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Stability Analysis and Robust Control of Time-Delay Systems

（时滞系统的鲁棒控制和稳定性分析）

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Preface

A system is said to have a delay when the rate of variation in the system state depends on past states. Such a system is called a time-delay system. Delays appear frequently in real-world engineering systems. They are often a source of instability and poor performance, and greatly increase the difficulty of stability analysis and control design. So, many researchers in the field of control theory and engineering study the robust control of time-delay systems. The study of such systems has been very active for the last 20 years; and new developments, such as fixed model transformations based on the Newton-Leibnitz formula and parameterized model transformations, are continually appearing. Although these methods are a great improvement over previous ones, they still have their limitations.

We recently devised a method called the free-weighting-matrix (FWM) approach for the stability analysis and control synthesis of various classes of time-delay systems; and we obtained a series of not so conservative delay-dependent stability criteria and controller design methods. This book is based primarily on our recent research. It focuses on the stability analysis and robust control of various time-delay systems, and includes such topics as stability analysis, stabilization, control design, and filtering. The main method employed is the FWM approach. The effectiveness of this method and its advantages over other existing ones are proven theoretically and illustrated by means of various examples. The book will give readers an overview of the latest advances in this active research area and equip them with a state-of-the-art method for studying time-delay systems.

This book is a useful reference for control theorists and mathematicians working with time-delay systems, engineering designing controllers for plants or systems with delays, and for graduate students interested in robust control theory and/or its application to time-delay systems.

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Abbreviations

inf	infimum
lim	limit
max	maximum
min	minimum
sup	supremum

BRL	bounded real lemma
CCL	cone complementarity linearization
DOF	dynamic output feedback
FWM	free weighting matrix
ICCL	improved cone complementarity linearization
IFWM	improved free weighting matrix
LFT	linear fractional transaction
LMI	linear matrix inequality
MADB	maximum allowable delay bound
MATI	maximum allowable transfer interval
NCS	networked control system
NFDE	neutral functional differential equation
NLMI	nonlinear matrix inequality
RFDE	retarded functional differential equation
SOF	static output feedback

Symbols

$\mathbb{R}$, $\mathbb{R}^n$, $\mathbb{R}^{n\times m}$	set of real numbers, set of n-dimensional real vectors, and set of $n \times m$ real matrices
$\mathbb{C}$, $\mathbb{C}^n$, $\mathbb{C}^{n\times m}$	set of complex numbers, set of n component complex vectors, and set of $n \times m$ complex matrices
$\bar{\mathbb{R}}_+$	set of non-negative real numbers
$\bar{\mathbb{Z}}_+$	set of non-negative integers
$\mathrm{Re}(s)$	real part of $s \in \mathbb{C}$
A^{T}	transpose of matrix A
A^{-1}	inverse of matrix A
$A^{-\mathrm{T}}$	shorthand for $(A^{-1})^{\mathrm{T}}$
I_n	$n \times n$ identity matrix (the subscript is omitted if no confusion will occur)
$\mathrm{diag}\{A_1, \cdots, A_n\}$	diagonal matrix with A_i as its ith diagonal element
$\begin{bmatrix} X & Y \\ * & Z \end{bmatrix}$	symmetric matrix $\begin{bmatrix} X & Y \\ Y^{\mathrm{T}} & Z \end{bmatrix}$
$A > 0$ (< 0)	symmetric positive (negative) definite matrix
$A \geqslant 0$ $(\leqslant 0)$	symmetric positive (negative) semi-definite matrix
$\det(A)$	determinant of matrix A
$\mathrm{Tr}\{A\}$	trace of matrix A
$\lambda(A)$	eigenvalue of matrix A
$\lambda_{\max}(A)$	largest eigenvalue of matrix A
$\lambda_{\min}(A)$	smallest eigenvalue of matrix A

