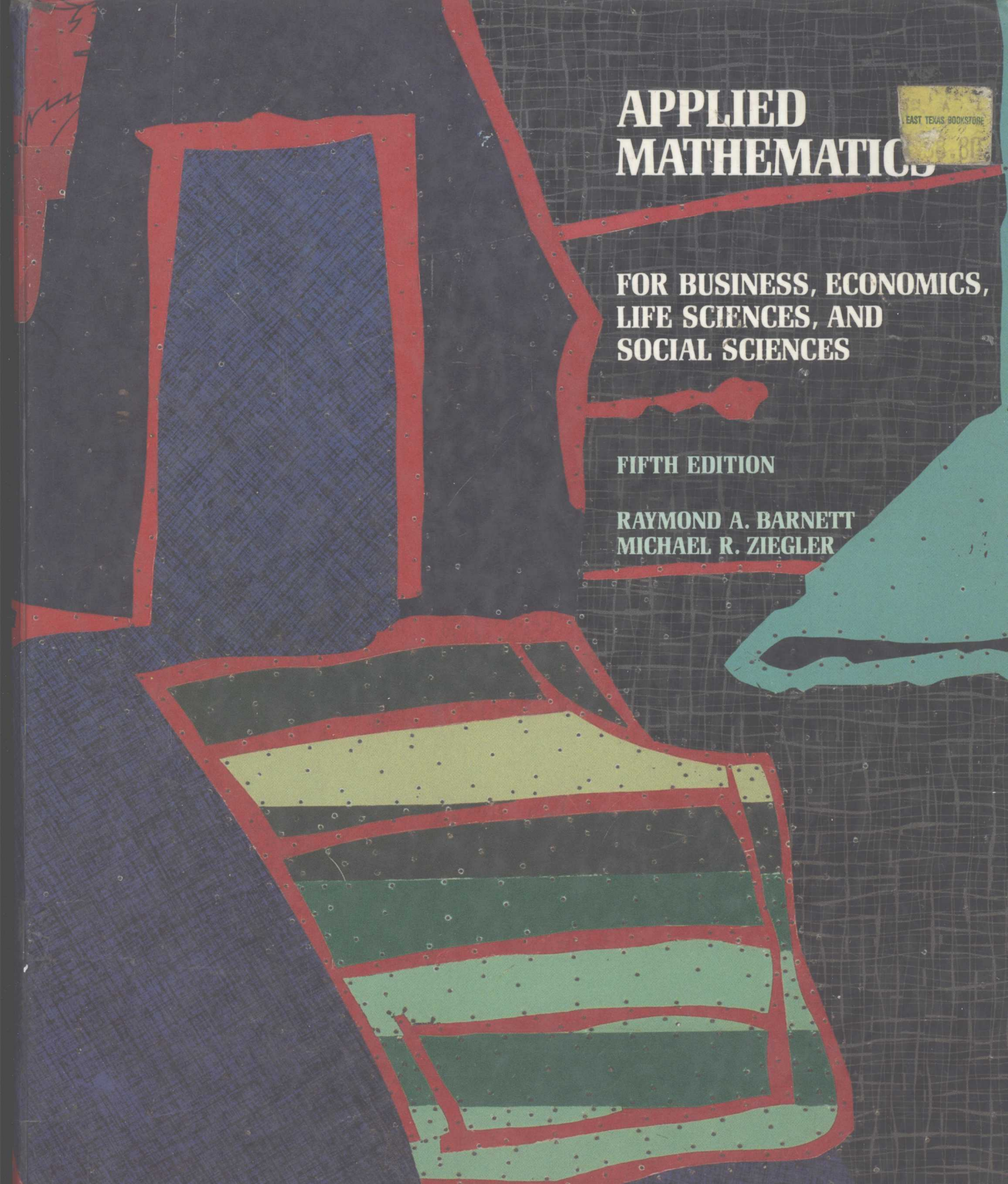

APPLIED MATHEMATICS

FOR BUSINESS, ECONOMICS, LIFE SCIENCES, AND SOCIAL SCIENCES

FIFTH EDITION

RAYMOND A. BARNETT
MICHAEL R. ZIEGLER

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Applied Mathematics

FOR BUSINESS, ECONOMICS,
LIFE SCIENCES, AND SOCIAL SCIENCES

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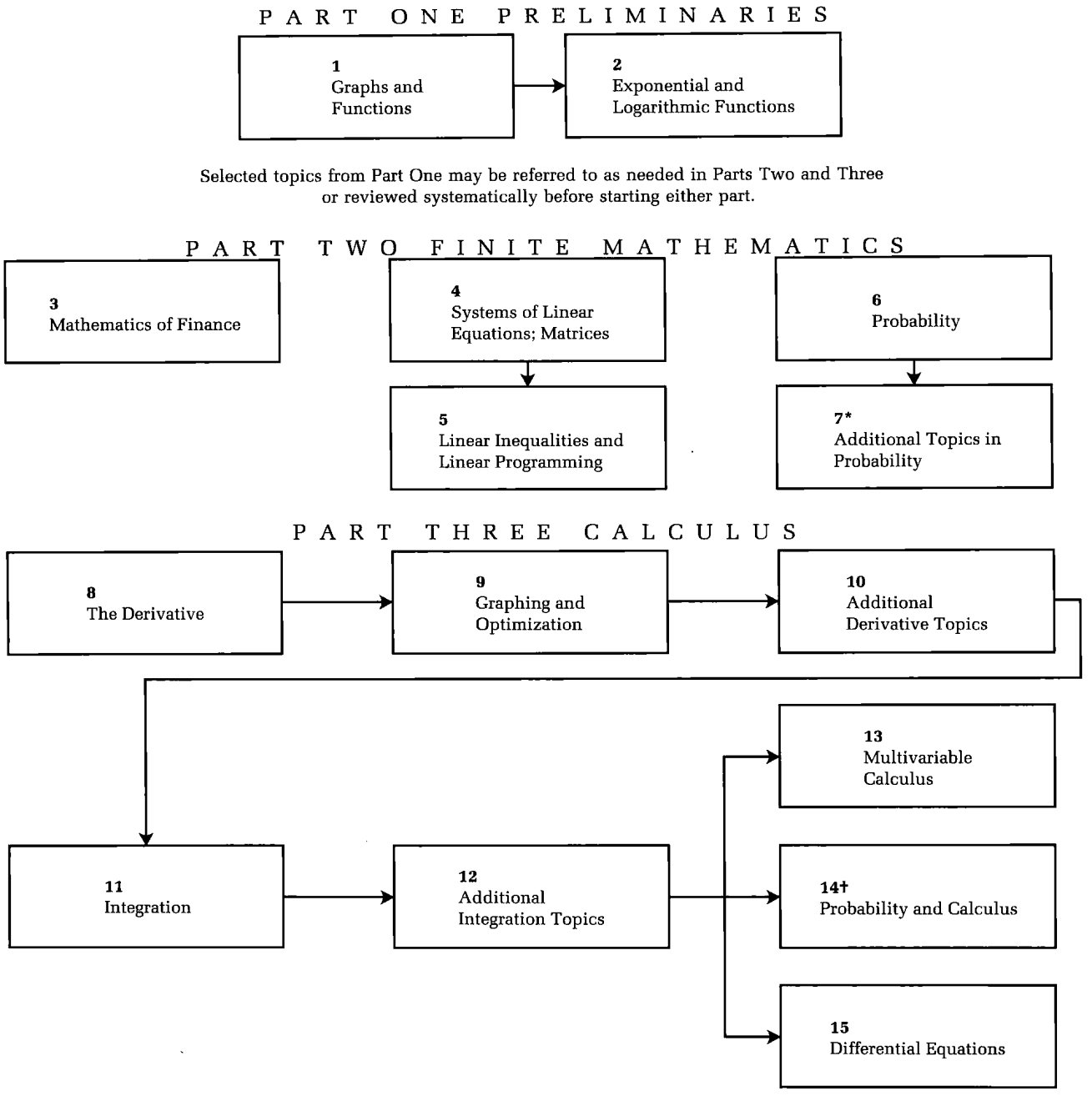

*Section 7-4 requires Sections 4-3 and 4-5.
†Chapter 6 is also a prerequisite for Chapter 14.
See the *Instructor's Resource Manual* for a more detailed discussion of topic dependencies.

PREFACE

The fifth edition of *Applied Mathematics for Business, Economics, Life Sciences, and Social Sciences* is designed for a two- or three-term course in finite mathematics and calculus and for students who have had $1\frac{1}{2}-2$ years of high school algebra or the equivalent. The choice and independence of topics make the text readily adaptable to a variety of courses (see the chapter dependency chart on page x). It is one of six books in the authors' College Mathematics series (see page ii for a brief comparison of all six books).

