GAUGE FIELD THEORIES

STEFAN POKORSKI

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Preface

This book has its origin in a long series of lectures given at the Institute for Theoretical Physics, Warsaw University. It is addressed to graduate students and to young research workers in theoretical physics who have some knowledge of quantum field theory in its canonical formulation, for instance at the level of two volumes by Bjorken & Drell (1964, 1965). The book is intended to be a relatively concise reference to some of the field theoretical tools used in contemporary research in the theory of fundamental interactions. It is a technical book and not easy reading. Physical problems are discussed only as illustrations of certain theoretical ideas and of computational methods. No attempt has been made to review systematically the present status of the theory of fundamental interactions.

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Warsaw, 1985 — Stefan Pokorski

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Introduction

1.1 Gauge invariance

It seems appropriate to begin this book by quoting the following experimental information (Review of Particle Properties 1984):

electron life-time	$> 2 \times 10^{22}$ years
neutron life-time for the electric charge nonconserving decays ($n \to p +$ neutrals)	$\gtrsim 10^{19}$ years
proton life-time	$> 10^{32}$ years
photon mass	$< 6 \times 10^{-22}$ MeV
neutrino (ν_e) mass	$< 46 \times 10^{-6}$ MeV

From the first three lines of this Table we see that the conservation of the electric charge is proved experimentally much worse than the conservation of the baryonic charge. Nevertheless, nobody is seriously contesting electric charge conservation whereas experiments searching for proton decay belong to the present frontiers in physics. The reason lies in the general conviction that the theory of electromagnetism has gauge symmetry[†] whereas no gauge invariance principle can be invoked to protect baryonic charge conservation. Exact gauge invariance protects the conservation of the electric charge. It also implies the masslessness of the photon and as seen from the Table the present experimental limit on the photon mass is indeed many orders of magnitude better than for the other 'massless' particle: the electron neutrino.

It should be stressed at this point that $U(1)$ gauge invariance implies global $U(1)$ invariance but, of course, the opposite is not true. A tiny mass for the photon would destroy the gauge invariance of electrodynamics but leave unaffected all its Earth-bound effects, including the quantum ones, as long as the electric charge

[†] The reader who is not familiar with notions of global symmetry and gauge symmetry is advised to read Section 1.3 first.

was conserved. In particular, such a theory is also renormalizable (Matthews 1949, Boulware 1970, Salam & Strathdee 1970) since longitudinally polarized photons decouple from the conserved current.

Thus one may ask what are the virtues of gauge invariance? We trust electric charge conservation, because we expect gauge invariance is behind it, and we doubt baryon charge conservation, though this has been much better proved experimentally than the former one. We do not trust global symmetries as candidates for underlying first principles! A global symmetry gives us a freedom of convention: choice of a reference frame (the phase of the electron wave function for the $U(1)$ symmetry of electrodynamics). It can be redefined freely, provided that all observers in the universe redefine it in exactly the same way. This sounds unphysical and we are led to propose that this freedom of convention is present independently at every space-time point or is not present at all as an exact law of nature. (Approximate global symmetries may, nevertheless, be and are very useful in describing the fundamental interactions.) This aesthetical argument may not convince everybody. Those who remain sceptical should then remember that gauge theories give an economical description of the laws of nature based on well-defined underlying principles which has been phenomenologically successful. As we know at present, this statement accounts not only for electrodynamics with its $U(1)$ abelian gauge symmetry, but also for weak and strong interactions successfully described by gauge field theories with non-abelian symmetry groups: $SU(2) \times U(1)$ and $SU(3)$, respectively. And non-abelian gauge symmetries are more restrictive and more profound than the $U(1)$ symmetry. In particular, non-abelian gauge bosons carry the group charges and their mass terms in the lagrangian would in general, unless introduced by spontaneous symmetry breaking, destroy not only the gauge symmetry but also the current conservation and therefore the renormalizability of the theory. The standard experimental evidence for gauge theories of weak and strong interactions is briefly summarized in the next Section.

We end this Section with a short historical 'footnote' (Pauli 1933). The terminology 'gauge invariance' can be traced back to Weyl's studies (Weyl 1919) of invariance under space-time-dependent changes of gauge (scale) in an attempt to unify gravity and electromagnetism. This attempt proved, however, unsuccessful. In 1926, Fock observed (Fock 1926) that one could base quantum electrodynamics (QED) of scalar particles on the operator

$$-i\hbar\frac{\partial}{\partial x^\mu}-\frac{e}{c}A_\mu$$

where A_μ is the electromagnetic four-potential and that the equations were invariant under the transformation

$$A_\mu \to A_\mu + \frac{\partial f(x)}{\partial x^\mu}, \qquad \Phi \to \Phi \exp\left[ief(x)/\hbar c\right]$$

which he called gradient transformation. London (1927) pointed out the similarity of Fock's to Weyl's earlier work: instead of Weyl's scale change a local phase change was considered by Fock. In 1929, Weyl studied invariance under this phase change but he kept unchanged his earlier terminology 'gauge invariance' (Weyl 1929). The concept of gauge transformations was generalized to non-abelian gauge groups by Yang & Mills (1954). Similar ideas were also proposed much earlier by Klein (1939) and by Shaw (1955).

