THE PRINCIPLES OF MECHANICS

by Heinrich Hertz

HEINRICH HERTZ

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PRESENTED IN A NEW FORM

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HERTZ'S PHILOSOPHY OF SCIENCE: AN INTRODUCTORY ESSAY

1. What is Philosophy of Science? Professor Braithwaite recently described Heinrich Hertz as the most philosophically profound of the great nineteenth century physicists (*Scientific Explanation*, p. 90). In the sixty years since Hertz died, philosophical clarification of physical knowledge has continued to be an expanding concern of philosophers, logicians, and physicists. Ordinarily, the plunge into experimental physics, like the practice of mathematical calculation, may leave the foundations of the subject far from view. Established notions serve as the basis for explanation until some crisis arises which cannot be evaded, and then contradictory elements in the conceptual foundations may have to be tolerated until scientific work of a logical character can be carried out. Thus, work on the foundations of physics becomes itself a branch of theoretical physics, treating in a technical way those questions which lie in the border area of physics, logic, and epistemology.

The crisis may take place in the thought of one man or in the thought of a generation, and it is not always resolved. Think of Berkeley's critique of the calculus, unanswered for two centuries, or of the scientific generation which puzzled over the spatial medium that transmits electromagnetic waves. The crisis may, moreover, arise as a puzzle over explanation itself. Here a classic instance is Newton's grappling with his own inescapable doctrine of gravitational force which acts at a distance. If we consider that forces acting at a distance are merely descriptive of factual circumstances and not further explanatory of those circumstances, we are left with so reduced a significance to the idea of force that Newton's Laws of Motion may seem to be a matter more of linguistic usage than of factual content. Yet forces, force-functions, and energies were essential to subsequent advances in physics.

For attacking these crises, philosophy, considered as logic and the analysis of science, has two principal techniques—analysis of concepts and analysis of theories.

Concept-analysis considers the meaning of terms and their definition, proposing criteria to which the terms may be subject. The criteria may be such as to relate the term to scientific experience or to everyday life. They may imbed the meaning of terms in a context of usage. Or, they may direct their attention only to fundamental or *primitive* terms within a science, criticizing the insufficiency or the impossibility of accepted scientific usage, and proposing new definitions.

Concept-analysis can proceed in general in only one of a limited number of ways:

(1) the term may arbitrarily be defined as denoting a phenomenon of the world or of the observer's experience, for example, "blue" as denoting a sensory experience;

(2) more likely, the term may be defined as an arbitrary symbol for a phenomenon which has been observed or perceived by means of apparatus or by theoretical interpretation, and hence it presumes the cognitive equipment of the interpretive and experimental technique, for example, "the satellites of Jupiter" as denoting the stellar objects telescopically observed by Galileo;

(3) via nominal definition the term may be reduced, in the fashion of a dictionary of synonyms, to serving as shorthand for other symbols, themselves of significance, for example, "average density" as synonymous with "ratio of mass in grams to volume in cubic centimeters";

(4) the term may be defined by its function in a theoretical calculus, gaining a clarity of meaning by virtue of logical relations but losing empirical reference except perhaps as derived through remote parts of the theory, for example, "point" used as a primitive term in Hilbert's formulation of Euclidean geometry.

Clearly it is more important to analyze theories than to analyze terms, for even the most denotative of terms enters into scientific usage by its role in theoretical and experimental analysis, i.e., through its role in a systematic theory or in a system of apparatus. In whatever way it may be undertaken, analysis of theories generally comes to be a rational reconstruction of an existing body of thought, formulating, in *logical* sequence, the natural laws and their consequences for the field in question.

