

Proceedings
of the tenth South African

**SYMPOSIUM ON
NUMERICAL MATHEMATICS**

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ON

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PREFACE

For the three days July 2-4, 1984 numerical mathematicians of South Africa as well as two leading numerical analysts from overseas gathered in Ballito, Natal, to celebrate the special event of the Tenth South African Symposium on Numerical Mathematics. This event was organised by the South African Society for Numerical Mathematics (SANUM) in conjunction with the University of Natal. A record total of 38 papers were read at this gathering. Invited papers were presented by Dr Sean McKee of the Oxford University Consortium for Industrial Numerical Analysis and by Dr GR Joubert of Philips, Eindhoven.

The Proceedings are a published record of 33 of the papers presented. The choice was left to the authors as to whether they wished to publish their papers in full or as a summary. All contributions included in the Proceedings are published as received from the authors.

This preface provides the opportunity to thank all the speakers, chairmen of sessions and participants for their contributions. Our thanks also to the University of Natal, in particular the head of the Department of Computer Science Prof Sartori-Angus and his very able assistant Ms Ethel Carte, for helping with the organisation of the Symposium.

Financial support for this meeting was again obtained from IBM (South Africa). In times of escalating costs it would not have been possible to present this Symposium without IBM's help and this support is gratefully acknowledged.

Jan Snyman
(Vice-Chairman : SANUM)

September 1984

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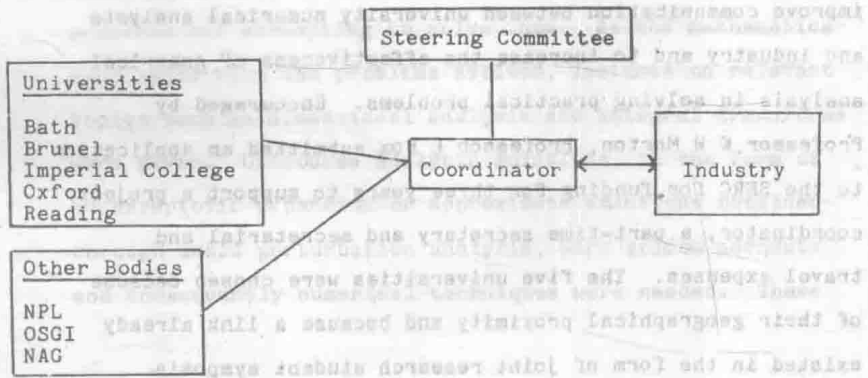
UCINA - A Vehicle for Academic-Industrial Collaboration

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1. Introduction

The University Consortium for Industrial Numerical Analysis, known by the acronym of UCINA, is an umbrella organisation consisting of numerical analysts from five universities in the south of England and those from the Division of Information Technology and Computing at the National Physical Laboratory at Teddington. The universities involved are Bath, Brunel, Oxford, Reading and Imperial College. The aims of UCINA are not only to bring to the attention of university mathematicians problems of current importance to industry but also to disseminate recent advances and new techniques to industrialists.

UCINA began 1 October 1979 and is funded by the SERC until 30 September 1984. The funding covers the salary and support costs of the coordinator. Diagrammatically the organisation of UCINA is



(NPL = National Physical Laboratory;

OSGI = Oxford Study Group with Industry;

NAG = Numerical Algorithms Group).

There is close liaison between the coordinator and Dr John Ockendon of the Oxford Study Group with Industry, built upon a good personal relationship and a free and helpful exchange of information and projects. UCINA's association with the Numerical Algorithms Group (NAG) is as yet at the informal stage. The presence of these two groups was, however, instrumental in the UCINA coordinator being based at Oxford.

The steering committee consists of senior representatives from the five universities, the coordinator and Mr Miller of NPL. The Committee sets the policy and the direction of UCINA; the coordinator is responsible for its operation.

2. Historical Background

In 1977 a report of the Numerical Analysis Panel, under the chairmanship of Professor K W Morton, (see Morton (1977)) recommended that proposals should be invited for regional groupings of universities whose objectives would be to improve communication between university numerical analysts and industry and to increase the effectiveness of numerical analysis in solving practical problems. Encouraged by Professor K W Morton, Professor L Fox submitted an application to the SERC for funding for three years to support a project coordinator, a part-time secretary and secretarial and travel expenses. The five universities were chosen because of their geographical proximity and because a link already existed in the form of joint research student symposia which took place, and still do, twice a year at one of the

five universities. Thus on 1 October 1979 the author was appointed as the coordinator. After three years the SERC extended the funding for a further two years on the condition that UCINA become self-funding thereafter. How this was done and the form it takes will be described briefly in Section 8.

