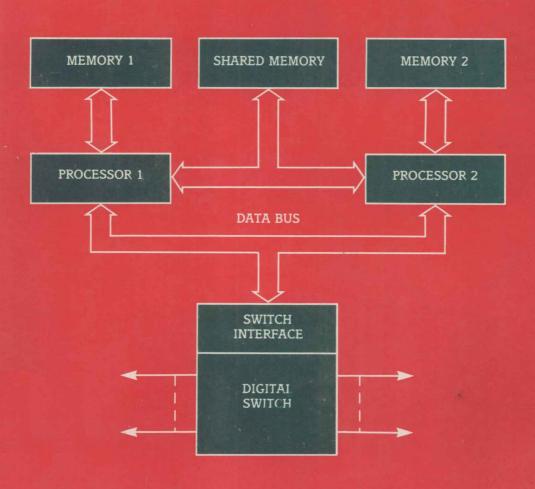
Modern Communication Systems Second Edition

R.F.W.Coates



Modern Communication Systems

R. F. W. Coates

Senior Lecturer in Electronic Systems Engineering School of Information Systems University of East Anglia

Second edition



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Preface to the First Edition

This text is intended to cover a two-year course on communication system engineering at B.Sc. level. The subject material is based upon courses which I have taught at both the University of Wales and the University of London.

Chapter 1 concentrates upon the use of Fourier methods in the analysis and processing of waveforms. It provides the mathematical groundwork upon which the rest of the book is based.

The communication channel and the problems it presents to the system designer are studied in chapter 2.

Chapters 3, 4 and 5 are concerned with the various methods of modulating a sinusoidal carrier. Much of the material presented in these chapters is standard and has been well-documented in other undergraduate texts. None the less, I have attempted to introduce modern techniques wherever possible. In particular, I have tended to stress such system implementations as lead naturally to integrated-circuit fabrications. For example, in chapter 4, the principle of frequency discrimination and the desirable features required of a practical discriminator are considered, leading to a description of the Travis discriminator. Instead of following the usual policy and covering the Foster-Seeley and ratio detectors in great detail, the available space has been devoted to introducing that quite-different frequency detector: the phase-locked loop. This system module is now available as an integrated circuit. It is cheap, requires little trimming and involves no transformers. It must, surely, become the major means of attaining frequency detection, even in domestic receivers, in the near future.

The titles of chapters 3, 4 and 5 are 'Envelope Modulation', 'Angle Modulation' and 'Composite Modulation', respectively. Single-sideband systems, being a combined envelope and phase modulation, therefore appear in chapter 5. It is common policy, in most texts, to include them with envelope (amplitude) modulation. I hope that the slightly unusual classification employed here will help the student to appreciate the nature of single-sideband waveforms more clearly.

Digital techniques are discussed in chapters 6, 7 and 8 in greater detail than has been customary in most undergraduate texts. Notwithstanding

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the small amount of time commonly devoted to these topics, they represent the major growth area in communication system installation at the present time; in particular, digital-data links are becoming an area of great importance. In this respect, I hope that the discussions of both digital coding and data transmission reflect modern practice with reasonable accuracy and provide an indication of trends in the immediate future.

In a text of this nature, space is at a premium and, if the cost of the final work is not to become excessive, some omissions are inevitable. It is with regret, for example, that I have not been able to include descriptions of differential pulse code modulation and delta modulation. The latter system, particularly, is of increasing interest to many engineers. However, it has not yet found extensive practical application in communication systems.

Let me reiterate the function of this book before defending the omission of one major topic. The objective is to teach undergraduates about communication systems. Communications is a vast field; one that is capital-intensive and makes heavy demands on available skilled manpower. The book, then, must attempt to present the principles upon which modern communication system design is based. It must also, perhaps to a lesser extent, present an outline of current practice. A conflict in aims, therefore, tends to arise when we come to consider the suitability of including such subjects as statistical decision theory and information theory. Practice makes it evident that profitable systems are not, in the main, designed as a consequence of the application of the principles which derive from these disciplines.

