SOLID MECHANICS AND ITS APPLICATIONS

Dominique François, André Pineau
and André Zaoui

Mechanical Behaviour of Materials

Volume II: Viscoplasticity, Damage, Fracture and Contact Mechanics

KLUWER ACADEMIC PUBLISHERS

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Aims and Scope of the Series

The fundamental questions arising in mechanics are: *Why?*, *How?*, and *How much?* The aim of this series is to provide lucid accounts written by authoritative researchers giving vision and insight in answering these questions on the subject of mechanics as it relates to solids.

The scope of the series covers the entire spectrum of solid mechanics. Thus it includes the foundation of mechanics; variational formulations; computational mechanics; statics, kinematics and dynamics of rigid and elastic bodies; vibrations of solids and structures; dynamical systems and chaos; the theories of elasticity, plasticity and viscoelasticity; composite materials; rods, beams, shells and membranes; structural control and stability; soils, rocks and geomechanics; fracture; tribology; experimental mechanics; biomechanics and machine design.

The median level of presentation is the first year graduate student. Some texts are monographs defining the current state of the field; others are accessible to final year undergraduates; but essentially the emphasis is on readability and clarity.

For a list of related mechanics titles, see final pages.

MECHANICAL BEHAVIOUR OF MATERIALS

FOREWORD

Man discovered a long time ago that quenching steel would increase its hardness; the reason for this was found much more recently, and the understanding, together with the finding of ways of exploiting the interactions between martensitic transformations and dislocations, precipitates and texture, has led to the development of new steels, new heat treatments and new alloys with unusual properties. The art of metallurgy had provided many recipes rooted in empiricism: the introduction of scientific thinking has made it possible to improve these, and the science of metals thus founded has opened the way to the wider subject of materials science. In the same way the practical problems of construction have led to the growth of solid mechanics as a branch of applied mathematics. For a long time the constitutive equations needed in materials science remained crude idealisations of actual behaviour. The pioneers in this field could correspond equally well with their peers about metallurgy (or alchemy) as about mechanics (or astrology); later, scientists have become more and more specialised, and there is now little overlap between materials science and solid mechanics.

As technical equipment of ever greater sophistication has become available, the risk of catastrophes, of a scale that can affect the environment and kill many people, has increased; and safety has become a major concern. Economic considerations press for longer lifetimes and smaller safety factors; these generate strong incentives to use more realistic constitutive equations and better failure criteria in the calculations, and the computer now makes this possible. Materials design has become much more of a practical possibility, and materials can be produced with better and more reliable properties.

All this shows that establishing relations, as quantitative as possible, between the microstructure of materials and their macroscopic properties is nowadays essential. Thanks to fruitful cooperation between materials scientists and solid-mechanics specialists, recent research has led to promising achievements in this direction; but the number of training programs which cover both fields has remained low. It was the awareness of the need for advanced courses here that led us, some ten years ago, to create in France what is called a Diplome d'Études Approfondies (DEA) - Advanced Studies Diploma - with the title Mécanique et Matériaux - Mechanics and Materials; and the present books stem from the notes provided for the courses. The need was probably greater in France than in English-speaking countries, where the famous book of McClintock and Argon, Mechanical Properties of Materials, was already much in use. This, however, was published in 1966 and so did not deal with recent developments; and this gave us the incentive to embark on these books, even though we felt that we could not match McClintock and Argon.

The organisation of the two volumes follows the main classes of mechanical behaviour: the first deals with elastic and plastic deformations and the second with viscoplastic, followed by treatments of damage mechanisms, fracture mechanics and contact mechanics. Throughout we attempt to describe the physical processes that are responsible for the kinds of behaviour studied, the way in which the constitutive equations can represent the behaviour and how they relate to the microstructures. We follow each chapter with a set of exercises, to which we give either the solutions or hints on how these are to be obtained. Understanding the subject matter requires a good knowledge of solid mechanics and materials science; we give the main elements of these fields in a set of Annexes at the end of the first volumes.

Whilst the books are addressed primarily to graduate students, they could possibly be used in undergraduate courses; and we hope that practising engineers and scientists will find the information they convey useful. We hope also that English-speaking readers will be interested in the aspects of French culture which our treatment will undoubtedly display.

The authors are very grateful to all their colleagues, in particular those who teach in the DEA Mécanique et Matériaux, for their contributions and encouragement; and wish to thank all those people who have provided photographs to illustrate the book. We also thank Professor Gladwell of the University of Waterloo, Canada, for his final proofreading. The English translation was done by Dr. Jack Howlett, whose frequent questions and suggestions have helped to improve many paragraphs significantly. We have found cooperation with him very stimulating and we thank him for his excellent work.

