A_M x_m$ to an n -dimensional quadratic optimization criterion J . A similar conversion result for the case $\rho \rightarrow 0$, in which case the plant and ideal model are both n -th order, is well known in linear-quadratic regulator theory [9]. An important generalization of the latter result has recently been developed [10, 11].

D. CONTRIBUTION OF THIS ARTICLE

In this article we present a new approach to multivariable model-reference adaptive control based on the ideas and techniques of disturbance-accommodating control theory [12-16]. This new approach leads to an adaptive controller design procedure that is systematic in nature and applies to a broad class of plants. Moreover, the new adaptive controllers obtained by this design procedure are strikingly simple in structure and often reduce to completely linear, time-invariant controllers. The remarkable degree of adaptive performance which these new controllers can achieve is demonstrated by numerous worked examples with computer simulation results.

II. CONCEPTUAL APPROACHES TO ADAPTIVE CONTROL: AN OVERVIEW

The idea of an "adaptive" control system has a long, colorful history and, over the years, has been the subject of numerous research efforts resulting in many technical papers [7-20] and books [1, 21-23, 50, 62]. As a result of this effort, basically two conceptual approaches to adaptive controller design have emerged.

A. THE 'ADAPTIVE-GAIN' SCHOOL OF THOUGHT

Most of the past and current research in adaptive control has focused on what we will call the "adaptive-gain" school of thought [22]. In that approach the adaptive control law is postulated in the form (linear, scalar control case shown)

$$u = k_1(\cdot)x_1 + k_2(\cdot)x_2 + \dots + k_n(\cdot)x_n, \quad (1)$$

where the control "gains" $k_i(\cdot)$ are automatically adjusted in real time, by an adaptive algorithm, in accordance with perceived perturbations in plant parameters, environmental conditions, etc. The adaptive algorithm is driven by the available plant outputs so that the $k_i(\cdot)$ in (1) are functions of (or functionals on) one or more of the plant state variables x_i . Moreover, the adaptive algorithm itself usually involves one or more nonlinear operations. As a consequence, adaptive controllers based on the adaptive-gain control law (1) are inherently nonlinear.

B. THE 'SIGNAL-SYNTHESIS' SCHOOL OF THOUGHT

Although the adaptive-gain school of thought continues to dominate most of the research in adaptive control, it is, in fact, only one of the possible con-

ceptual approaches to adaptive controller design. Another conceptual approach to adaptive control, which received some attention in the 1950–1965 period but is rarely given more than casual mention now, is called the "signal-synthesis" (or "auxiliary-input signal") school of thought [1; pp. 13, 16–19]. This latter approach is based on the concept that expression (1) is nothing more than a particular decomposition (expansion) of the needed control action $u(t)$ and, as such, expression (1) is only one of many possible ways to synthesize the (essentially) same adaptive control time signal $u(t)$, $t_0 \leq t \leq T$. In particular, one can envision the adaptive control *signal* $u(t)$ in (1) to be generated alternatively by a real-time weighted linear combination of "basis functions" of the form

$$u(t) = c_1 f_1(t) + c_2 f_2(t) + \dots + c_k f_k(t), \quad (2)$$

where the set $\{f_1(t), \dots, f_k(t)\}$ of basis functions is chosen a priori by the designer to provide a qualitative fit to the likely waveform of $u(t)$ and the "constant" weighting coefficients c_i are automatically adjusted in real time to achieve a quantitatively good approximation to $u(t)$. Some possible choices for the basis functions $\{f_i(t)\}$ are those associated with: power series in t , Fourier series, or any of the classical orthogonal polynomials such as Chebyshev, Legendre, Hermite, etc. The two challenges in the signal-synthesis approach to adaptive control are to determine what value $u(t)$ should be at each t and to devise an effective real-time procedure for automatically adjusting the weighting coefficients c_i in (2) to continually realize the required adaptive control signal $u(t)$, $t_0 \leq t \leq T$. Some attempts to design and implement such signal-synthesis adaptive controllers are described in Refs. [24, 25].

The fact that one can achieve essentially the same adaptive control *signals* $u(t)$ in (1) and (2), using adaptive controllers that are very different in structure, is a point which seems to have been overlooked by many researchers and educators in the adaptive control field; see remarks at the end of Section VII.

C. SIGNAL SYNTHESIS REVISITED

The signal-synthesis approach to model-reference adaptive control for linear plants was re-examined [26–29] using the relatively new concepts and tools of disturbance-accommodating control (DAC) theory [12–16]. The result of that effort was the development of a new, easily implemented and remarkably effective version of multivariable signal-synthesis adaptive control. These new adaptive controllers can achieve effective adaptive control for both linear and nonlinear plants and in many cases the controllers are *entirely linear* and have *constant coefficients*. The remainder of this article is a tutorial presentation of this new approach to multivariable model-reference adaptive control, including worked examples and results of computer simulation studies.

