International Society for Analysis, Applications and Computation

Recent Developments in Complex Analysis and Computer Algebra

Edited by
Robert P. Gilbert,
Joji Kajiwara
and Yongzhi S. Xu

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PREFACE

This volume consists of papers presented in the special sessions on "Complex and Numerical Analysis", "Value Distribution Theory and Complex Domains", and "Use of Symbolic Computation in Mathematics Education" of the ISAAC'97 Congress held at the University of Delaware, during June 2-7, 1997. The ISAAC Congress coincided with a U.S.-Japan Seminar also held at the University of Delaware. The latter was supported by the National Science Foundation through Grant INT-9603029 and the Japan Society for the Promotion of Science through Grant MTCS-134.

It was natural that the participants of both meetings should interact and consequently several persons attending the Congress also presented papers in the Seminar. The success of the ISAAC Congress and the U.S.-Japan Seminar has led to the ISAAC'99 Congress being held in Fukuoka, Japan during August 1999. Many of the same participants will return to this Seminar. Indeed, it appears that the spirit of the U.S.-Japan Seminar will be continued every second year as part of the ISAAC Congresses. We decided to include with the papers presented in the ISAAC Congress and the U.S.-Japan Seminar several very good papers by colleagues from the former Soviet Union. These participants in the ISAAC Congress attended at their own expense.

This volume has the title **Recent Developments in Complex Analysis and Computer Algebra.** These papers contain treatments of Nevanlinna theory, Fatou-Julia theory, entire and meromorphic functions, Kähler manifolds, kernel functions, extensions of holomorphic and meromorphic functions, several complex variables, computer applications to complex analysis, line bundles, and collocation methods.

We would like to thank the National Science Foundation and the Japanese Science Foundation who so generously supported our seminar. We would like to thank Ms. Pamela Irwin and Ginger Moore who helped in the organization of the Conference. Professors Wenbo Li, Rakesh, and Shangyou Zhang served on the organization committee. The following graduate students Min Fang, Zhongshan Lin, Nilima Nigam, Yvonne Ou, Alexander Panchenko and his wife Elena who helped at the registration desk. Finally, most of all we want to thank Pamela Irwin for her tireless effort with the preparation and formatting of the manuscripts. Without this help these proceedings would not have made it to publication.

CONTENTS

H^P AND L^P EXTENSIONS OF HOLOMORPHIC FUNCTIONS FROM SUBVARIETIES TO SOME CONVEX DOMAINS

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Abstract: Using the integral formula for holomorphic functions in subvarieties, we study the extension of holomorphic functions from subvarieties to convex domains in some function spaces.

1.1 INTRODUCTION

Let D be a bounded domain in $\boldsymbol{C}^n$ with C^1 boundary. We denote by $C^{k,\infty}(\partial D \times D)$, $k \geq 0$, the space of all C^k functions $f(\zeta, z)$ in $\partial D \times D$ which are of class C^∞ with respect to the second variable. We say a $(1,0)$-form $W = \sum_{j=1}^n w_j(\zeta, z) d\zeta_j$ is a **generating form** with coefficients in $C^{k,\infty}(\partial D \times D)$ if W satisfies the following conditions (i) and (ii):

(i) $w_j(\zeta, z) \in C^{k,\infty}(\partial D \times D)$.

(ii) $\sum_{j=1}^n w_j(\zeta, z)(\zeta_j - z_j) = 1$.

R. P. Gilbert et al. (eds.), Recent Developments in Complex Analysis and Computer Algebra, 1–12.

Let D be a bounded convex domain with C^2 boundary defined by a defining function $\rho(z)$. If we set

$$w_j(\zeta, z) = \frac{\frac{\partial \rho}{\partial \zeta_j}(\zeta)}{\sum_{k=1}^n \frac{\partial \rho}{\partial \zeta_k}(\zeta)(\zeta_k - z_k)}.$$