However the background goes back even further than the 1977 Numerical Analysis Panel report. In November 1965, the Council of the Royal Society decided to set up an ad hoc committee to examine postgraduate training in science and technology in the United Kingdom. The report of the mathematic's sub-committee which appeared in 1968 (see Blackett (1968)), stressed the necessity for the orientation of research in applied mathematics towards relevant real problems; it also argued that graduate students and faculty members should have experience in formulating problems.

It was partly in response to this report that in 1968 Professor L Fox and Dr A Tayler (Fox and Tayler (1969)) set up an experiment in academic-industrial co-operation, which later became known as the Oxford Study Group with Industry (OSGI). Problems were solicited from industrial mathematicians, engineers and scientists and during one week at the end of Hilary term faculty members, graduate students and industrialists worked together at Oxford formulating the problems and attempting to solve them. As the mathematics associated with the problems evolved, lectures on relevant topics such as dimensional analysis and integral transforms were given. Of course analytic solutions, in the form of an asymptotic expansion or approximate solutions obtained through small perturbation analysis, were seldom adequate and consequently numerical techniques were needed. These

meetings have been held every year since and numerical analysts became more involved. It was therefore natural, when UCINA came into being, that the project coordinator should be based at Oxford.

3. Contact with industry

The first task of the coordinator was to build up contacts with industry. Dr Ockendon provided an initial mailing list which has now been extensively increased. This was done in part through Computer Survey which details all United Kingdom firms with computers and the uses to which they put them. Conference mailing lists and lists of new members of the Institute of Mathematics and its Applications were also helpful. However old students and word-of-mouth contacts have invariably proved to be the most useful. Over one hundred visits have been made to firms and Government Establishments. The purpose of these visits is to solicit problems for the two to three open meetings held generally at Oxford or Imperial College. At these meetings the industrialists present their problems while the academics offer their solutions to earlier problems. Initial formulation, so that a problem is suitable for subsequent numerical analysis, is usually done by the coordinator. The industrialist is not encouraged to believe that his problem can be solved immediately. Rather a contact is made and the coordinator arranges follow-up meetings with interested colleagues. On occasions firms or Government Establishments do not like to give a public presentation. In this case the coordinator selects the appropriate academic colleagues and a private meeting is arranged.

4. Industrially orientated M.Sc. courses

The universities of Brunel, Oxford and Reading offer M.Sc. courses in applied numerical analysis. Brunel's emphasis is on continuum mechanics while Reading tends to specialize in computational fluid dynamics. The early experience with OSGI confirmed original expectations that the solution of problems in the applied sciences and industry requires a combination, in various degrees, of skills in problem formulation, analytical treatment and numerical work and this led in 1979 to the formulation of a new M.Sc. course in mathematical modelling and numerical analysis. This vocationally orientated M.Sc. requires all students to sit two core papers, one in mathematical modelling and one in numerical analysis, in addition to two further optional papers in either one area or the other. A dissertation is also written often on some real industrial problem arising from OSGI or UCINA's contacts. These theses frequently provide an initial preliminary study prior to tendering for a major contract.

5. UCINA's educational role

UCINA's role is primarily educational in a broad sense. A large proportion of the M.Sc. and Ph.D. students from the five universities write dissertations on some application of numerical analysis to industrial problems. The problems that the coordinator obtains from industry are therefore fed back into the educational programme. The essence of some of these dissertations are given in M.Sc. lectures in subsequent years and occasionally find their way into the undergraduate course. The graduates thus produced are therefore well prepared for a life in industry and do not have to be "re-educated".

Industrial problems also provide faculty staff with interesting problems and new lines of research. If the academic is unable to tackle it himself he may, perhaps with some help from the coordinator, apply for a CASE studentship or for funding for a research assistant through one of the many SERC schemes. Occasionally industry is happy to meet the total cost of a project. In this way faculty staff are kept in touch with real problems. On the other hand because of the number of numerical analysts involved UCINA is able to offer advice on a wide range of topics. Informal consultative meetings are held frequently and it has been found that academic staff are happy to listen to the industrialist and offer him their advice freely.