III. FORMULATION OF A GENERAL CLASS OF ADAPTIVE CONTROL PROBLEMS FOR LINEARIZED DYNAMICAL SYSTEMS

The class of adaptive control problems considered in this article is formulated around a general type of controlled, possibly nonlinear, dynamical system modeled by the state/output equations

$$\dot{x} = \mathcal{F}(x, t, u, w), \quad x = (x_1, \dots, x_n) \quad (3a)$$

$$u = (u_1, \dots, u_r)$$

$$w = (w_1, \dots, w_p)$$

$$y = \mathcal{G}(x, t), \quad y = (y_1, \dots, y_m), \quad (3b)$$

where x is the system state vector, y is the system output vector, and u, w are, respectively, the system control input vector and external disturbance input vector. It is assumed, for convenience only, that the operating regime \mathcal{R} for (3) is sufficiently well known and the functions $\mathcal{F}(\cdot), \mathcal{G}(\cdot)$ are sufficiently smooth in \mathcal{R} to permit the effective linearization of (3) in \mathcal{R} so that

$$\dot{x} \big|_{\mathcal{R}} = \mathcal{F}(x, t, u, w) = A(t)x + B(t)u + F(t)w + \eta(x, t, u, w) \quad (4a)$$

$$y \big|_{\mathcal{R}} = \mathcal{G}(x, t) = C(t)x + v(x, t), \quad (4b)$$

where the expressions $\eta(x, t, u, w), v(x, t)$ represent the collection of all higher-order (e.g., not linear) terms associated with the series expansions on the right sides of (4).

The matrices $A(t), B(t), F(t), C(t)$ of partial derivatives $\partial \mathcal{F} / \partial x_j, \partial \mathcal{G} / \partial x_k$, etc., are assumed to be evaluated at a specified (given) "nominal operating point" $\pi \in \mathcal{R}$. Thus the nominal behavior (value) of each element in $A(t), B(t)$, etc., in (4) is assumed known a priori. The expressions $\eta(\cdot), v(\cdot)$ representing the higher-order terms in (4) are not assumed known but, in keeping with tradition, are assumed negligibly small in comparison with the linear terms of (4), at least in a neighborhood of the nominal operating point $\pi \in \mathcal{R}$. Actually, none of these assumptions is essential because the adaptive controller to be derived here can, in principle, accommodate even nonsmooth, unknown systems (3) and values of $\eta(\cdot)$ which are not negligibly small, as will be shown later in the examples of Section V. Thus, in a neighborhood of the specified operating point π the dynamical system (1) being controlled is represented by the linearized model

$$\dot{x} = A(t)x + B(t)u + F(t)w \quad (5a)$$

$$y = C(t)x, \quad (5b)$$

where the nominal behavior of $A(t)$, $B(t)$, $F(t)$, and $C(t)$ is known. The results developed here can be generalized to the case $y = Cx + Eu + Hw$ by the techniques used in Refs. [13, Eqs. (42) and (49); 30].

A. CHARACTERIZATION OF PLANT UNCERTAINTY

The uncertainty associated with the mathematical model (5) is presumed to be in the off-nominal behavior of $A(t)$, $B(t)$, $C(t)$, $F(t)$ and in the behavior of the external disturbance $w(t)$. In regard to the plant uncertainty, the parameter elements $a_{ij}(t)$, $b_{ik}(t)$, $c_{si}(t)$, $f_{il}(t)$ of the matrices $A(t)$, $B(t)$, $C(t)$, $F(t)$ are assumed to deviate from their known nominal behavior in a manner which is uncertain. Thus, the behavior of $A(t)$, $B(t)$, $C(t)$, $F(t)$ in (5) can be represented as

$$A(t) = A_N(t) + [\delta A(t)]; \quad C(t) = C_N(t) + [\delta C(t)] \quad (6)$$

$$B(t) = B_N(t) + [\delta B(t)]; \quad F(t) = F_N(t) + [\delta F(t)],$$

where $A_N(t)$, $B_N(t)$, $C_N(t)$, $F_N(t)$ are known and the parameter perturbation matrices $[\delta A(t)]$, $[\delta B(t)]$, $[\delta C(t)]$, $[\delta F(t)]$ are completely unknown. In the controller design procedures to be derived below, it will be mathematically convenient to model the time behavior of the unknown perturbations $[\delta A(t)]$, $[\delta B(t)]$, $[\delta C(t)]$, $[\delta F(t)]$ as either slowly varying (time derivative ≈ 0) or as piecewise-constant with random jumps occurring in a once-in-a-while fashion. However, the final adaptive controller designs can, in fact, accommodate unknown parameter perturbations with a significant degree of nonconstant behavior, as illustrated by the worked examples presented in Section V.

B. CHARACTERIZATION OF DISTURBANCE UNCERTAINTY

The uncertainty associated with the behavior of measurable and unmeasurable external disturbances $w(t)$ in (5) is not modeled statistically, but rather is represented by a semideterministic waveform-model description of the generalized spline-function type. Namely, each independent element $w_i(t)$ of w is modeled as

$$w_i(t) = c_{i1}f_{i1}(t) + c_{i2}f_{i2}(t) + \dots + c_{im_i}f_{im_i}(t), \quad (7)$$