Then $W = \sum_{j=1}^n w_j(\zeta, z) d\zeta_j$ is a generating form with coefficients in $C^{1,\infty}(\partial D \times D)$. We define

$$\beta = |\zeta - z|^2, \qquad B = \frac{\partial \beta}{\beta}, \qquad I = [0, 1].$$

The homotopy form $\hat{W}(\zeta, \lambda, z)$ on $(\partial D \times I) \times D$ associated to W is defined by

$$\hat{W}(\zeta, \lambda, z) = \lambda W(\zeta, z) + (1 - \lambda) B(\zeta, z).$$

Cauchy-Fantappiè kernel $\Omega_q(\hat{W})$ of order q generated by $\hat{W}$ is defined by

$$\Omega_q(\hat{W}) = \frac{(-1)^{q(q-1)/2}}{(2\pi i)^n} \binom{n-1}{q} \hat{W} \wedge (\bar{\partial}_{\zeta,\lambda} \hat{W})^{n-q-1} \wedge (\bar{\partial}_z \hat{W})^q,$$

for $0 \leq q \leq n - 1$. We define $K_q = \Omega_q(B)$. Then we have the following integral formula (cf. Range[26]).

Theorm 1. *For $1 \leq q \leq n$, define the linear operator*

$$T_q^W : C_{0,q}(\overline{D}) \to C_{0,q-1}(\overline{D})$$

by

$$T_q^W f = \int_{\partial D \times I} f \wedge \Omega_{q-1}(\hat{W}) - \int_D f \wedge K_{q-1}$$

and set $T_0^W = T_{n+1}^W = 0$. Then the following holds.

(a) *For $k \geq 0$, if $f \in C^k_{0,q}(D) \cap C_{0,q}(\overline{D})$, then $T_q^W f \in C^k_{0,q-1}(D)$.*

(b) *For $0 \leq q \leq n$, if $f \in C^1_{0,q}(\overline{D})$, then*

$$f = \int_{\partial D} f \wedge \Omega_q(W) + \bar{\partial} T_q^W f + T_{q+1}^W \bar{\partial} f \qquad \text{on} \qquad D.$$

Remark 1. If $W = \sum_{j=1}^n w_j(\zeta, z) d\zeta_j$ is holomorphic in z, then $\Omega_q(W) = 0$ for $q \geq 1$. In this case, if f is a $\bar{\partial}$ closed $(0, q)$-form, then it holds that $f = \bar{\partial}(T_q^W f)$.

Henkin[18] and Ramirez[25] obtained the following theorem independently.

Theorem 2. *Let $D = \{z : \rho(z) < 0\}$ be a bounded strictly pseudoconvex domain in $\boldsymbol{C}^n$ with C^4-boundary. Then there exist a pseudoconvex domain $\widetilde{D} \supset \overline{D}$ and functions $K(\zeta, z)$ and $g(\zeta, z)$ defined for $\zeta \in \partial D$ and $z \in \widetilde{D}$ such that*

(a) *$K(\zeta, z)$ and $g(\zeta, z)$ are holomorphic in $z \in \widetilde{D}$ and of class C^1 in $\zeta \in \partial D$.*

(b) *For every $\zeta \in \partial D$ the function $g(\zeta, z)$ vanishes on the closure $\overline{D}$ only at the point $z = \zeta$.*

(c) *For any function $f \in \mathcal{O}(D) \cap C^0(\overline{D})$ and any $z \in D$, the integral formula*

$$f(z) = \int_{\partial D} f(\zeta) \frac{K(\zeta, z)}{g(\zeta, z)^n} d\sigma(\zeta)$$

holds, where $\mathcal{O}(D)$ is the space of holomorphic functions in D and $d\sigma$ is the $(2n-1)$ dimensional Lebesgue measure on ∂D.

In view of the properties (a) and (b), we obtain the division

$$g(\zeta, z) = \sum_{i=1}^{n} P_i(\zeta, z)(\zeta_i - z_i),$$

where $P_i(\zeta, z)$ are of class C^1 on $\partial D \times \widetilde{D}$ and holomorphic in z.