Axiomatic reformulation, apart from responding to the immediate stimulus of logical inconsistency or experimental inadequacy, can give five-fold service:

(1) it provides a technically respectable test for philosophical doctrines in the theory of knowledge, for example, the thesis of conventionalism that Euclidean geometry is always a possible mode of formulating dynamical laws;

(2) it provides a technical means for assessing relations among concepts, such as the relations of reduction and coordination between thermal and mechanical terms in the dynamical theory of gases;

(3) it makes possible a comparison among different theories of the same phenomena, showing their identity, compatibility, or incompatibility, and, thereby, eliminating fruitless controversy which has been masked by linguistic differences, or stimulating controversy over hitherto unsuspected differences;

(4) it provides a technical way of discussing certain classic problems of philosophy by showing the needs, as set by the facts of nature, for primitive entities, basic qualities, and logical relations in our scientific theories, as well as showing those areas in which epistemological questions must remain unsolved, at least on the basis of the particular science which was axiomatized; and

(5) it uncovers and makes possible the evaluation of hidden problems, and it provides the chance of making an inventory of possible and impossible answers to problems.

Thus axiomatic formulation of a body of scientific knowledge enables us to know more exactly what we are talking about; perhaps it is best to put this negatively, by saying that axiomatic formulation reveals what we do not know but about which we are in danger of self-deception. The great strength of this deductive procedure is that the primitive terms and fundamental axioms of the system form a model-system to which the natural processes are akin. Hertz wrote of this: "We form for ourselves (internal) images or symbols of external objects; and the form which we give them is such that the necessary consequents of the images in thought are always the images of the necessary consequents in nature of the things pictured" (page 1).

Hertz thus saw that the hostile reaction of modern science against the formal character of medieval Scholastic philosophy of nature needed to be compensated by an explicit return to the understanding of formal or structural aspects of scientific knowledge. Axiomatic analysis is no substitute for empirical investigation; only rarely is it a new way of theoretical investigation, for example, in the discovery of non-Euclidean geometry. But it may be, at many stages of scientific inquiry, a necessary activity of self-criticism.

2. Hertz's Theory of Knowledge. Hertz set himself just such a task of criticism. Dissatisfied with the obscurity lurking in the force-concept, especially with the assumption of Newton's Laws that "force" denotes properties of both internal inertia and external causal influence, Hertz was equally dissatisfied with the fashionable new energy formulation of dynamics in terms of Hamilton's Principle. The undoubted insight provided by the doctrine of conservation of energy was obscured, in its turn, by uncertainty as to (a) the meaning of the concept of energy, especially of potential energy, and (b) the difference between kinetic and potential energy. Potential energy was defined in terms of masses in motion, while kinetic energy was defined in terms of gravitational, electrical, or other forces acting at a distance. Hertz constructed, as an alternative, a system which permits purely *nominal* definitions of force and energy, and in which the sole primitive terms with physical significance are mass, space, and time. Just as d'Alembert reduced dynamics to statics, so Hertz reduced dynamics to kinematics, with the necessary significance that all motions are natural, free, un-constrained. This conceptual simplicity is achieved by postulating unobserved masses which undergo motions of their own, each apparently constrained observable system being part of a larger Hertzian system of rigidly bound atomic masses which move freely as a system. Similar to that of Gauss, Hertz's law of such free motions is a minimal principle: Every free system persists in its state of rest or of uniform motion in a straightest path (p. 144).

What results is a lucid and compact theory with a minimum of physical assumptions and a sparseness of anthropomorphic language. In the course of his work, Hertz formulated criteria for axiomatic reconstructions of empirical science. He distinguished, first, "in our pictures" between accuracy of formal structure and unattainable accuracy of those theoretical models which are taken in some literal sense to be complete duplicates of reality. Second, he distinguished "between what arises from necessity in thought, what from experience, and what from our arbitrary choice" (page 8). The latter three aspects of scientific work are sharply revealed in his work; they are frequently entangled in science and in the philosophy of science. To Hertz, it was crucial that they be distinguished in physical thought. First, a theory should be logically *clear*, in the sense of free from contradiction; second, the theory must be *correct*, agreeing with the observed material motions; third, the theory should be *appropriate*, utilizing notions which are neither ambiguous nor extraneous.