1.2 Reasons for gauge theories of strong and electroweak interactions

We summarize very briefly the standard arguments in favour of gauge theories in elementary particle physics. Both quantum chromodynamics (QCD) and the Glashow–Salam–Weinberg theory are syntheses of our understanding of fundamental interactions progressing over many past years.

QCD

QCD emerged as a development of the Gell-Mann–Zweig quark model for hadrons (Gell-Mann 1964, Zweig 1964). The latter was postulated as a rationale for the successful $SU(3)$ classification of hadrons (today one should say flavour $SU(3)$). Assigning quarks q to the fundamental representation of $SU(3)$, not realized by any known hadrons, and giving them spin one-half one obtains the phenomenologically successful $SU(3)$ and $SU(6)$ schemes. $SU(6)$ is obtained by adjoining the group $SU(2)$ of spin rotation to the internal symmetry group $SU(3)$ for baryons (qqq) and mesons ($q\bar{q}$). In particular, the known hadrons indeed realize only those representations of $SU(3)$ which are given by the composite model. The quark model for hadrons, successful as it was, appeared, however, to have difficulties in reconciling the Fermi statistics for quarks with the most natural assumption that in the lowest-lying hadronic states all the relative angular momenta among constituent quarks vanish (s-wave states). Thus, baryon wave functions should be antisymmetric in spin and flavour degrees of freedom. This is not the case in the original quark model as can be immediately seen from inspection of the $\Delta^{++}(\frac{3}{2}^+)$ wave function which must be $u\uparrow u\uparrow u\uparrow$; u denotes the quark with electric charge $Q=\frac{2}{3}$, the arrow denotes spin $S_z=\frac{1}{2}$ for each quark.

The difficulty can be resolved by postulating a new internal quantum number for quarks which has been called colour (Greenberg 1964, Han & Nambu 1965, Nambu 1966 and Bardeen, Fritzsch & Gell-Mann 1973). If a quark of each flavour has three, otherwise indistinguishable, colour states, Fermi statistics is saved by using a totally antisymmetric colour wave function $\varepsilon_{abc}u_a\uparrow u_b\uparrow u_c\uparrow$. Assuming furthermore that (i) strong interactions are invariant under global $SU(3)_{\text{colour}}$ transformations (the states may then be classified by their $SU(3)_{\text{colour}}$ representation) (ii) physical hadrons are colourless i.e. they are singlets under $SU(3)_{\text{colour}}$ (quark confinement) we can understand why only qqq and $q\bar{q}$ states, and not qq or qqqq etc., exist in nature: the singlet representation appears only in the $3\times 3\times 3$ and $3\times\bar{3}$ products.

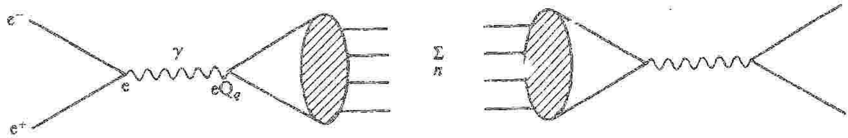

Fig. 1.1 The process $e^+e^- \to \gamma \to$ hadrons in the parton model. The sum is taken over all hadronic states in the reaction $e^+e^- \to \gamma \to \bar{q}q \to$ hadrons.

The concept of colour is supported also by at least two other, strong arguments. One is based on the parton model (Feynman 1972) approach to the reaction $e^+e^- \to$ hadrons. The total cross section for this process is then given by the diagram in Fig. 1.1 and the ratio

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \tag{1.1}$$

is predicted to be

$$R = \frac{e^2 \sum_q Q_q^2}{e^2} = \sum_q Q_q^2 = 3 \times (\tfrac{4}{9} + \tfrac{1}{9} + \tfrac{4}{9} + \tfrac{1}{9} + \tfrac{1}{9} + \cdots)$$
$$= \tfrac{11}{3} \text{ (including quarks up to b)} \tag{1.2}$$

The experimental value of R is in good agreement with this prediction and in poor agreement with the colourless prediction $\frac{11}{9}$.

Yet another reason for colour is provided by the decay $\pi^0 \to 2\gamma$. Here again the number of quark states matters in explaining the width of $\pi^0 \to 2\gamma$. This problem will be discussed in more detail in Chapter 12.