1.2 EXTENSION OF HOLOMORPHIC FUNCTIONS FROM SUBVARIETIES

Let D be a bounded domain in $\boldsymbol{C}^n$ with smooth boundary and $\widetilde{M}$ a submanifold in a neighborhood of $\overline{D}$ which meets ∂D transversally. Define $M = \widetilde{M} \cap D$. We denote by $A^k(D)$(resp. $A^k(M)$) the space of all holomorphic functions which belong to $C^k(\overline{D})$ (resp. $C^k(\overline{M})$, $k = 0, 1, \cdots, \infty$). We set $A(D) = A^0(D), A(M) = A^0(M)$.

Henkin[19] obtained the bounded extension of holomorphic functions from submanifolds of strictly pseudoconvex domains in $\boldsymbol{C}^n$.

Theorem 3. *Suppose that D is a bounded strictly pseudoconvex domain in $\boldsymbol{C}^n$ with smooth boundary. Then there exists a linear extension operator $E : H^\infty(M) \to H^\infty(D)$ such that $Ef \in A(D)$ if $f \in A(M)$.*

Remark 2. Amar[8] proved that Theorem 3 holds in the bounded case without assuming the transversality.

Remark 3. Adachi[1] and Elgueta[14] proved that $Ef \in A^\infty(D)$ if $f \in A^\infty(M)$.

Amar[9] obtained A^∞ extension in weakly pseudoconvex domain in $\boldsymbol{C}^n$ with smooth boundary.

Theorem 4. *Let D be a bounded pseudoconvex domain in $\boldsymbol{C}^n$ with smooth boundary and M a submanifold in D which has bounded order of contact. Then any holomorphic function in $A^\infty(M)$ has a holomorphic extension in $A^\infty(D)$.*

Remark 4. Let $\Omega = \{(z,w) \in \boldsymbol{C}^2 : |z|^2 < |w|\}$, $H = \{(z,w) \in \boldsymbol{C}^2 : w = 1\}$. Define $g(z) = z$ for $(z,1) \in \Omega \cap H$. Then Tsuji[28] proved that there is no bounded holomorphic function in Ω such that the restriction $f|\Omega \cap H$ of f to $\Omega \cap H$ coinsides with g on $\Omega \cap H$. Hamada and Tsuji[16] gave the counterexample in a bounded pseudoconvex domain.

Remark 5. Mazzilli[22] proved that there exist a complex ellipsoid D of $\boldsymbol{C}^{2p+1}$ $(p \geq 1)$, a complex subvariety V of $\boldsymbol{C}^{2p+1}$ of codimension p transverse to ∂D and a bounded holomorphic function f in $V \cap D$ such that there does not exist bounded holomorphic function F in D with $F = f$ on $V \cap D$.

The integral representations of holomorphic functions on subvarieties are studied by Stout and Hatziafratis.

Let D be a bounded strictly pseudoconvex domain in $\boldsymbol{C}^n$ with smooth boundary and F a holomorphic function in a neighborhood $\widetilde{D}$ of $\overline{D}$. Define

$$\widetilde{V} = \{z \in \widetilde{D} : F(z) = 0\}, \qquad V = \widetilde{V} \cap D.$$

Then Stout[27] proved the following:

Theorem 5. *Suppose that $\widetilde{V}$ meets ∂D transversally. Let $dF \neq 0$ on ∂V. Then there are a smooth function $K : \partial V \times \overline{V} \to C$ with $K(\zeta, z)$ holomorophic on $\overline{V}$ for each $\zeta \in \partial V$ and a smooth $(2n-3)$-form $d\theta$ on ∂V such that if $f \in \mathcal{O}(\overline{V})$, then for all $z \in V$,*

$$f(z) = \int_{\partial V} f(\zeta) \frac{K(\zeta, z)}{g(\zeta, z)^{n-1}} d\theta,$$

where $g(\zeta, z)$ is the function obtained by Henkin and Ramirez.

Hatziafratis[17] extended Stout's result to pseudoconvex domains and subvarieties of arbitrary dimension.