MECHANICAL BEHAVIOUR OF MATERIALS

VOLUME II: VISCOPLASTICITY, DAMAGE, FRACTURE and CONTACT MECHANICS

CONTENTS

MECHANICAL BEHAVIOUR OF MATERIALS

CHAPTER 1 ELASTOVISCOPLASTICITY

1.1 INTRODUCTION

The type of behaviour to be discussed in this chapter has in common with elastoplasticity the fact that a permanent deformation remains after the load has been removed. On the other hand, viscosity now prevents the occurrence of instantaneous plastic deformations: time controls the inelastic deformation and we have what is called *rate-dependent plasticity*. Apart from this, viscoplastic behaviour is very similar to ordinary plasticity, already treated in Chapter 3 of Volume 1; but it is distinguished from this by the separation of the deformation into an elastic part and an inelastic viscoplastic part, say $\varepsilon = \varepsilon^e + \varepsilon^V$, with, in general, absence of reversibility, even delayed, for the inelastic part.

Viscoplastic behaviour occurs especially in metals and alloys when at high temperatures, not less than one-third of the melting point. It is then accompanied by diffusion phenomena with different characteristic distances: those related to dislocations are seen in the *creep* which results from the competition between work-hardening and recovery, or at greater distances those related to grain boundaries in *diffusional creep*. Low-temperature viscoplasticity also is possible, related to thermal activation of the plastic deformation. We define these various terms at the appropriate points later in the chapter. As well as to metals, viscoplasticity theory applies to resins and polymers when these are loaded beyond the point at which their behaviour can continue to be viscoelastic, and also, in studies of the evolution of mechanical behaviour over very long periods, to soils, rocks and ice.

As in the study of plasticity, we make the simplifying assumptions of linear elasticity, small deformations and absence of damage. This chapter gives:

- a description of the tests that enable the basic features of viscoplastic behaviour and the associated phenomena to be seen,

- an account of the physical models of viscoplasticity (thermal activation, creep). Particular attention is paid to the field of validity of each equation derived, so that the limits of the models can be predicted - for example, when results are to be extrapolated to longer times or higher temperatures,

- an account of phenomenological mechanical models of viscoplasticity, bringing out the physical significance of each term introduced; here we discuss the theories of plasticity and viscoplasticity in combination, and also, for reasons of computational convenience, the use of viscoplasticity rather than time-independent plasticity,

- a discussion of methods for reinforcing against creep, showing how, once the deformation mechanisms have been understood, means for guarding against excessive deformation and rupture can be found.

Only viscoplasticity is considered here, damage being left to the next chapter.

1.2 TYPICAL EXPERIMENTAL RESULTS

The basic features of viscoplasticity appear under steady loading, with either the force or the deformation held constant after the load is applied - corresponding to creep and relaxation experiments respectively. The importance of such tests is their simplicity, but in general they do not show the complete behaviour since either the stress or the plastic deformation remains practically constant each time. We must therefore consider more elaborate tests, with changes made incrementally and showing changes in the strain rate, or with the load increasing in steps. One of the most important requirements for a good characterisation of the mechanism is the separation of the effects of deformation from those of time on the state of the material.

1.2.1 One-dimensional response

1.2.1.1 Creep

Creep tests are carried out on cylindrical or flat test pieces under constant load; the time for a test can be anything from a few hours to several years, according to the application. The results are given as a time-deformation curve (Fig. 1.1). To the extent that the load is usually applied as a dead weight the stress is not constant throughout the test, but this is usually ignored and the test characterised by the initial value of the axial stress, $\sigma_0 = F/S_0$, where F is the applied force and S_0 the initial cross-section of the test piece. There are three main stages in the response to the load, more or less clearly distinguished according to the material and the temperature:

primary, during which the strain rate falls; this corresponds to an increase in the resistance of the material,

secondary, in which the strain rate remains constant,

tertiary, when the rate increases; significant mechanical damage appears in this stage, related to cavitation for example, or to a softening of the material, induced by localisation of the strain on the scale of the microstructure.

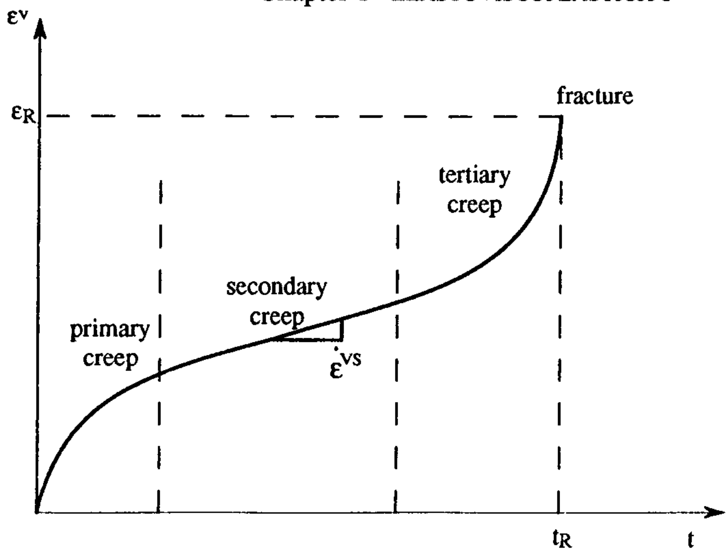

Fig. 1.1 The three phases in a creep test.