Thus the techniques used in the design of data modems do not, in practice, derive from decision-making strategies suggested by statistical decision theory. Nor does the use of frequency modulation result from the application of the methods of information theory. It is true that information theory, and in particular Shannon's well-known theorem

information transfer = channel bandwidth × transmission time

 $\times \log (1 + \text{signal/noise ratio})$

throws light on the efficacy of the tradeoff between bandwidth and signal-to-noise ratio evidenced by frequency-modulation systems. It is also arguable that Shannon's theorem provides us with upper bounds beyond which we cannot expect to improve the performance of communication systems; perhaps it does. Unfortunately, it is rarely the case that the postulates used in deriving the theorem are all encountered in a real system. Also, most real systems exhibit a performance level well below the Shannon bound. Finally, information theory suggests no way of designing systems so that the bound is approached. Hence its omission in favour of topics of greater practical importance to the young engineer about to seek employment.

Let me conclude by extending my appreciation to the people who have helped me during the preparation of this book, I would like to thank my colleagues, David Armitage, David Everett and Robert Newton for many helpful discussions and for reading and criticising the manuscript with such diligence. For his assistance in preparing and executing the diagrams, PREFACE Xi

particularly some of the more difficult, I would like to thank my father, Stafford Coates, Principal Lecturer, Southampton College of Technology. Finally, thanks are due to my mother, Mollie, and my wife, Gillian, for their support and encouragement during the writing of this book.

October 1974

RODNEY COATES

Preface to the Second Edition

In the short period of seven years since the publication of the first edition of this book, many significant and, in some respects unforseeable, changes in communication system technology have emerged. In particular the 'microprocessor revolution', the rapid escalation in complexity of computing microcircuitry, has provided the communication engineer with powerful, yet flexible and easily implemented processing capabilities. Taken in conjunction with fibre optic and satellite technology and the need to sustain ever-expanding computer data-bases and communication facilities, such capabilities pave the way for a second revolution: the 'information revolution' predicted for the closing decade of this century.

Respecting the pace and nature of communication system developments, I have included a new chapter: 'The Integrated Services Digital Network'. This will I hope provide an orientation for the engineer in a field of some complexity—namely, the structure and organisation of the future international telephony and data-transfer network—which, in social terms, must profoundly affect us all.

In chapters 6, 7 and 8 I have added new material emphasising the importance of digital techniques in the communications industry. Indeed, much of the material included in chapter 6 (in particular, that concerning differential pulse code and delta modulation) had to be deliberately omitted from the first edition, being at that time insufficiently important to warrant inclusion. Now, both techniques are essential material for the understanding of the integrated circuit 'CODEC'. This device (described in detail in chapter 6) will, it has been predicted, account for a major proportion of the revenue of the integrated circuit fabrication industry by the end of the present decade.

Changing social attitudes and the volume and sensitivity of commercial information transfer have persuaded me to include in chapter 7 discussions of the use of coding techniques in speech and data communication to achieve security and privacy. In chapter 8, consideration is given to advanced data keying systems and modem techniques. Finally, new material providing a resumé of the significance of fibre optic communications in modern communication networks is to be found in chapter 2.

Despite the inclusion of considerable new material, much has, inevitably, been left unsaid. It is my hope, however, that within the following pages, the engineer will find a helpful overview of this vast, expanding and vitally important field of human endeavour. Finally, I am indebted to Larry Lind for reading and correcting the manuscript and for his many useful suggestions throughout the production of this new edition.