Let D be a bounded pseudoconvex domain in $\boldsymbol{C}^n$ with smooth boundary. Let $\gamma = (\gamma_1, \cdots, \gamma_n)$: $\partial D \times D \to \boldsymbol{C}^n$ be a smooth mapping such that

$$(\zeta - z, \gamma) = \sum_{j=1}^{n} (\zeta_j - z_j)\gamma_j(\zeta, z) \neq 0 \qquad \text{on} \qquad \partial D \times D.$$

Let $h_1, \cdots, h_m (m < n)$ be holomorphic functions in a neighborhood $\widetilde{D}$ of $\overline{D}$. Define

$$\widetilde{V} = \{z \in \widetilde{D} : h_1(z) = \cdots = h_m(z) = 0\}, \qquad V = \widetilde{V} \cap D.$$

We impose the tansversal assumption that

$$d\rho \wedge \partial h_1 \wedge \cdots \wedge \partial h_m \neq 0 \qquad \text{on} \qquad \partial V.$$

Then Hatziafratis[17] proved the following:

Theorem 6. *There is a smooth form $K_V(\zeta, z)$ on $\partial V \times \overline{V}$ which is of type $(0,0)$ in z and $(n-m-1, n-m)$ in ζ such that if $f \in A(V)$, then for all $z \in V$*

$$f(z) = \int_{\zeta \in \partial V} f(\zeta) \frac{K_V(\zeta, z)}{(\zeta - z, \gamma(\zeta, z))^{n-m}}.$$

Moreover $K_V(\zeta, z)$ is holomorphic in $z \in D$ provided that $\gamma(\zeta, z)$ is holomorphic in $z \in D$.

If D is strictly pseudoconvex, then we can take $\gamma(\zeta, z) = P(\zeta, z)$, where $P(\zeta, z) = (P_1(\zeta, z), \cdots, P_n(\zeta, z))$ is the mapping defined in Theorem 2. For $f \in L^1(\partial V)$ and $z \in D$, define

$$Ef(z) = \int_{\zeta \in \partial V} f(\zeta) \frac{K_V(\zeta, z)}{g(\zeta, z)^{n-m}}.$$

Then $Ef \in \mathcal{O}(D)$. Moreover, if $f \in A(V)$, then $Ef|_V = f$.

By using the tequniques of Henkin[19] and Ahern-Schneider[7], the operator E satisfies the following(cf. Adachi[2],[3], Jakobczak[21]).

Theorem 7. *Under the above assumption it holds that*

(a) *if $f \in H^\infty(V)$, then $Ef \in H^\infty(D)$.*

(b) *if $f \in A^m(V)$, then $Ef \in A^m(D)$, where m is a non-negative integer.*

(c) *if $f \in Lip(\alpha, \partial V)$, $0 < \alpha < 1$, then $Ef \in \mathcal{O}(D) \cap Lip(\alpha, D)$.*

Remark 6. By Fornaess embedding theorem[15], Theorem 7 is still valid under the assumption that D is a bounded pseudoconvex domain with C^∞-boundary and ∂V consists of strictly pseudoconvex boundary points.

Now we consider the extendability of functions defined on ∂V to holomorphic functions in D.

For any Lipschitz function f on ∂V and $z \in \partial V$, define

$$P.V. \int_{\partial V} f(\zeta) \frac{K_V(\zeta, z)}{g(\zeta, z)^{n-m}} = \lim_{\varepsilon \to +0} \int_{\partial V \cap \{\zeta : |g(\zeta, z)| > \varepsilon\}} f(\zeta) \frac{K_V(\zeta, z)}{g(\zeta, z)^{n-m}}.$$

Using the principal value integral, Adachi and Kajimoto[6] gave the condition for a Lipschitz function on ∂V to be the boundary value of a holomorphic function in D.