At low temperatures it is usually the primary and secondary creep that dominates; as the temperature rises the secondary phase becomes established sooner, and tertiary creep becomes more important.

The results of a set of tests can be presented as a body by plotting all the curves together on the same sheet; but it can be useful to plot lines of constant deformation in the time-stress plane, showing, for each initial stress, the time required to reach a certain deformation. Fig. 1.2, for example, shows these lines for deformations of *0.2%*, *1%* and *2%*. This has the advantage of making it easy to compare different materials, or to assess the effect of temperature: thus material A will have a "*50°C* creep difference" from material B if their curves are separated by this amount.

At temperatures below $0.3T_f$ (T_f being the melting point) only primary creep occurs, with the reduction in strain rate with time given by either a power or a logarithmic law, e.g.:

$$\varepsilon = At^{1/3} \qquad \text{(Andrade's law)} \qquad (1.1a)$$

$$\varepsilon = A\log(1 + t/t_0) \qquad \text{(logarithmic creep)} \qquad (1.1b)$$

where A and t_0 depend on the material.

These expressions should not be confused with the constitutive equations, since they cannot give a correct description of the way the deformation changes in response to a real variation of loading, such as partial or total unloading. We shall discuss later how work-hardening can be expressed, in terms of viscoplastic deformation for example.

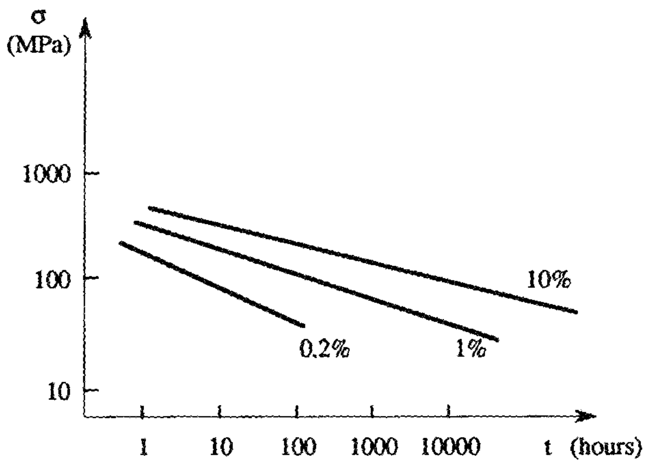

Fig. 1.2 Curves of constant deformation in creep.

Another way to present the results, restricted to secondary creep, is to show on a log-log plot of strain rate vs. time the minimum rate measured in each test. In any small interval of stress the points will lie on a straight line, enabling the tests to be interpreted with the aid of Norton's law:

$$\dot{\varepsilon}^{vs} = \left(\frac{\sigma}{K}\right)^{M} \tag{1.1c}$$

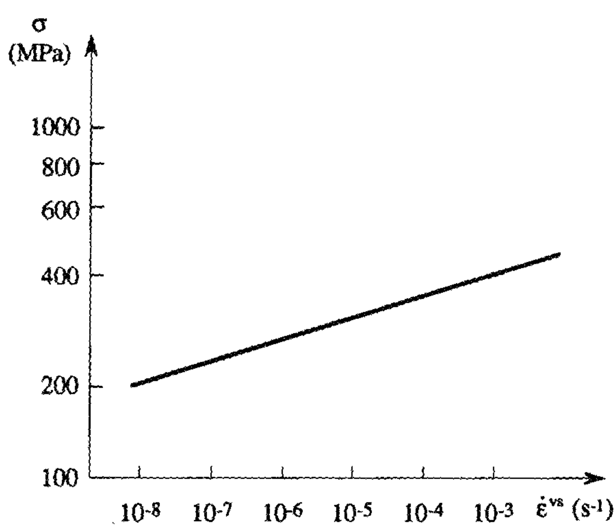

Fig. 1.3 Interpretation of creep test by Norton's law.

Whilst there is only one relation that is valid for describing the steady state, and which, in particular, takes no account of work-hardening, (1.1c) is often used in calculations as though it were an established constitutive equation. The coefficient M, the numerical value of which decreases with increasing temperature, is "Norton's exponent"; Table 1.1 gives the values for a number of pure metals. Physical models, as we shall show in our later discussion (§1.3 below), predict values of *1* for diffusional creep and *4-5*, according to the mechanism involved, for dislocation creep. However, these models are valid only over a limited range, and for low temperatures an internal stress has to be introduced; applying the rule as it is leads to exponents that can reach *40-50*, particularly in the case of complex alloys that are not simple solid solutions.