The concept of colour certainly underlies what we believe to be the true theory of strong interactions, namely QCD. However the theory also has several other basic features which are partly suggested by experimental observations and partly required by theoretical consistency. Firstly, it is assumed that strong interactions act on the colour quantum numbers and only on them. Experimentally there is no evidence for any flavour dependence of strong forces; all flavour-dependent effects can be explained by quark mass differences and the origin of the quark masses, though not satisfactorily understood yet, is expected to be outside of QCD. In addition, only colour symmetry can be assumed to be an exact symmetry (flavour symmetry is evidently broken) and this, combined with the assumption that it is a gauge symmetry (Han & Nambu 1965, Fritzsch, Gell-Mann & Leutwyler 1973), has profound implications: asymptotic freedom (Gross & Wilczek 1973, Politzer 1973) and presumably, though not proven, confinement of quarks. Both are welcome features. Asymptotic freedom means that the forces become negligible at short distances and consequently the interaction between quarks by exchange of non-abelian gauge fields (gluons) is consistent with the successful, as the first approximation, description of the deep inelastic scattering in the frame-work of the parton model. It has been shown that only non-abelian gauge theories are asymptotically free (Coleman & Gross 1973). Confinement of the colour quantum numbers, i.e. of quarks and gluons, has not yet been proved to follow from QCD but it is likely to be true, reflecting strongly singular structure of the non-abelian gauge theory in the IR region. Once we assume colourful quarks as elementary objects in hadrons, confinement of colour is desirable in view of the so far unsuccessful experimental search for free quarks and to avoid a proliferation of unwanted states.

An important line of argument in favour of gluons as vector bosons begins with approximate chiral symmetry of strong interactions (Chapter 9). Coupling of fermions with vector and axial-vector fields, but not with scalars or pseudoscalars, is chirally invariant. A theory with axial-vector gluons based on the $SU(3)$ group cannot be consistently renormalized because of anomalies (Chapter 12). Thus we arrive at vector gluon interaction.

In recent years there has been a lot of research in QCD perturbation theory neglecting the unsolved confinement problem. Of course this cannot be fully satisfactory since experimenters collide hadrons and not quarks and gluons. One can nevertheless argue that it is a justifiable approximation at short distances. Thus, at its present stage the theory provides us with calculable corrections to the free-field behaviour of quarks and gluons in the parton model and can be tested in the deep inelastic region. Given the accuracy of the calculations and of the experimental data one cannot claim yet to have strongly positive experimental verification of perturbative QCD predictions. However, experimental results are certainly consistent with QCD (in particular jet physics provides us already with good evidence for the vector nature of gluons) and in view of its elegance and self-consistency there are few sceptical of its chance of being the theory of strong interactions.

Electroweak theory

There is, at present, impressive experimental evidence for the electroweak gauge theory with the gauge symmetry spontaneously broken. To introduce the Glashow–Salam–Weinberg theory we recall first that the effective Fermi lagrangian for the charged-current weak interactions, valid at low energies, has conventionally been taken to be (for a systematic account of the weak interactions phenomenology see e.g. Gasiorowicz 1966 and more recently Abers & Lee 1973 and Taylor 1976)

$$\mathscr{L}_{\text{eff}}(x) = \sqrt{2}G_F j_\mu^\dagger(x) j^\mu(x) \tag{1.3}$$

where the Fermi β-decay constant $G_F = 1.165 \times 10^{-5}\,\text{GeV}^{-2}$ ($\hbar = c = 1$) and the charged current $j_\mu(x)$ is composed of several pieces, each with V–A structure. In terms of the lepton and quark fields it can be written as follows

$$j^\mu = \sum_i \bar{\Psi}_L^i \gamma^\mu T^- \Psi_L^i \tag{1.4}$$

with

$$T^\pm = \tfrac{1}{2}(\tau^1 \pm i\tau^2) = T^1 \pm iT^2$$

where τ^i are Pauli matrices and for weak interactions, as known in 1960s,

$$\Psi^i_L = \tfrac{1}{2}(1-\gamma_5)\left\{\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}, \begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}, \begin{pmatrix}u\\ d'\end{pmatrix}\right\} \tag{1.5}$$

The subscript L stands for the left-handed fermions. The prime superscript indicates the existence of mixing of the quark fields observed in strong interactions (mass eigenstates):

$$d' = d\cos\Theta_C + s\sin\Theta_C$$

where the angle Θ_C is known as the Cabibbo angle and has been measured in weak decays of strange particles.