Theorem 8. *Let $f \in Lip(\alpha, \partial V), 0 < \alpha$. If f satisfies for any $z \in \partial V$*

$$P.V. \int_{\partial V} f(\zeta) \frac{K_V(\zeta, z)}{g(\zeta, z)^{n-m}} = \frac{f(z)}{2},$$

Then there exists a function $F \in \mathcal{O}(D) \cap Lip(\alpha, D)$ such that $F|_{\partial V} = f$. Moreover by applying the method of Chen[12], if D is strictly convex domain with real analytic boundary and f is real analytic on ∂V, then $F \in \mathcal{O}(\overline{D})$.

Let Ω be a relatively compact open subset with smooth boundary in a complex manifold $\widetilde{\Omega}$. For $z \in \Omega$ let $\delta(z)$ denote the distance from z to the boundary of Ω with respect to the some Riemannian metric on $\widetilde{\Omega}$. Define

$$A^p_s(\Omega) = \{f \in \mathcal{O}(\Omega) : \int_\Omega |f|^p \delta^s dV < \infty\}, \qquad 0 < p \leq \infty, \qquad s \geq -1.$$

We denote by $A^p_{-1}(\Omega)$ the usual Hardy space $H^p(\Omega)$. Then Beatrous[10] obtained the following(Cumenge[13] also obtained similar results):

Theorem 9. *Let D be a bounded strictly pseudoconvex domain in $\mathbf{C}^n$ withs mooth boundary and $\widetilde{M}$ a submanifold in a neighborhood of $\overline{D}$ which meets ∂D transversally. Let $M = \widetilde{M} \cap D$. Then there is a linear operator*

$$E : A^p_{n-m+s}(M) \to A^p_s(D), \qquad s \geq -1,$$

such that $Ef|_V = f$, where $m = dim_{\mathbf{C}} M$.

Ohsawa and Takegoshi[24] and Ohsawa[23] obtaind the L^2 extension of holomorphic functions from submanifolds of bounded pseudoconvex domains.

Theorem 10. *Suppose that D is a bounded pseudoconvex domain in $\mathbf{C}^n$, ψ a plurisubharmonic function on D and $\widetilde{M}$ a pure dimensional closed complex submanifold of $\mathbf{C}^n$. Let $M = \widetilde{M} \cap D$. Then for any holomorphic function f on M satisfying $\int_M e^{-\psi}|f|^2 dV_M < \infty$, there exists a holomorphic function F on D satisfyingF$|_M = f$ and*

$$\int_D e^{-\psi}|F|^2 dV_n \leq c \int_M e^{-\psi}|f|^2 dV_M.$$

1.3 SOME RESULTS

Let $\psi \in C^2[0, 1]$ be a real function satisfying

(A) $\psi(0) = 0, \qquad$ and $\qquad \psi(1) = 1.$

(B) $\psi'(t) > 0, \qquad 0 < t \leq 1.$

(C) $(1-\psi(t))(\psi'(t)+t\psi''(t))+t^2\psi'^2(t)>0, \qquad 0<t\le 1.$

(D) there exists $\tau\in(0,1)$ such that $\psi''(t)>0, \qquad 0<t<\tau.$

(E) $\int_0^1 \log\psi(t)t^{-1/2}dt>-\infty.$

Define

$$\rho(z)=|z_1|^2+\psi(|z_2|^2)-1, \qquad D_\psi=\{z\in \boldsymbol{C}^2 : |z_1|,|z_2|<1, \rho(z)<0\},$$

so that D_ψ is a bounded pseudoconvex Reinhardt domain. D_ψ is not necessarily of finite type. Verdera[29] obtained the following inequality for ψ.

Lemma 1. *There exists a constant $\eta>0$ such that for $N=1/16$, the following inequality holds.*

$$\psi(|\zeta+v|^2)-\psi(|\zeta|^2)-2\mathrm{Re}\left(\frac{\partial\psi}{\partial\zeta}(|\zeta|^2)v\right)\ge c\psi(N|v|^2)$$

for $\zeta, v\in \boldsymbol{C}, |\zeta|<\eta, |v|<\eta$.

Verdera[29] obtained the uniform estimate for solutions of the $\bar\partial$-problem in D_ψ.

