



COLLOQUIA MATHEMATICA  
SOCIETATIS JÁNOS BOLYAI, 30.

**QUALITATIVE THEORY OF  
DIFFERENTIAL EQUATIONS**

**Edited by:**

**M. FARKAS**

**Vol. I.**



**NORTH-HOLLAND PUBLISHING COMPANY**  
AMSTERDAM — OXFORD — NEW YORK

© BOLYAI JÁNOS MATEMATIKAI TÁRSULAT

Budapest, Hungary, 1981



*ISBN North-Holland:* 0444 86173 4

*ISSN Bolyai:* 0139 3383

*ISBN Bolyai:* 963 8021 43 8 Vol. I-II

*ISBN Bolyai:* 963 8021 44 6 Vol. I

Joint edition published by

**JÁNOS BOLYAI MATHEMATICAL SOCIETY**

and

**NORTH-HOLLAND PUBLISHING COMPANY**

Amsterdam — Oxford — New York

In the U.S.A. and Canada:

**NORTH-HOLLAND PUBLISHING COMPANY**

52 Vanderbilt Avenue

New York, N.Y. 10017

Printed in Hungary

**ÁFÉSZ, VÁC**

Sokszorosító üzeme

## PREFACE

The Colloquium on "Qualitative Theory of Differential Equations" Szeged, August 1979 was organized by the Bolyai János Mathematical Society and the Mathematical Institute of the József Attila University of Szeged. The honorary chairman was B. Szőkefalvi-Nagy, the members, secretaries and the chairman of the organizing committee were Á. Elbert, L. Pintér, E. Vincze, P. Moson and the technical editor and the editor of the present volume. The number of participants who came from sixteen countries was 112.

The majority of the talks read have dealt with problems of stability of ordinary, functional and partial differential equations and this is characteristic to this volume too. However, there were interesting talks about oscillation theory, non linear vibrations, applications and other topics as well. Discussions centered about generalization of Ljapunov's second method including vector Ljapunov functions, and bifurcation theory. The present volume containing papers of most distinguished specialists on the one hand and first papers of young mathematicians on the other seems to reflect the state of affairs in qualitative theory

adequately.

I am extremely grateful to Mrs. C. Szabados and Miss M. Berczik for the enormous organizational work done, to Mrs. Zs. Ürmös for the excellent typing and to North Holland Publishing Co. for making the publication of this volume possible.

M. Farkas

## CONTENTS

Vol. I.

PREFACE .....	3
CONTENTS .....	5
SCIENTIFIC PROGRAM .....	13
LIST OF PARTICIPANTS .....	27
 D.D. Bainov - S.D. Milusheva, Justification of an averaging method for systems of differential equations with fast and slow variables and impulses .....	37
K. Balla, On singular boundary value problems for systems of nonlinear ordinary differential equations .....	63
Ju.N. Bibikov, The Hopf bifurcation for quasi- -periodic motions .....	71
H.G. Bothe, Generic properties of ordinary differ- ential equations of higher order .....	77
G.J. Butler, Oscillation criteria for second order nonlinear ordinary differential equations .....	93
L.A. Cherkas, On the structure of the bifurcations of the analytic autonomous systems on the	

plane with a parameter rotating the field .....	111
R. Conti, On linear autonomous controllability .....	125
Ch. Djaja, On Poisson quasi-stable and uniformly quasi-stable motions .....	141
A. Elbert, A half-linear second order differential equation .....	153
J. Elias, On the solutions of $n$ -th order differential equation in $L^2(0, \infty)$ .....	181
L. Erbe, Comparison theorems for linear differen- tial equations .....	193
P. Érdi - J. Tóth - V. Hárs, Some kinds of exotic phenomena in chemical systems .....	205
J.O.C. Ezeilo, Some properties of the differen- tial equation $d^p u/dt^p = f(u)$ of arbitrary order $p \geq 1$ .....	231
I. Foltynska - J. Werbowski, On the oscillatory behavior of solutions of systems of differential equations with deviating arguments .....	243
A. Galántai, Discrete convergence to generalized solution of Cauchy problems .....	257
B.M. Garay, Controllably periodic perturbations of autonomous functional differential equations ..	267

M. Gregus, On some asymptotic properties of the solutions of a third order linear differential equation .....	277
P.S. Gromova, On comparison methods for systems with delay .....	293
D. Gronau, Meromorphic solutions and solutions by differential operators of linear partial differential equations with constant coefficients .....	307
I. Györi, On existence of the limits of solutions of functional differential equations .....	325
V. Hárs - J. Tóth, On the inverse problem of reaction kinetics .....	363
L. Hatvani, A generalization of the Barbashin - Krasovskij theorems to the partial stability in nonautonomous systems .....	381
E. Heil, Oscillatory and non-oscillatory systems of two first order linear differential equations .....	411
J. Kalas, Asymptotic behaviour of the solutions of the equation $dz/dt=f(t,z)$ with a complex- valued function $f$ .....	431



F. Kappel, Approximation of neutral functional differential equations in the state space $R^n \times L^2$ .....	463
I.T. Kiguradze, On asymptotic behavior of solutions of nonlinear non-autonomous ordinary differential equations .....	507

Vol. II.