Metal	*M*	*Q* *kcal/mole*	Self-diffusion energy *kcal/mole*
Al	*4.4*	*34*	*34*
Cu	*4.8*	*48.4*	*47.1*
Au	*5.5*	*48 ± 5*	*41.7*
Ni	*4.6*	*66.5*	*66.8*
Pb	*4.2*	*24.2 ± 2.5*	*24.2*
Ta	*4.2*	*114 ± 4*	*110*
Cd	*4.3*	*19 ± 2*	*19.1*
Zn	*6.1*	*21.6*	*24.3*

Table 1.1 Values of Norton's exponent *M*, *Q* and self-diffusion energy for pure metals (Mukherjee et al (1969) p.155).

It can be useful to keep to the same type of law, modified to include both stress and temperature; such a law, involving an activation energy and the temperature (in °*K*) is

$$\dot{\varepsilon}^{vs} = \left(\frac{\sigma}{K}\right)^M \exp\left(-\frac{Q}{kT}\right) \tag{1.2}$$

As Table 1.1 shows, the activation energy is the same as the energy of *self diffusion* (see Vol. I, Annex 2) in the case of pure metals. For more complex materials the term cannot be given a precise physical meaning: *Q* can depend, for example, on the stress applied. Nevertheless, it continues to be used, in particular to establish time-temperature equivalences; these have a bearing on lifetime calculations and can be helpful in using the results of short-time tests at high temperatures to estimate times to failure at lower temperatures for which when the relevant times are very long, beyond the reach of experiment. If we accept the Monkman-Grant law (Ch. 2 §2.12) that states that the lifetime t_R is a power function of the rate of steady-state creep, that is, $\dot{\varepsilon}^{vs} t_R^{\alpha} = \text{constant}$, we find $t_R = A \exp(Q/kT)$. A parametric representation is then possible provided that *A* and *Q* do not both depend on the stress: this is the case, for example, for diffusional creep, where only A varies with stress and the parameter $P' = \log t_R$

- *const./T* can be used to represent the creep data. If only Q varies with stress the Larson-Miller parameter $P = T(\log t_R + const.)$ can be used, as shown in Fig. 1.4: see Larson and Miller (1952) and McClintock and Argon (1966) p.639.

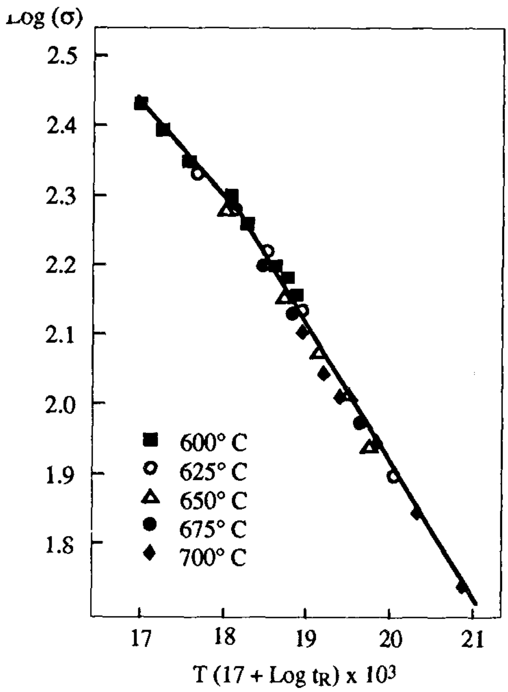

Fig. 1.4 Larson-Miller plot for the austenitic stainless steel Z03CND17-12, containing 2.5% molybdenum.

1.2.1.1 Tensile curves

In contrast to the case in which the plasticity does not depend on time, no single stress-strain curve represents tensile loading at different rates. The "rate effect" is usually to increase the stress for a given deformation, as shown in Fig. 1.5. There is a tendency to saturation at very high and at very low rates, suggesting limiting cases of instantaneous plasticity at very high rates and at zero rate. If we define a *critical stress* $\sigma_c(\varepsilon^v, \alpha)$, the stress corresponding to the zero-rate case, where α is the conventional representation of the work-hardening variables, and a *viscous stress* $\sigma_v(\dot{\varepsilon}^v, \alpha)$, which depends on the viscoplastic strain rate and is zero at zero rate, we can write the stress-strain relation for different strain rates v as

$$\sigma = \sigma_c(\varepsilon^v, \alpha) + \sigma_v(\dot{\varepsilon}^v, \alpha) \tag{1.3}$$

The existence of this "viscous stress" can be made evident by making a temporary change in the rate of loading in the course of a test. As Fig. 1.6 shows, there is a tendency to rejoin the undisturbed curve after the return to the original rate.