Theorem 11. *There exists a constant c such that for each $\bar\partial$ closed $f\in L^\infty_{0,1}(D_\psi)$, there exists a bounded function u on D_ψ satisfying $\bar\partial u=f$ and $||u||_\infty\le c||f||_\infty$.*

Next we study the extension problem of holomorphic functions from subvarieties to some convex domain D. The domain D is obtained by modifying the Verdera's domain D_ψ.

Let $\Psi_j\in C^\infty[0,1], j=1,\cdots,n$, be real functions satisfying

(A) $\Psi_j(0)=0, \qquad \Psi_j(1)=1;$

(B) $\psi_j'(t)>0, \qquad 0<t<1;$

(C) $2\Psi_j''(t)t+\Psi_j'(t)>0, \qquad 0<t<1;$

(D) there exists $\tau>0$ such that $\Psi_j''(t)>0, \qquad 0<t<\tau;$

(E) there exists $\lambda\ge 1$ such that

$$\int_0^1 |\log\Psi_1(t)|^\lambda t^{-1/2}dt<\infty, \qquad \int_0^1 |\log\Psi_2(t)|^\lambda dt<\infty.$$

We set

$$\rho(z)=\sum_{j=1}^n \Psi_j(|z_j|^2)-1, \qquad D=\{z : |z_j|<1, j=1\cdots,n, \rho(z)<0\}.$$

and

$$F_0(\zeta, z) = \sum_{i=1}^{n} \frac{\partial \rho}{\partial \zeta_i}(\zeta)(\zeta_i - z_i).$$

For example, define

$$\Psi_\alpha(t) = 2e^A \exp\{-At^{-\alpha}\}, \qquad A = 2 + \frac{1}{\alpha}.$$

Then Ψ_α satidfies (A),(B),(C),(D), and for $\lambda \geq 1$,

$$\int_0^1 |\log \Psi_\alpha(t)|^\lambda t^{-1/2} dt < \infty \qquad \text{if and only if} \qquad 0 < \alpha < \frac{1}{2\lambda}.$$

Let $h_1, \cdots, h_m, 1 \leq m < n$, be holomorphic functions in a neighborhood $\widetilde{D}$ of $\bar{D}$. Define

$$\widetilde{V} = \{z \in \widetilde{D} : h_1(z) = \cdots = h_m(z) = 0\}, \qquad V = \widetilde{V} \cap D.$$

We impose the assumption that

$$\partial h_1 \wedge \cdots \wedge \partial h_m \wedge \partial \rho \neq 0 \qquad \text{on} \qquad \partial V. \tag{1}$$

Then we have the following[4]:

Theorem 12. *Let V be a one dimensional subvariety of D satisfying* (1). *Suppose that $1 \leq p \leq \lambda$. Then there exists a linear extension operator $E : H^1(V) \to H^1(D)$ such that*

(a) *If $f \in H^p(V)$, then $E(f) \in H^p(D)$.*

(b) *Let V have no singular points. If $f \in \mathcal{O}(V) \cap L^p(V)$, then $E(f) \in \mathcal{O}(D) \cap L^p(D)$.*

In the case of the polydisc Henkin and Polyakov[20] obtained the bounded extension of holomorphic functions:

Theorem 13. *Let D be a unit polydisc in $\mathbf{C}^n$. Let $\sigma_I = \{\zeta \in \overline{D} : |\zeta_{i_k}| = 1, k = 1, \cdots, p\}$. Let $\widetilde{V}$ be a subvariety in a neighborhood of $\overline{D}$ and $V = \widetilde{V} \cap D$. Suppose that $\widetilde{V}$ satisfies the following conditions.*

(i) *For any point $z \in \partial V$, there exists a neighborhood $\mathcal{U}$ in $\mathbf{C}^n$ such that*

$$\widetilde{V} \cap \mathcal{U} = \{\zeta \in \mathcal{U} : g_r(\zeta) = 0, r = 1, \cdots, m\}$$

where g_r are holomorphic functions in $\mathcal{U}$.