Improvements in this edition evolved out of the generous response from a large number of users of the last and previous editions as well as survey results from instructors, mathematics departments, course outlines, and college catalogs. Fundamental to a book's growth and effectiveness is classroom use and feedback. Now in its fifth edition, *Applied Mathematics for Business, Economics, Life Sciences, and Social Sciences* has had the benefit of a substantial amount of both.

◆ PRINCIPAL CHANGES FROM THE FIFTH EDITION

Many reviewers indicated a strong interest in using graphic calculators or computers to increase the effectiveness of their instruction and to stimulate student interest in applications of mathematics. Our response to this desire to incorporate technology into the classroom is threefold.

First, problems requiring the use of a **graphic calculator or computer** have been added to many exercise sets. These problems have been carefully selected to illustrate methods for using technology to increase student understanding of mathematical concepts and to provide an opportunity for the instructor to discuss interesting applications that are not easily solved by hand. Graphic calculator or computer problems are clearly identified with the icon shown in the margin and may be omitted without loss of continuity.

Second, a **graphics calculator manual** by Carolyn L. Meitler is now available for those who want to make more extensive use of these calculators. This manual provides a great deal of support for both the instructor and the student, including a flexible organization that permits instructors to select the portions of the course where they wish to emphasize calculator use. Detailed examples of the specific keystrokes required (on three of the most popular calculator

models) to solve problems in the text are included. While well-suited for a class where all students purchase the same calculator, this manual is especially effective where students are using a variety of different calculators—an important consideration as more and more students arrive at college having already purchased one of these calculators. See Student Aids and Instructor Aids, later in this preface, for more information about this manual.

Third, *Explorations in Finite Mathematics* and *Visual Calculus*, by David Schneider, are two new **computer software** packages available to those who want to incorporate computers into the instructional process. This extensive collection of interactive and very user-friendly programs can be used to provide additional insight into many of the mathematical concepts discussed in the text and to stimulate discussion of applications that are suited to computer solutions. See Student Aids and Instructor Aids, later in this preface, for more information about this software.

All **exercise sets** were carefully reviewed, and additional **application problems** were added. **New areas of applications** introduced in this edition include traffic flow, incidence matrices, and cryptology.

Specific improvements are as follows:

1. At the request of many reviewers, the material on **equations and inequalities** was moved to Appendix A. This permits the course to start at a higher level with **graphs and functions,** yet still provides a **complete review of pertinent algebra topics** in Appendix A for those students who need it.
2. Several of the sections in Appendix A were revised. In the section dealing with basic operations on polynomials, the FOIL method was deleted, by popular demand, and more emphasis is now placed on the use of distributive properties to accomplish multiplication. The section on **factoring polynomials** was completely rewritten. It is now less formal and has **a systematic approach to factoring** that does not depend on trial-and-error. In the section on equations and inequalities, formal set notation was generally replaced with less formal verbal statements. A new application of the quadratic formula for **factoring second-degree polynomials** was added to the section on quadratic equations.
3. In Chapter 1, the set definition of function was deleted.
4. In Chapter 2, more material on solving exponential equations and the **change-of-base formula** for logarithmic functions were added.
5. In Chapter 4, the review of methods for solving linear systems was shortened to cover just 2×2 systems in order to facilitate earlier introduction of matrix methods. At the request of many reviewers, the definition of **matrix multiplication** was stated without reference to the dot product and the section on **matrix inverses** was split into two sections. Section 4-6 now discusses **computation of inverses,** and Section 4-7 considers **matrix inverse solutions for systems of equations**
6. In Chapter 6, the section on **counting techniques** was divided into two sections. Section 6-1 contains a discussion of the **multiplication principle,**

as before, and a new treatment of the **addition principle** and **Venn diagram** counting techniques. **Permutations** and **combinations** are now covered in Section 6-2.

7. In Chapter 9, exercise sets were expanded to include more problems dealing with **fundamental graph properties** and **sign charts.**
8. In Chapter 14, the discussion of the **normal probability distribution** of a continuous random variable, as well as its use in **approximating a binomial distribution** of a discrete random variable, were expanded.
9. Chapter 15, a **new chapter on differential equations,** discusses two techniques for solving differential equations: separation of variables and integrating factors. The importance of this topic is emphasized by the **numerous applications of differential equations** included in this chapter.

◆ IMPORTANT FEATURES

Emphasis and Style

The text is **written for student comprehension.** Great care has been taken to write a book that is mathematically correct and accessible to students. Emphasis is on computational skills, ideas, and problem solving rather than mathematical theory. Most derivations and proofs are omitted except where their inclusion adds significant insight into a particular concept. General concepts and results are usually presented only after particular cases have been discussed.