$\sigma_{\max}(A)$	largest singular value of matrix A
$L_2[0, +\infty)$	set of square integrable functions on $[0, +\infty)$
$l_2[0, +\infty)$	set of square multipliable functions on $[0, +\infty)$
$\mathcal{C}([a, b], \mathbb{R}^n)$	family of continuous functions ϕ from $[a, b]$ to $\mathbb{R}^n$
$\mathcal{C}^b_{\mathcal{F}_0}([a, b], \mathbb{R}^n)$	family of all bounded $\mathcal{F}_0$-measurable $\mathcal{C}([a, b], \mathbb{R}^n)$-valued random variables
$L^2_{\mathcal{F}_0}([a, b], \mathbb{R}^n)$	family of all bounded $\mathcal{F}_0$-measurable $\mathcal{C}([a, b], \mathbb{R}^n)$-valued random variables $\xi = \left\{\xi(t) : \sup_{a \leqslant t \leqslant b} \mathcal{E}\|\xi(t)\|^2 < \infty\right\}$
$\lvert \cdot \rvert$	absolute value (or modulus)
$\| \cdot \|$	Euclidean norm of a vector or spectral norm of a matrix
$\| \cdot \|_\infty$	induced l_∞-norm
$\|\phi\|_c$	continuous norm $\sup_{a \leqslant t \leqslant b} \|\phi(t)\|$ for $\phi \in \mathcal{C}([a, b], \mathbb{R}^n)$
$\mathcal{L}$	weak infinitesimal of a stochastic process
$\mathcal{D}x_t$	operator that maps $\mathcal{C}([-h, 0], \mathbb{R}^n) \to \mathbb{R}^n$; that is, $\mathcal{D}x_t = x(t) - Cx(t-h)$
$\mathcal{E}$	mathematical expectation
$\left[\begin{array}{c\|c} A & B \\ \hline C & D \end{array}\right]$	shorthand for state space realization $C(sI - A)^{-1}B + D$ for a continuous-time system or $C(zI - A)^{-1}B + D$ for a discrete-time system
$\forall$	for all
$\in$	belongs to
$\exists$	there exists
$\subseteq$	is a subset of
$\cup$	union
$\rightarrow$	tends toward or is mapped into (case sensitive)
$\Rightarrow$	implies
$:=$	is defined as
$\square$	end of proof

Contents

1. Introduction

In many physical and biological phenomena, the rate of variation in the system state depends on past states. This characteristic is called a delay or a time delay, and a system with a time delay is called a time-delay system. Time-delay phenomena were first discovered in biological systems and were later found in many engineering systems, such as mechanical transmissions, fluid transmissions, metallurgical processes, and networked control systems. They are often a source of instability and poor control performance. Time-delay systems have attracted the attention of many researchers [1–3] because of their importance and widespread occurrence. Basic theories describing such systems were established in the 1950s and 1960s; they covered topics such as the existence and uniqueness of solutions to dynamic equations, stability theory for trivial solutions, etc. That work laid the foundation for the later analysis and design of time-delay systems.

The robust control of time-delay systems has been a very active field for the last 20 years and has spawned many branches, for example, stability analysis, stabilization design, H_∞ control, passive and dissipative control, reliable control, guaranteed-cost control, H_∞ filtering, Kalman filtering, and stochastic control. Regardless of the branch, stability is the foundation. So, important developments in the field of time-delay systems that explore new directions have generally been launched from a consideration of stability as the starting point. This chapter reviews methods of studying the stability of time-delay systems and points out their limitations, and then goes on to describe a new method called the free-weighting-matrix (FWM) approach.