May, 1982

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1 The Analysis and Synthesis of Waveforms

Speech is in many ways typical of communication waveforms. When converted into an electrical signal and displayed on an oscilloscope it is seen to have a continuously fluctuating waveshape which is extremely complicated. Faced with the problem of analysing and manipulating a complicated waveform, we naturally look for some 'lowest common denominator', something which allows us to classify all such signals. Then we may be able to isolate categories of signals, or processing techniques, which are of some particular benefit to us. In this first chapter, we shall examine one waveform, the sinewave or sinusoid, which is very often used as the basis for analysis and synthesis. It is not the only one which could be adopted but it is the most convenient for our purposes, since it is relatively easy to handle, both mathematically and electronically.

Following from our discussion of the sinusoid, we shall consider the problems of specifying and analysing both periodic and aperiodic waveforms. This will provide us with the essential mathematical groundwork upon which we may base our examination of communication systems and their operation.

We shall also examine the manner in which the content of a waveform may be altered by the process of linear filtering. Finally, we shall consider the determination of power- and energy-density spectra, since these properties tell us how the waveform occupies its transmission bandwidth.

1.1 The Sinusoid

In our investigations into the nature and development of communication systems, we shall repeatedly come into contact with the sinusoidal time function. There is indeed good reason why this should be so. Many natural and physical systems principally contain energy-storage and dissipation elements. Often these elements may be regarded as *lumped* or individual components of the system. When this is the case, we can write first- or

second-order differential equations describing the system. When solved, such equations exhibit natural modes of response which are exponential, or damped sinusoidal functions of time.

The sinusoid, as we shall see, is of great value both as a fundamental waveform 'type' from which other, more complicated waveforms may be built up and as a signalling waveform in its own right. We may express the sinusoid as

$$v(t) = A\cos(\omega t + \phi) \quad \text{all } t \tag{1.1}$$

This equation is a *time domain* mathematical specification of the sinusoid. The phrase 'time domain' is used because the independent variable in the equation is time, t. Notice that t is unrestricted. The condition 'all t' merely states that the sinusoid, as defined, was always 'switched on', so that no transient conditions need be considered. The sinusoid is specified by three parameters: A, the amplitude, measured typically in volts or amps; ω , the angular velocity, sometimes also referred to by communication engineers as the radian frequency, measured in radian s⁻¹ and ϕ , the phase angle, measured in radians. Note that the radian frequency is related to the period, T, of the sinusoid and to its frequency, f, measured in hertz (Hz)

$$f = \omega/2\pi$$
$$T = 1/f$$

Another important mathematical (not pictorial) representation of the sinusoid is as a rotating *phasor*. In order to facilitate our discussion of the phasor, let us first recollect de Moivre's theorem. That is

$$\exp(j\theta) = \cos(\theta) + j\sin(\theta)$$

We refer to $\exp(j\theta)$ as a *cisoid*, in the same sense that we refer to $\cos \theta$ and $\sin \theta$ as sinusoids. $\exp j\theta$ may be represented on the complex plane as a line of unit length inclined at an angle θ to the real axis; see figure 1.1a. 'cisoid' simply stands for $\cos + i$ sinusoid. The precise definition of the term 'phasor' depends on the area of application. In circuit theory, the phasor is a complex

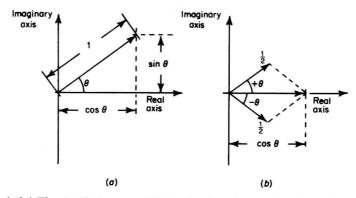


Figure 1.1 (a) The cisoid phasor $\exp(j\theta)$ depicted on the complex plane. (b) Generation of the sinusoid $\cos(\theta)$ as the sum of two cisoids