Computational modelling is increasing at the expense of physical modelling and, as industry attempts to solve larger and more difficult problems, their need for efficient and robust algorithms grows. It is here that numerical analysis can contribute most and UCINA has the opportunity to promulgate new techniques such as sparse matrix methods, conjugate gradients and multigrid. Instructional courses, specially tailored to meet a firm's particular requirements are given to industry. This usually involves a team of two or three academics. They visit the firm, having read some of their internal reports, give a one hour talk or two 40 minute talks on their speciality, answer questions and during long tea, coffee and lunch breaks talk informally with the industrial participants.

6. Areas of Research

UCINA is not only involved in trouble-shooting but also in co-ordinated research programmes. One-off problems can be interesting but resources need to be replenished and so co-ordinated research programmes are necessary. Thus at Brunel the major effort is in continuum mechanics and in particular in non-linear elasticity whereas at Reading the expertise is in computational fluid dynamics and optimal control. The recent formation of the Institute of Computational Fluid Dynamics (ICFD), now shared between Oxford and Reading, has added to Oxford's research capabilities which before were scattered across most of numerical analysis with perhaps special emphasis on the numerical solution of free and moving boundary value problems.

UCINA has received a wide variety of problems. The Central Electricity Generating Board (C.E.G.B.) has provided problems on fluid flow, galvanic corrosion, non-destructive testing, ageing in steel, particles flowing in a turbulent fluid, optimal control and vibration in a gear box. Rolls Royce presented problems in turbulent fluid flow, numerical coordinate generation and eigensolution techniques. Other firms have mathematical problems in a specific area; for instance, Pilkington Brothers are almost exclusively concerned with the flow of molten glass while the Admiralty Marine Technology Establishment (A.M.T.E.) are most interested in flow past smooth bodies.

Studies associated with energy have played an important part. For example, the United Kingdom Atomic Energy Authority placed a contract through UCINA at Oxford University to look after, run and interpret the results produced by an American code, modelling the behaviour of nuclear fuel elements

under various cooling regimes. Meanwhile at Reading University Dr Nancy Nichols was, and still is, studying control strategies for power generation from tidal flow associated with the proposed Severn barrage. This work is continuing for a further three years funded by a joint SERC/CEGB grant. Natural Gas from the North Sea will eventually become exhausted and there is, consequently, an interest in considering the possibility of building large plants to extract gas from coal. The coal is burnt at the bottom of a tall cylindrical drum and a study of the slow granular flow of the coal is a necessary part of understanding the process with a view to predicting what will happen when the small pilot plant is increased ten fold. A preliminary study of numerical methods for solving Spencer's granular flow equations is underway.

UCINA is also involved with the new technologies. Numerous problems in the field of semiconductors are being considered. In particular, UCINA has a joint contract with the General Electric Company (G.E.C.), Plessey and the Standard Telecommunication Laboratories (S.T.L.) to study process modelling of semiconductor devices. Mathematically this requires the solution of coupled highly nonlinear diffusion equations on a difficult geometry with two moving boundaries. The intention is to put together a sophisticated computer programme which will help engineers to design smaller silicon chips. This work is being done mainly at Reading under the supervision of Dr Michael Baines with a little input from the coordinator. Another aspect of the new technologies is the manufacture of optical fibres. Mathematically this requires the solution of the low Reynolds number Navier-Stokes equations with a time dependent free surface. This has been successfully accomplished by Dr Joyce Aitchison and one of her students

at Oxford and has led to a better assessment of the process. Oxford Research Systems manufacture superconducting magnets which are then used in scanners in hospitals. Screening of these magnets is important since a pacemaker even two rooms away or in the floor above can be thrown out of order. The major difficulty here arises from the requirement that the metal plate, which is the screen, must necessarily be thin which in itself, causes numerical difficulties. This work is being undertaken at Imperial College by Dr Michael Bernal and his student. UCINA has an interest and expertise in the broad area of elasticity. A substantial contract with the National Gas Turbine Establishment (now part of the Royal Aircraft Establishment) to study real time simulation of the stress on turbine blades is in progress at Brunel University under the direction of Professor John Whiteman of BICOM. A problem from BICC concerned the vertical unreeling of a wire from a coil on the ground as fast as possible without snagging. Although this can be modelling as a two point boundary value problem, the boundary conditions are not clear and all the standard NAG routines break down. This was the starting point that led to considerable analysis and the development of further models. This problem is not yet fully resolved. Stress in timber is another important area and UCINA acts as a consultant to the Timber Research and Development Association. Optimisation is also a field in which UCINA is involved. The problem of optimal tool packing was tackled. This concerned the best way to pack a given consignment into boxes of a given size subject to certain constraints. While simple integer programming models were derived, the real problem could only be solved in a number of steps which grew exponentially with the number of tools. As is well known