1.1 Review of Stability Analysis for Time-Delay Systems

Stability is a very basic issue in control theory and has been extensively discussed in many monographs [4–6]. Research on the stability of time-delay systems began in the 1950s, first using frequency-domain methods and later also using time-domain methods. Frequency-domain methods determine the stability of a system from the distribution of the roots of its characteristic equation [7] or from the solutions of a complex Lyapunov matrix function equation [8]. They are suitable only for systems with constant delays. The main time-domain methods are the Lyapunov-Krasovskii functional and Razumikhin function methods [1]. They are the most common approaches to the stability analysis of time-delay systems. Since it was very difficult to construct Lyapunov-Krasovskii functionals and Lyapunov functions until the 1990s, the stability criteria obtained were generally in the form of existence conditions; and it was impossible to derive a general solution. Then, Riccati equations, linear matrix inequalities (LMIs) [9], and Matlab toolboxes came into use; and the solutions they provided were used to construct Lyapunov-Krasovskii functionals and Lyapunov functions. These time-domain methods are now very important in the stability analysis of linear systems. This section reviews methods of examining stability and their limitations.

Consider the following linear system with a delay:

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t-h), \\ x(t) = \varphi(t), \quad t \in [-h, 0], \end{cases} \tag{1.1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $h > 0$ is a delay in the state of the system, that is, it is a discrete delay; $\varphi(t)$ is the initial condition; and $A \in \mathbb{R}^{n\times n}$ and $A_d \in \mathbb{R}^{n\times n}$ are the system matrices. The future evolution of this system depends not only on its present state, but also on its history. The main methods of examining its stability can be classified into two types: frequency-domain and time-domain.

Frequency-domain methods: Frequency-domain methods provide the most sophisticated approach to analyzing the stability of a system with no delay ($h = 0$). The necessary and sufficient condition for the stability of such a system is $\lambda(A + A_d) < 0$. When $h > 0$, frequency-domain methods yield the result that system (1.1) is stable if and only if all the roots of its characteristic function,

$$f(\lambda) = \det(\lambda I - A - A_d e^{-h\lambda}) = 0, \tag{1.2}$$

have negative real parts. However, this equation is transcendental, which makes it difficult to solve. Moreover, if the system has uncertainties and a time-varying delay, the solution is even more complicated. So the use of a frequency-domain method to study time-delay systems has serious limitations.

Time-domain methods: Time-domain methods are based primarily on two famous theorems: the Lyapunov-Krasovskii stability theorem and the Razumikhin theorem. They were established in the 1950s by the Russian mathematicians Krasovskii and Razumikhin, respectively. The main idea is to obtain a sufficient condition for the stability of system (1.1) by constructing an appropriate Lyapunov-Krasovskii functional or an appropriate Lyapunov function. This idea is theoretically very important; but until the 1990s, there was no good way to implement it. Then the Matlab toolboxes appeared and made it easy to construct Lyapunov-Krasovskii functionals and Lyapunov functions, thus greatly promoting the development and application of these methods. Since then, significant results have continued to appear one after another (see [10] and references therein). Among them, two classes of sufficient conditions have received a great deal of attention. One class is independent of the length of the delay, and its members are called delay-independent conditions. The other class makes use of information on the length of the delay, and its members are called delay-dependent conditions.

The Lyapunov-Krasovskii functional candidate is generally chosen to be

$$V_1(x_t) = x^{\mathrm{T}}(t)Px(t) + \int_{t-h}^{t} x^{\mathrm{T}}(s)Qx(s)\mathrm{d}s, \tag{1.3}$$

where $P > 0$ and $Q > 0$ are to be determined and are called Lyapunov matrices; and x_t denotes the translation operator acting on the trajectory: $x_t(\theta) = x(t+\theta)$ for some (non-zero) interval $[-h, 0]$ ($\theta \in [-h, 0]$). Calculating the derivative of $V_1(x_t)$ along the solutions of system (1.1) and restricting it to less than zero yield the delay-independent stability condition of the system:

$$\begin{bmatrix} PA + A^{\mathrm{T}}P + Q & PA_d \\ * & -Q \end{bmatrix} < 0. \tag{1.4}$$

Since this inequality is linear with respect to the matrix variables P and Q, it is called an LMI. If the LMI toolbox of Matlab yields solutions to LMI (1.4) for these variables, then according to the Lyapunov-Krasovskii stability theorem, system (1.1) is asymptotically stable for all $h \geqslant 0$; and furthermore, an appropriate Lyapunov-Krasovskii functional is obtained.