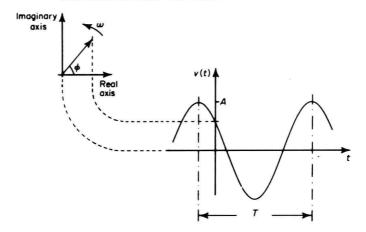


Figure 1.2 Generation of the sinusoid $v(t) = A \cos(\omega t + \phi)$ as the real part (horizontal projection) of a rotating phasor of length A

quantity, V, chosen such that

$$Re{V \exp(i\omega t)} = A \cos(\omega t + \phi)$$

It follows that

$$V = A \exp(i\phi)$$

Thus the phasor conveys the essential information regarding amplitude and phase of a fixed frequency sinusoid. Being a complex quantity, the phasor may be depicted on the complex plane as a fixed line of length A, inclined at an angle ϕ to the real axis. The term 'vector' may, in some texts be synonymous with the term 'phasor' defined as above. However, modern usage reserves the former expression for quantities defined spatially, such as electromagnetic fields.

For communication applications, the phasor is defined to be the entire quantity

$$\mathbf{V} \exp(\mathrm{j}\omega t)$$

This, when represented on the complex plane, consists of a rotating line, inclined at an angle ϕ at time t=0, the real part of which generates the sinusoid $A\cos(\omega t+\phi)$ as a function of time. Figure 1.2 illustrates this effect, which is analogous to the familiar process of generation of a sinusoid as the projection onto the horizontal of a uniformly rotating rigid member pivoted at one end.

An alternative definition of the sinusoid makes use of de Moivre's theorem, quoted above. It is easily shown that

$$\cos(\theta) = \frac{1}{2} \{ \exp(+j\theta) + \exp(-j\theta) \}$$

Figure 1.1b illustrates this complex addition, or 'phasor sum'. It follows directly from this relationship that

$$A\cos(\omega t + \phi) = \frac{1}{2}A\exp\{+j(\omega t + \phi)\} + \frac{1}{2}A\exp\{-j(\omega t + \phi)\}$$
 (1.2)

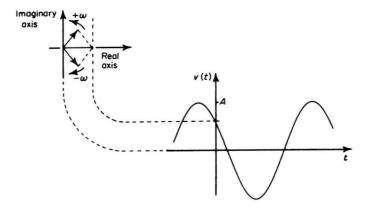


Figure 1.3 Generation of the sinusoid $v(t) = A \cos(\omega t + \phi)$ as the sum of two rotating phasors

This equation describes the phasor sum of two rotating phasors. Their respective rotations are of opposite sense $\pm \omega$ and they start at time t=0 symmetrically disposed about the real axis with initial phase angles $\pm \phi$. Consequently, the imaginary parts of the phasors cancel on phasor addition, generating the purely real sinusoid, as figure 1.3 shows.

We may extend both phasor representations of the sinusoid, figures 1.2 and 1.3, to create spectral representations which are of great value in the analysis of systems. Considering first figure 1.2, we see that the phasor itself may be defined in terms of its length, A, initial phase angle, ϕ , and angular velocity, ω . We find it convenient to depict the sinusoid graphically, plotting A and ϕ versus ω . Thus a specific sine wave

$$A_0 \cos(\omega_0 t + \phi_0)$$

for which $A = A_0$ and $\phi = \phi_0$ when $\omega = \omega_0$ is shown in the manner illustrated in figure 1.4a.

In contrast, when we examine figure 1.3, we see that we must spectrally identify two phasors. Because they rotate in opposite directions, we draw a 'two-sided' spectrum encompassing both positive and negative angular velocities. The spectral lines in this case are of length $\frac{1}{2}A_0$ and initial phase angles $\pm \phi_0$; see figure 1.4b.

'Frequency domain' diagrams such as those shown in figures 1.4a and b may be referred to as 'sinusoid-based' and 'cisoid-based' spectra, respectively. Either of these forms may occur in the literature. As might be expected, the cisoid representation tells us no more than the sinusoid representation. Indeed, it merely appears to complicate the issue. While this is a valid argument when we are concerned only with the analysis of simple waveforms, the cisoid representation offers definite advantages when we come to study complex systems and waveforms. In fact its use can make the mathematics very much easier.

Our general sinusoid

$$v(t) = A \cos(\omega t + \phi)$$