We now extend the Fermi–Cabibbo theory by several additional assumptions. Firstly, we postulate the existence of a symmetry group, global for the time being, for the weak interactions. Having the current (1.4) it is natural to postulate $SU(2)$ symmetry and consequently the existence of the neutral current corresponding to the third generator of the $SU(2)$

$$[T^+, T^-] = 2T^3 \tag{1.6}$$

which would induce transitions, such as, for instance, those in Fig. 1.2 occurring with a similar strength to the charged-current reactions. Neutral-current weak transitions with the expected strength have been discovered at CERN (Hasert *et al.* 1973). However, they (i) are not of purely V–A character as expected from the $SU(2)$ model, and (ii) always conserve strangeness to a very good accuracy. According to the existing experimental limits the strangeness non-conserving neutral-current transitions (like $K^0_L \to \mu^+\mu^-$ or $K^0 \leftrightarrow \bar{K}^0$) are suppressed by many orders of magnitude as compared to the standard weak processes. Both factors call for further invention in searching for a realistic theory of weak interactions. Glashow, Iliopoulos & Maiani (1970) have discovered that the problem of the strangeness non-conserving neutral current is solved if the set of fermionic doublets Ψ^i_L is completed with a fourth one

$$\begin{pmatrix}c\\ s'\end{pmatrix}, \quad s' = -d\sin\Theta_C + s\cos\Theta_C.$$

One can immediately check that with the s' orthogonal to d' the neutral current is diagonal in flavour. Thus, they have predicted the existence of the charm quark discovered later at Slac (Aubert *et al.* 1974, Augustin *et al.* 1974). Also the doublet classification of the left-handed fermions with the equal number of lepton and quark doublets, now further confirmed by the experimental discovery of the

$$\begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix} \text{ and } \begin{pmatrix}t\\ b\end{pmatrix}$$

doublets†, has emerged as an important property of weak interactions. This has

† For the t quark the experimental situation is still unclear.

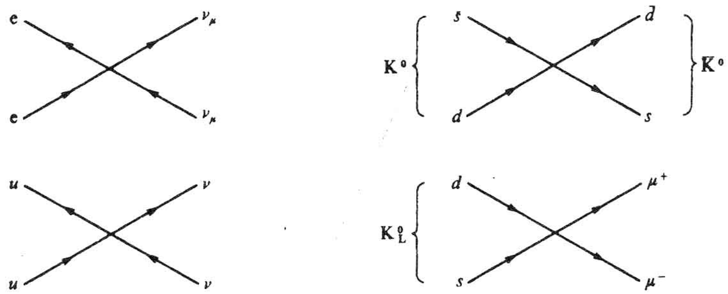

Fig. 1.2 Some neutral-current weak transitions.

profound implications for a successful extension of the effective model into a non-abelian local gauge field theory: it assures the cancellation of the chiral anomaly and consequently the renormalizability of the theory. A highly consistent scheme begins to be expected.

The next step towards the final form of the Glashow–Salam–Weinberg theory is a 'unification' of weak and electromagnetic interactions (Schwinger 1957, Glashow 1961, Salam & Ward 1964). Thus, we want the electric charge Q also to be the generator of the symmetry group of our theory. To achieve this in the most economical way we notice that for the left-handed doublets of fermions we can define a new quantum number Y (weak hypercharge)

$$\tfrac{1}{2}Y = Q - T^3 \tag{1.7}$$

such that each doublet of the left-handed fermions is an eigenvector of the operator Y (e.g. $Y_{\nu_L} = Y_{e_L} = -1$). Therefore Y commutes with the generators of $SU(2)$ and including the right-handed fermions by the prescription $\frac{1}{2}Y = Q$ (they are singlets with respects to $SU(2)$) we arrive at the $SU(2) \times U(1)$ symmetry group for the electroweak interactions. The $U(1)$ current reads

$$2j^\mu_Y = \sum_i \bar{\Psi}^i_L \gamma^\mu Y \Psi^i_L + \sum_i \bar{l}^i_R \gamma^\mu Y l^i_R + \sum_i \bar{q}^i_R \gamma^\mu Y q^i_R \tag{1.8}$$

where $l^i_R = \frac{1}{2}(1+\gamma_5)l^i$ and l^i and q^i are lepton and quark fields, respectively. According to (1.7), the electromagnetic current can be written as follows

$$\begin{aligned} j^\mu_{em} &= j^\mu_Y + \sum_i \bar{\Psi}^i_L \gamma^\mu T^3 \Psi^i_L \\ &= \sum_i \bar{l}^i Q \gamma^\mu l^i + \sum_i \bar{q}^i Q \gamma^\mu q^i \end{aligned} \tag{1.9}$$

The $SU(2) \times U(1)$ group is the minimal one which contains the electromagnetic and weak currents. With electromagnetism being described by a gauge field A_μ our minimal model 'unifying' electromagnetic and weak interactions requires the Yang–Mills gauge fields W^α_μ and B_μ to couple to the $SU(2) \times U(1)$ currents giving