In this succinct way, Hertz elucidates the contributions to the understanding of science which are offered by three traditional philosophical approaches. His first requirement is frankly *metaphysical*, nor would he preclude any other metaphysical queries for he says: "A doubt which makes an impression on our mind cannot be removed by calling it metaphysical . . . we cannot *a priori* demand from nature simplicity . . . But with regard to images of our own creation we can lay down requirements" (page 23). While he was Kantian with regard to formal necessities of thought, he was also, with Kant, ruthlessly *empirical* with regard to the coordinating relations of thoughts to facts; "that which is derived from experience can again be annulled by experience"; for Hertz the test of truth is ultimately an experimental matter (page 9). Finally, he devoted himself to *analysis* of the symbols used in scientific discourse, searching out their formal and their factual meanings, questioning the overtones of analogical meaning which other physicists may hear, repudiating those senseless questions which arise from illogical usage of the symbols rather than from legitimate puzzlement over the facts. In this third mode, linguistic analysis, he answers such questions as "what is electricity?" by noting that "we have accumulated around the terms 'force' and 'electricity' more relations than can be completely reconciled among themselves . . . by removing the contradictions, the question will not have been answered but our minds, no longer vexed, will cease to ask illegitimate questions" (pp. 7–8). What he seeks is an explication, in Carnap's sense, i.e. he has the "task of making more exact a vague or not quite exact concept used in everyday life or in an earlier stage of scientific or logical development, or rather of replacing it by a newly constructed, more exact concept . . ." (*Meaning and Necessity*, p. 7).

The isomorphism of thought and fact, or as he might prefer to phrase it, of thinking and natural process, must be carefully distinguished from isomorphism or equivalence of symbolic systems. What is the logical structure of reality? Hertz answered, speaking of electromagnetic phenomena: "To the question 'What is Maxwell's Theory?' I know of no shorter or more definite answer than the following: Maxwell's Theory is Maxwell's system of equations. Every theory which leads to the same system of equations, and therefore comprises the same possible phenomena, I would consider as being a form or special case of Maxwell's theory; every theory which leads to different equations, and therefore to different possible phenomena is a different theory" (*Electric Waves*, German ed., p. 23). It is curious how far his research anticipates later attention to the language of science. He even uses the key metaphor of the "systematic grammar" of mechanics as his goal (p. 40).

When he analyzes the word "force" and shows that it is unclear, it is not because of its hypothetical and unobservable status. He not merely retains it later as a clarified shorthand symbol or "intervening variable," but he seeks explanations for the phenomena of motion by extended and hypothetical use of his three primitive terms, mass, space, time. Bridgman recognized (with grudging approval) that Hertz posed the question of the existential meaning of hypothetical entities, i.e. of entities which are not operationally defined. To Hertz it was epistemologically legitimate to use hypothetical entities which were unobservable so long as the entities were essentially similar to observed entities; in his view, the meaning of hypothetical terms would be understandable if, apart from space-time locations and space-time differentials, they were the same as the meanings of purely empirical terms. According to these standards, the hypothetical entities of mechanics could only be masses which interact by rigid connections; there could be no forces acting at a distance. Extension of such a kinematics to a theory of the ether was implicit, and it may be argued that Hertz's mechanics is but a generalization from his work with the hidden causal sequences of electromagnetic waves (see p. 26).

As we see then, Hertz separated the question of the meaning of concepts from the question of evidence. He accepted both *a priori* and pure empirical characteristics as meaningful for science and, along with these, he presumed that existential significance is independent of observability; on the issue of evidence and confirmation, he accepted the deductive-inferential mode of empirical thought which is characteristic of a sophisticated natural science.

The position of Hertz's *Principles of Mechanics* in the history of modern physical thought has been set forth and appraised by Cassirer, Duhem, Voss, and Dugas. What may deserve brief emphasis here is the contrast of Hertz's theory of scientific knowledge with that of Mach, and of Hertz's geometrization of physical reality with the proposals of Descartes.