Examples and Matched Problems

Over 450 completely worked examples are included. Each example is followed by a similar problem for the student to work while reading the material. This actively involves the student in the learning process. The answers to these matched problems are included at the end of each section for easy reference.

Exercise Sets

The book contains over 5,200 problems. Each exercise set is designed so that an average or below-average student will experience success and a very capable student will be challenged. Exercise sets are mostly divided into A (routine, easy mechanics), B (more difficult mechanics), and C (difficult mechanics and some theory) levels.

Applications

Enough applications are included to convince even the most skeptical student that mathematics is really useful. The majority of the applications are included at the end of exercise sets and are generally divided into business and economics, life science, and social science groupings. An instructor with students from all three disciplines can let them choose applications from their own field of interest; but if most students are from one of the three areas, then special emphasis can be placed there. Most of the applications are simplified versions of actual real-world problems taken from professional journals and books. No specialized experience is required to solve any of the applications.

◆ STUDENT AIDS

1. **Think boxes** (dashed boxes) are used to enclose steps that are usually performed mentally (see Sections 1-1 and 8-4).

2. **Annotation** of examples and developments, in color type, is found throughout the text to help students through critical stages (see Sections 2-2 and 4-3).
3. **Functional use of color** improves the clarity of many illustrations, graphs, and developments, and guides students through certain critical steps (see Sections 2-2 and 4-3).
4. **Boldface type** is used to introduce new terms and highlight important comments.
5. **Shaded boxes** are used to highlight important definitions, theorems, results, and step-by-step processes.
6. **Answers** to odd-numbered problems are included in the back of the book.
7. **Chapter review** sections include a review of all important terms and symbols and a comprehensive review exercise. Answers to all review exercises are included in the back of the book.
8. A **student's solution manual** is available at a nominal cost through a book store. The manual includes detailed solutions to all odd-numbered problems and all review exercises.
9. **Computer software** for IBM-compatible computers is available at a nominal cost through a book store. *Explorations in Finite Mathematics* and *Visual Calculus*, by David Schneider, each contain over twenty routines that provide additional insight into the topics discussed in the text. Although these software packages have much of the computing power of standard mathematical software packages, they are primarily teaching tools that focus on understanding mathematical concepts, rather than on computing. All the routines in these software packages are menu-driven and very easy to use. Included in *Explorations in Finite Mathematics* are routines for Gaussian elimination, matrix inversion, solution of linear programming problems by both the geometric and simplex methods, Markov chains, probability and statistics, and mathematics of finance. The matrix routines use and display rational numbers, and matrices may be saved and printed. The *Visual Calculus* routines incorporate graphics whenever possible to illustrate topics such as secant lines, tangent lines, velocity, optimization, the relationship between the graphs of f, f', and f'', and the various approaches to approximating definite integrals. Both software packages are accompanied by manuals with instructions and additional exercises for the student. Hardware requirements are an IBM-compatible computer with at least 640 K of memory and a graphics adapter: CGA, EGA, VGA, or Hercules.
10. A **graphics calculator manual** by Carolyn L. Meitler is available at a nominal cost through a book store. This manual contains examples illustrating the use of a graphics calculator to solve problems similar to those discussed in the text. The manual is organized in terms of the topics in the text, making it easy to find examples in the manual illustrating appropriate calculator solution methods for corresponding problems in the text.
11. A **Supplemental Applications and Topics** manual by Jon E. Baum is available at a nominal cost through a book store. Part I of the manual expands

the application exercises in the text and reinforces the important role of the mathematics presented. These exercises provide the student with a richer and more varied experience in solving real-world problems. Part II of the manual presents some applications that are not covered in the text, including transportation problems, assignment problems, sensitivity analysis, and a variety of finance topics. After completing the prerequisite material in the text, students interested in these more specialized topics will realize substantial benefits by studying this portion of the manual.

◆ INSTRUCTOR AIDS

For additional information concerning the instructor aids described below, contact:

Dellen Publishing Company
400 Pacific Avenue
San Francisco, CA 94133
(415) 433-9900

For a summary of all available supplementary materials and detailed information regarding examination copy requests and orders, see page xx.