(ii) $dg_1 \wedge \cdots \wedge dg_m \wedge d\zeta_{i_1} \wedge \cdots \wedge d\zeta_{i_p} \neq 0 \qquad$ *on* $\qquad \mathcal{U} \cap \widetilde{V} \cap \sigma_I,\ p + m \leq n.$

Then there exists a continuous linear operator $E : H^\infty(V) \to H^\infty(D)$ *such that*

$$E(f)|_V = f \qquad \textit{and} \qquad ||E(f)||_{H^\infty(D)} \leq c||f||_{H^\infty(V)}.$$

A bounded domain $\Omega \subset \boldsymbol{C}^n$ is an analytic polyhedron with defining functions ϕ_j if

$$\Omega = \{z \in \widetilde{\Omega} : |\phi_j(z)| < 1, j = 1, \cdots, N\},$$

where the defining functions ϕ_j are holomorphic in some neighborhood $\widetilde{\Omega}$ of $\overline{\Omega}$. For a multiindex $I \subset \{1, \cdots, N\}$ we let $\sigma_I = \{z \in \overline{\Omega} : |\phi_j(z)| = 1, j \in J\}$. The skeleton of Ω is the subset

$$\sigma = \bigcup_{|I|=n} \sigma_I$$

of $\partial\Omega$. We say that Ω is non-degenerate if $\partial\phi_{I_1} \wedge \cdots \wedge \partial\phi_{I_k}\ neq0$ on σ_I for every multiindex $I = \{I_1, \cdots, I_k\}$ such that $|I| = k \leq n$.

We say that the analytic polyhedron Ω is strongly non-degenerate if $\partial\phi_{I_1} \wedge \cdots \wedge \partial\phi_{I_k}\ neq0$ on σ_I for all multiindices I.

Let $\widetilde{V}$ be a regular subvariety of $\widetilde{\Omega}$ of codimension m given as

$$\widetilde{V} = \{z \in \widetilde{\Omega} : h_1(z) = \cdots = h_m(z) = 0\},$$

where $h_j \in \mathcal{O}(\widetilde{\Omega})$, and $\partial h_1 \wedge \cdots \wedge \partial h_m \neq 0$ on $\widetilde{V}$. If we impose the transversal assumption that

$$(2) \qquad \partial h_1 \wedge \cdots \wedge \partial h_m \wedge \partial\phi_{I_1} \wedge \cdots \wedge \partial\phi_{I_k} \neq 0 \qquad \text{on} \qquad \overline{V} \cap \sigma_I,$$

for every multiindex I such that $|I| = k \leq n - m$, then V is a non-degenerate analytic polyhedron on the manifold $\widetilde{V}$.

For $\epsilon > 0$ small enough, we define $\Omega_\epsilon = \{z \in \widetilde{\Omega} : |\phi_j(z)| \leq 1 - \epsilon, j = 1, \cdots, N\}$. Let σ_ϵ be the skelton of Ω_ϵ. For a strongly non-degenerate polyhedron Ω we can define the Hardy spaces

$$H^p(\Omega) = \{f \in \mathcal{O}(\Omega) : \sup_{\epsilon>0} ||f||_{L^p(\sigma_\epsilon)} < \ infty\}.$$

Let f be holomorphic in some neighborhood of $\overline{V}$. Then using Berndtsson integral formula[11], f can be extended to a holomorphic function F in Ω such that

$$F(z) = \sum_{|\alpha|=n-m} \int_{\sigma_\alpha} f(\ zeta) \frac{\omega_\alpha(\zeta, z)}{\prod_{j\in\alpha}(\phi_j(\zeta) - \phi_j(z))}, \qquad z \in \Omega,$$

where ω_α are $(n-m, 0)$-forms in $d\zeta$ which are smooth in a neighborhood of $\sigma_\alpha \times \overline{\Omega}$ and holomorphic in $z \in \Omega$ and $\sigma_\alpha = \{\zeta \in \overline{V} : |\phi_j(\zeta)| = 1, j \in \alpha\}$.