3. Hertz and Mach. Thermodynamics had its great classic period in the early years of the nineteenth century, arising from the study of heat phenomena but coming to be the general science of energy transformation. Its challenge to mechanics was stated by Fourier as early as 1822: ". . . a very extensive class of phenomena exists, not produced by mechanical forces, but resulting simply from the presence and accumulation of heat. This part of natural philosophy cannot be connected with dynamical theories, it has principles peculiar to itself, and is founded on a method similar to that of other exact sciences" (*The Analytical Theory of Heat*, p. 23). By the mid-century the laws of thermodynamics had, of course, been given a dynamical interpretation by Clausius, Maxwell and others, but it was realized that the mathematical equations of energy transfer and transformation did not require such interpretation. Indeed, as Carnot's research on heat engines made clear, the operation of heat devices was intrinsically independent of the working substance involved; in fact, thermodynamics needed no picture of the nature of matter at all. To those who embraced this new science, then called *energetics*, it was a matter of philosophical importance that natural phenomena might be described as varying appearances of energy. No need to penetrate into the causes of apparent motion; no need to restrict the description of the natural entities which undergo motions to material particles or rigid bodies so long as there are observed numerical correlations between the energy manifestations. Passage to an attitude hostile to hypothesis was easily taken, the nature of energy itself coming to be as much a discredited subject as the nature of matter. Robert Mayer wrote to a friend that "one single number has more real and permanent value than an expensive library of hypotheses . . . the attempt to penetrate by hypothesis to the inner recesses of the world order is of a piece with the efforts of the alchemists" (quoted by Cassirer, *The Problem of Knowledge*, p. 99).

Ernst Mach provided a philosophical account of this science of correlated observations, linking energetics with a purely sensationist view of scientific observation. For a scientific object to exist means, in Mach's view, that its symbol is the name of a set of perceptions; for it to persist as an entity, the perceptions must persist as a correlated set through the observer's flux of sensations. This phenomenalistic basis for science served several purposes: first, and most important to Mach, it provided a means whereby the various sciences of inanimate, animate and psychic nature might be unified into a general science of sensed experiences; second, it eliminated certain metaphysical (i.e. unobservable) aspects of scientific theories; third, it seemed faithful to the trend toward structural isomorphism in contemporary physics, as contrasted with previous picture-thinking; fourth, it provided an account of scientific speculation which might satisfy the current demand for evolutionary interpretation of all phenomena, human knowledge included; finally, it dissolved some old problems of philosophy, freeing experimental science from their fetters by carrying a long tradition of nominalism and empiricism to its most refined conclusion.

By so doing, Mach's phenomenalistic positivism returned the philosophy of science to an ancient position, one which natural philosophers had distrusted at least since Plato, namely to a dependence on sensuous appearances. Newton, *à propos* of the mystery of gravitational force acting at a distance, had expressed the modesty of the cognitive claim of physics when he said he had described natural processes simply and completely but disclaimed any *ultimate* explanation. Echoed by Kirchhoff, the modesty grew severely restrictive, such that the legitimate critique of sense-perception, patent in classical physics, was abandoned. In 1888, Boltzmann neatly summarized Kirchhoff's view by writing: "The aim is not to produce bold hypothesis as to the essence of matter, or to explain the movements of a body from that of molecules, but to present equations, which, free from hypothesis, are as far as possible true and quantitatively correct correspondents of the phenomenal world, careless of the essence of things and forces. In his book on Mechanics, Kirchhoff will ban all metaphysical concepts, such as force, the cause of a motion; he seeks only the equations which correspond so far as possible to observed motions" (quoted by Hoffding, ***Modern Philosophers***, p. 309).