1. A unique **computer-generated random test system** for either IBM-compatible or Apple Macintosh® computers is available to instructors without cost. The test system has been greatly expanded and now contains over 450 different problem algorithms directly related to material in the text. These carefully constructed algorithms use random number generators to produce different, yet equivalent, versions of each of these problems. The test system is available now in both **free-response and multiple-choice editions.** An almost unlimited number of quizzes, review exercises, chapter tests, midterms, and final examinations, each different from the other, can be generated quickly and easily. At the same time, the system will produce answer keys and student work sheets, if desired. In addition, the Macintosh® version incorporates a unique **editing function** that allows the instructor to alter any of the existing problems in the test and to create new problems. The editor includes a complete set of mathematical notation and also supports importation of text and graphics from other Macintosh® applications. Upon request, the publisher will supply institutions using this text with **DellenTest 4.0 (MS-DOS Free-Response or Multiple-Choice Edition)** or **DellenTest MAC 2.0 (Apple Macintosh® Free-Response or Multiple-Choice Edition)** on 3.5 inch floppy disks. **User notes** and **annotated problem printouts** are included with the disks. The notes provide step-by-step instructions for using the testing system and a complete description of the options in this menu-driven program. The annotated printouts identify by chapter and number each question the system is capable of generating, and also correlate each question with the prerequisite section from the text. When used in conjunction with the user notes, the annotated printouts enable instructors to select any combination of questions for an examina-

tion. The MS-DOS editions will produce high-quality output on IBM-compatible dot-matrix printers and on Hewlett-Packard Laserjet II®-compatible laser printers or Deskjet® printers. The Macintosh® editions require a minimum of two megabytes of RAM and System 6.0.5 or higher.

2. An **instructor's test battery** is also available to instructors without cost. The battery, organized by chapter, contains three equivalent versions (with answers) of over 450 different problems.
3. An **instructor's resource manual** provides over 200 transparency masters, a detailed discussion of chapter and topic dependencies, a comparison of this edition with the previous edition, and a detailed topic chart for comparing this book with other books in the author's College Mathematics series.
4. An **instructor's answer manual** containing all the answers not included in the text is available to instructors without charge.
5. A **student's solution manual** (see Student Aids) is available to instructors without charge from the publisher.
6. **Computer software and accompanying manuals** for *Explorations in Finite Mathematics* and *Visual Calculus*, by David Schneider (see Student Aids), are available to instructors without charge. The manuals contain complete instructions for using the software (eliminating the need to spend class time discussing these details) and examples and exercises for the student. In addition to providing students with the opportunity to use the computer as an effective tool in the learning process, instructors will find the software very useful for preparing examples for class, constructing test questions, classroom demonstrations, and similar activities.
7. A **graphics calculator manual** by Carolyn L. Meitler (see Student Aids) is available to instructors without charge from the publisher. The flexible organization of this manual allows the instructor to select the portions of the course where graphics calculator use will be emphasized. The manual contains all the necessary information for a student with no previous experience with these calculators, eliminating the need for the instructor to prepare materials related to calculator use. In particular, separate appendixes for the TI-81, TI-85, and Casio fx-7700G graphics calculators contain detailed instructions, including calculator-specific keystrokes, for performing the various operations required to effectively use each of these calculators to solve problems in the text. While well-suited for a class where all students purchase the same calculator, this manual is especially effective where students are using a variety of different calculators — an important consideration as more and more students arrive at college having already purchased one of these calculators.
8. A **Supplemental Applications and Topics** manual by Jon E. Baum (see Student Aids) is available to instructors without charge from the publisher. Instructors can use Part I of this manual to supplement the exercise sets in the text, providing students with additional experience in solving applications utilizing the mathematics presented. Part II of the manual can be used to provide coverage of applications not covered in the text, such as transpor-

tation problems, assignment problems, sensitivity analysis, and a variety of finance topics, either as part of the syllabus for a course or as subjects for independent study.

9. **Z-graph,** a HyperCard® graphing stack for the Apple Macintosh® computer, allows a user to graph most of the mathematical functions likely to be encountered quickly, accurately, and with considerable control over axes, scales, graph size, and labeling. In addition to graphing functions, this program will perform a variety of mathematical operations related to numerical integration, root approximation, interpolating polynomials, least-square polynomials, and approximate solutions of differential equations. Instructors will find this program useful for preparing examination material, transparency masters, and handouts. The publisher will supply this program free of charge to instructors using this book, and the program may be freely distributed to students.

◆ ERROR CHECK

Because of the careful checking and proofing by a number of mathematics instructors (acting independently), the authors and publisher believe this book to be substantially error-free. For any errors remaining, the authors would be grateful if they were sent to: Dellen Publishing Company, 400 Pacific Avenue, San Francisco, CA 94133.

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