M. Klincsik, On the stability of solutions of differential equations in Banach space by second general exponent .....	561
M.M. Konstantinov - D.D. Bainov, Stability and approximate solution of regularly perturbed boundary value problems for differential delayed equations .....	591
P. Krbec, Parasitic and nonparasitic solutions of differential equations and stability .....	615
Zs. Lipcsey, On the synthesis of families of locally given dynamical systems .....	621
W.S. Loud, Branching phenomena for nonlinear boundary-value problems .....	641
S. Manolov, Periodic approximation in nonautonomous systems .....	677
V.M. Matrosov - R.I. Kozlov, The method of vector Ljapunov functions in the qualitative theory of differential equations in Banach spaces ....	685
J. Mawhin, The Bernstein - Nagumo problem and two-point boundary value problems for ordinary differential equations .....	709

B. Mehri, A certain boundary value problem .....	741
Ju.A. Mitropol'skij, Integral manifolds in the theory of differential equations .....	749
F. Neuman, Global theory of linear differential equations of the $n$ -th order .....	777
M. Ôtani, Existence and asymptotic stability of strong solutions of nonlinear evolution equa- tions with a difference term of subdiffer- ential .....	795
A.S. Oziraner, Some theorems on the partial stability and stabilization .....	811
T. Radzikowski - W. Sadkowski, The properties of solutions of some hyperbolic system .....	827
A.A. Rejnfeld, Dynamical equivalence of dynamical systems in a neighbourhood of a torus .....	857
V.V. Rumjancev, On stability of motions of conservative systems .....	865
T. Rzeżuchowski, On the set where all the solutions satisfy a differential inclusion .....	903
L. Salvadori, Generalized Hopf bifurcation and related stability problems .....	915
A.-M. Sändig, The classical Dirichlet-problem for elliptic differential equations of higher	

order in bounded non-smooth domains in $R^N$ .....	933
K.R. Schneider, Hopf bifurcation and center manifolds .....	953
G. Stépán, On the stability of linear differential equations with delay .....	971
H.O. Tejumola, Periodic solutions of certain non- -dissipative systems of third-order differ- ential equations .....	985
J. Terjéki, On convergence of the solutions of ordinary and functional differential equations .....	1003
G. Tóth, On the reduction of differential systems to triangular form .....	1031
J. Werbowski - A. Wyrwinska, On functional integrability of solutions of differential equations with deviating argument .....	1045
M. Yamaguchi, Almost periodic solutions of some wave equations with Dirichlet boundary values .	1061
O.A. Zhautykov, Dichotomy and asymptotic behaviour of solutions of differential equations .....	1075



## SCIENTIFIC PROGRAM

AUGUST 27, MONDAY

Morning

### Plenary session

- 9.00 - 9.30 Opening of the Colloquium
- 9.30 - 10.20 W.S. Loud: Branching phenomena for non-linear boundary value problems
- 10.30 - 11.20 F. Neuman: Global theory of linear differential equations of the  $n$ -th order

AUGUST 27, MONDAY

Afternoon

### Section A

- 14.00 - 14.15 D. Bobrowski: On the inequalities of the Opial type for random differential equations
- 14.20 - 14.35 G.J. Butler: Oscillation theory for the equation  $x''(t) + p(t)f(x(t)) = 0$
- 14.40 - 14.55 Á. Elbert: A half-linear second order differential equation

- 15.00 - 15.15 L. Erbe: Comparison theorems for linear differential equations
- 15.20 - 15.35 E. Heil: Oscillation criteria for systems of ordinary differential equations
- 15.40 - 15.55 L. Lorch: On some Stourm-Liouville functions
- 16.00 - 16.15 J. Rovder: Asymptotic and oscillatory behaviour of the equation  $y^{(iv)} + qy' + ry = 0$
- 16.20 - 16.35 J. Vosmanský: On zeros of derivatives of solutions of linear differential equations
- 16.40 - 16.55 H.G. Boethe: Generic properties of differential equations of higher order

#### Section B

- 14.20 - 14.35 D. Gronau: Meromorphic solutions and differential operator solutions of linear partial differential equations with constant coefficients
- 14.40 - 14.55 M. Chanak: Stabilität der Lösungen des elliptischen Systems partieller Gleichungen erster Ordnung mit analytischen Koeffizienten in Bezug auf die Koeffizienten

- 15.00 - 15.15 A.-M. Sändig: Boundary behaviour of weak solutions of elliptic equations of higher order
- 15.20 - 15.35 F. Juhász - R. Kersner: Diffusion equation and dynamic systems
- 16.00 - 16.15 M. Yamaguchi: Almost periodic solutions of some wave equations with Dirichlet boundary values
- 16.20 - 16.35 W. Sadkowski: Boundedness and stability of solutions of some hyperbolic equations

AUGUST 28, TUESDAY

Morning

Plenary session

- 9.00 - 9.50 I.T. Kiguradze: On asymptotic behaviour of the solutions of nonlinear nonautonomous ordinary differential equations
- 10.00 - 10.50 J. Mawhin: Boundary value problems, including periodic solutions, for second order nonlinear differential systems
- 11.00 - 11.50 L. Salvadori: Generalized Hopf bifurcation and related stability problems



AUGUST 28, TUESDAY

Afternoon

Section A

- 14.20 - 14.35 M.M. Konstantinov: Structural stability of boundary value problems for ordinary differential equations
- 14.40 - 14.55 J.S. Muldowney: Invertibility of linear differential boundary value problems
- 15.00 - 15.15 L. Hacia: On the boundary value problem for linear ordinary differential equations
- 15.20 - 15.35 G. Anger: Some inverse problems for ordinary differential equations
- 15.40 - 15.55 M. Greguš: On certain asymptotic properties of the differential equation of the third order
- 16.20 - 16.35 J.O.C. Ezeilo: Some properties of the differential equations  $d^p u/dt^p = f(u)$  of arbitrary order  $p \geq 1$  (Presented by Á. Lőkös)
- 16.40 - 16.55 J. Kalas: Asymptotic behaviour of the solutions of the equation  $\dot{z} = f(t, z)$  with a complex-valued function  $f$