Just as Bacon, the elder empiricist, rejected the Copernican astronomy because it violated the testimony of the senses, so the new empiricists, Mach and Ostwald, going beyond Kirchhoff, rejected the atomic theory because it, too, violated sense perception. Ostwald's victory over the physics of mechanical explanation (he called it the "conquest of scientific materialism") was expressed in a notable injunction against pictorial thinking and model making: "Thou shalt not make unto thee any graven image, or any likeness of anything . . ."! Functional relations which relate phenomena to other phenomena, together with an epistemological requirement that the phenomena concerned can only be perceived sensations, are the essentials of scientific explanation in Mach's view.

Such pure empiricism has much in common with Kant's theory of science. Scientific entities seem to both Kant and Mach to be collections of sensed perceptions; to Kant, the perceptions are put in order by a synthetic procedure due to the intrinsic conceptual techniques of the understanding, founded in logic; to Mach, perceptions are conveniently wrapped into bundles, a process rooted in largely unexplored laws of physiological psychology. But their moods differ. Evidently Kant praises the creative role of the mind while Mach laments the mental weakness which requires logical ordering of individual perceptions. Mach's view of the shorthand nature of all theory is expressed in his earliest philosophical essay: "If all the individual facts—all the individual phenomena, knowledge of which we desire—were immediately accessible to us, a science would never have arisen" (*History and Root of the Principle of the Conservation of Energy*, p. 54).

There are difficulties in these two approaches to the relativity of knowledge. The theoretical symbols, which serve so conveniently, whether logical or psychological, are not arbitrary. Furthermore, it can scarcely be denied that these symbols have some meaningful significance, referring to existing properties or relations of world entities. If we proceed beyond Kant and Mach, Hertz's statement that the meaning of Maxwell's theory is in the equations entails that we must deal with the meaning of symbols. This was, for Hertz, a quasi-empirical investigation into the philosophy of science, in the sense that his *Introduction* to the *Principles of Mechanics*—a classic work in the philosophy of science—is the result of his reflections upon his own distinguished experimental demonstrations of the wave theorems which are derivable from Maxwell's electromagnetic field equations; in that context, it seemed essential that the electromagnetic *theory* had been verified as a relational whole, not termwise, neither as single entities nor simple theorems nor isolated symbols. In the context of electromagnetic waves, it is plausible that Hertz should have had little difficulty in supposing that there are hidden masses with unobserved motions. To Mach, our basic scientific concepts are either copies or names of *actual* perceptions; to Hertz, they are systems of possible events which are linked by subsumptive logical deductions to statements about *possible* perceptions. Hertz's recognition that Maxwell's theory is open to alternative modes of formulation might apply by analogy to the primitive foundations of science. He knew that different axiomatic reconstructions might be offered.

Hertz is a neo-Kantian whose *a priori* speculations function as the fundamental axioms of an axiom system. They are either open to empirical confirmation of deduced theorems or they are, in a way he did not explain, open to non-physical explanation, perhaps in a Kantian philosophical manner (see p. 145, sec. 314). In another way, Hertz breaks cleanly with Mach for Hertz's laws of nature are less *descriptive* shorthand for experientially correlated perceptions than *prescriptive* interpretive symbolic systems in the Kantian sense. Though extraordinarily powerful when successful, scientific *concepts* are, in what seems to be Hertz's most acute and haunting phrase, "subjective illusory images of external objects" or "inner phantoms." (Here the translators have done us a disservice; they render "innere Scheinbilder oder Symbole" as "images or symbols" on p. 1. Professor Braithwaite renders it "internal pictures.")

One issue to which later philosophers devoted care and of which Hertz made only passing mention, is the existence of a "certain conformity between nature and our thought" (p. 1), the problem of induction. Yet it is of special importance in assessing Hertz's system of physics. For Hertz, symbols taken individually are hardly subject to direct sensuous confirmation and hence he made an easy transition from Maxwell's and Kelvin's individual ether mechanisms to Poincaré's doctrine of conventional and systematic reconstructions of reality by theoretical science. For example, a straight path in higher dimensional space was of no greater conceptual difficulty, nor any further from reality, than mass itself.

4. Geometry and Physical Reality. Hertz strongly defends the use of symbolic systems of every kind, picture-models and abstract-models alike, so long as the empirical truth-value is seen to reside in the formal resemblance of the behavior of symbols to the behavior of external objects. But he has a preference which is due to his demand for clarity of symbolism. What is force? What is potential energy? His reply is to reconstruct the role of force and energy in dynamics, furnishing an adequate, though not uniquely necessary, explication of their meaning in the system of dynamical equations. What are the minimum number of qualitative entities he can get along with, and still have classical dynamics? His proposal has three primitive terms and one axiom. Furthermore, his use of identical mass particles, rigidly connected, depends solely on the motions of the particles, or on their identity through space-time variation (see Meyerson's *Identity and Reality*). Particles are individuals consisting of space-time sequences of events which are linked in unspecified ways (see sec. 3, pp. 45–46). The linkages, postulated to be the simplest of rigid connections, take the place of the immense variety of forces and constraints in the usual formulation of mechanics; they are supplemented by Hertz's unobserved masses, identical in all but observability to ordinary mass-particles. The straightest path in such a system of linked atomic masses is a geodesic in a space of many dimensions; Hamilton's Principle is a geometrical axiom for such a space.

In the writings of Descartes, there is sketched just such an efficiently running world-machine, devised without forces or energies, built of rigidly connected space-time atomic entities. Although he was an advocate of the mathematization of physical reality, Descartes was driven to admit the inadequacy of ordinary space-time geometry to explain inertia and gravitation. These two properties of bodies, perhaps to be supplemented by other non-geometric properties in later investigations, seemed to require more than the pure mathematics of Euclidean geometry, but Descartes asserted that they should be analyzed by a higher geometry of many dimensions. "By dimension, I understand nothing else than the mode and aspect (*modum et rationem*) in respect of which a subject is considered to be measurable. Thus, it is not only length, breadth and depth which are dimensions; gravity is also a dimension, speed is a dimension of motion . . . and so on with innumerable other dimensions of this sort" (*Rules for the Guidance of our Mental Powers*, Rule xiv, tr. N. K. Smith). Here is a program which is explicit about velocity space and which implicitly recognizes the need for a geometry whose relational properties are representative of physical observables. A straight path in such a multi-dimensional property space would be a geodesic; and the reduction of physics to geometry would be accomplished. Even Descartes' theory of invisible vortex-centers in the ether turns upon his kinematic reinterpretation of observed forces. In this proposal, he sought to derive gravitational attraction at a distance by postulating unobserved whirling motions in the ubiquitous invisible ether. The vortical elements are defined in terms of qualities of motion whose meanings are known by observational kinematics; their existence is postulated, their arrangements and relations are also postulated; but new qualities or meanings are not invoked.

Although he makes no deliberate reference, Hertz carried out the Cartesian proposal, even to the point of rejecting discontinuities (see pp. 36–37). While Duhem seems correct in calling Hertz's axiomatic reconstruction less a doctrine than a program for establishing a doctrine (*L'Évolution de la Mécanique*, p. 167), it was nevertheless a program which succeeded in bringing physicists to the utmost critical re-appraisal of their intellectual tools. As such it not only helped to stimulate the epistemological advances of Poincaré, the axiomatic researches of recent years, such as the work of Hamel and of McKinsey and Suppes, and the linguistic analytic program of Wittgenstein and Carnap, but also it set a pattern for the *n*-dimensional geometrization of reality in Einstein's theory of relativity and in the aptly named kinematic relativity of E. A. Milne.

5. Supplementary Biographical Note. In his introductory preface to this book, Helmholtz has written a notable tribute to his illustrious student. It is melancholy in mood, for Hertz, dead at 37, had been at the beginning of his career. Like Carnot, Clifford, Fresnel, and Galois, Hertz succeeded in winning important advances within a few years, only to die with a host of ideas and programs conceived but unborn. Hertz's youth was one of contented achievement. He was a bright pupil, a linguist, a hobbyist in the woodworking shop, and a devoted reader of literature and philosophy. He read Plato and Darwin and David Strauss, economics, history of mathematics, and the corpus of physics. He would recite Homer or the Greek tragedies freely, and turned seriously to Arabic and Sanskrit for a while. His modest interest in an engineering or architectural career was turned, by the force of intrinsic interest, to pure science; at 21 he went to Helmholtz and Kirchhoff at Berlin, there to become a prize-winning investigator. At 28 he went to teach at the technological institute at Karlsruhe. His early loneliness there was dissolved by a rapid courtship; he met his wife in mid-April, they were married in July. His extraordinary series of researches on electric waves commenced forthwith!

After four happy years of scientific and married life, he moved to the University at Bonn, into Clausius' old house, a young man with international honors and an eminent chair. With little warning, however, he was afflicted with a malignant growth. In ill health after an unsuccessful operation, he was depressed, unable to pursue his experiments to a successful conclusion (with one exception, the important paper of 1892 on cathode rays, number 21 in *Miscellaneous Papers*). Harassed by illness and melancholy, he turned to the examination of physical concepts which led to the *Principles of Mechanics*. The years of despondency are typified by his diary entry for February 26, 1891: "Unerfreulich Zeit, Ermüdung, Überdruss" (An unhappy time, fatigue, disgust).

His work on the *Principles* was relaxing and comforting. He was almost the ideal scientist, an extraordinary experimenter and a master of conceptual thinking. In his work, theory and experimental practice were of mutual influence, his theoretical discussions leading to experiments and his experimental results leading to his acute theoretical discussions. His like is rare enough within science—perhaps Enrico Fermi is the most distinguished example in our own day—but his fusion of theory and experiment with a creative interest in philosophical and logical foundations is nearly unique. Only Helmholtz comes readily to mind, and one can only wonder, regretfully, how far Hertz might have gone beyond his master and friend.

6. On the Translation. There are very few errors. In Helmholtz's *Preface*, p. xiv, the word "closed" in the second line should be "unclosed." A different version of line 10 on p. 1 is noted in sec. 3 above. And finally the term "Männigfaltigkeit," which is translated as "manifold," had best be associated with "variety" and "diversity" also; see, for example, the second paragraph on p. 25.

7. Bibliography. This list consists of seven parts: (A) Hertz's works, (B) Scientific appraisals of the *Principles of Mechanics*, (C) Technical applications and extensions of Hertz's work on mechanics, (D) Philosophical and historical works on Hertz's mechanics and related matters, (E) Some important works on axiomatic foundations of classical mechanics, (F) Some examples of axiomatic analysis in other fields, (G) Biographical materials on Hertz. Within each part the items are listed alphabetically.

A. Heinrich Hertz's *Collected Works*, edited by P. Lenard.

1. *Miscellaneous Papers.* (English translation, London, 1896; original German ed. 1895.)
   The early research papers, mainly prior to his research on electromagnetic waves, and also his summary lecture of 1889 "On the relations between light and electricity" and his tribute to Helmholtz of 1891 on the latter's 70th birthday.
2. *Electric Waves.* (English translation, London, 1893; orig. German ed. 1892.)
   The entire body of experimental and interpretive papers on Maxwellian waves, with a long general introduction.
3. *The Principles of Mechanics.* (English translation, London, 1899; reprinted New York, 1955; orig. German ed. 1894.)

B. Scientific Appraisals of the *Principles of Mechanics*.

4. A. Brill, "Über die Mechanik von Hertz," *Math.-Naturwiss. Mitt. Wurtemmburg*, **2**, 1–16 (1900).
5. A. Brill, "Über ein Beispiel des Herrn Boltzmann zur Mechanik von Hertz," *Jahresber. deutsch. Math.-Verein*, **8**, 200 (1900).