

EXCURSIONS

IN MODERN

MATHEMATICS

THIRD EDITION

PETER
TANNENBAUM
and
ROBERT
ARNOLD



01

Excursions in Modern Mathematics

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Preface



To most outsiders, modern mathematics is unknown territory. Its borders are protected by dense thickets of technical terms; its landscapes are a mass of indecipherable equations and incomprehensible concepts. Few realize that the world of modern mathematics is rich with vivid images and provocative ideas.

Ivars Peterson, The Mathematical Tourist

Excursions in Modern Mathematics is, as we hope the name might suggest, a collection of “trips” into that vast and alien frontier that many people perceive mathematics to be. While the purpose of this book is quite conventional--it is intended to serve as a textbook for a college-level liberal arts mathematics course--its contents are not. We have made a concerted effort to introduce the reader to a view of mathematics entirely different from the traditional algebra-geometry-trigonometry-finite math curriculum which so many people have learned to dread, fear, and occasionally abhor. The notion that general education mathematics must be dull, unrelated to the real world, highly technical, and deal mostly with concepts that are historically ancient is totally unfounded.

The excursions in this book represent a collection of topics chosen to meet a few simple criteria.

Applicability.

The connection between the mathematics presented here and down-to-earth, concrete real-life problems is direct and immediate. The often heard question, “What is this stuff good for?” is a legitimate one and deserves to be met head on. The often heard answer, “Well, you need to learn the material in Math 101 so that you can understand Math 102 which you will need to know if you plan to take Math 201 which will teach you the real applications.” is less than persuasive and in many cases reinforces students’ convictions that mathematics is remote, labyrinthine, and ultimately useless to them.

Accessibility.

Interesting mathematics need not always be highly technical and built on layers upon layers of concepts. As a general rule, the choice of topics in this book is such that a heavy mathematical infrastructure is not needed. We have found Intermediate Algebra to be an appropriate and sufficient prerequisite. (In the few instances in which more advanced concepts are unavoidable we have endeavored to provide enough background to make the material self-contained.) A word of caution—this does not mean that the material is easy! In mathematics, as in many other walks of life, simple and straightforward is not synonymous with easy and superficial.

Age.

Much of the mathematics in this book has been discovered in this century, some as recently as 20 years ago. Modern mathematical discoveries do not have to be only within the grasp of experts.

Aesthetics.

The notion that there is such a thing as beauty in mathematics is surprising to most casual observers. There is an important aesthetic component in mathematics and, just as in art and music (which mathematics very much resembles), it often surfaces in the simplest ideas. A fundamental objective of this book is to develop an appreciation for the aesthetic elements of mathematics. It is not necessary that the reader love everything in the book—it is sufficient that he or she find one topic about which they can say, “I really enjoyed learning this stuff!” We believe that anyone coming in with an open mind almost certainly will.

Outline

The material in the book is divided into four independent parts. Each of these parts in turn contains four chapters dealing with interrelated topics.

Part 1

(Chapters 1 through 4). The Mathematics of Social Choice. This part deals with mathematical applications in social science. How do groups make decisions? How are elections decided? What is power? How can power be measured? What is fairness? How are competing claims on property resolved in a fair and equitable way?

Part 2

(Chapters 5 through 8). Management Science. This part deals with methods for solving problems involving the organization and management of complex activities—that is, activities involving either a large number of steps and/or a large number of variables (routing the delivery of packages, landing a spaceship on Mars, organizing a banquet, scheduling classrooms at a big university, etc.). Efficiency is the name of the game in all these problems. Some limited or precious

resource (time, money, raw materials) must be managed in such a way that waste is minimized. We deal with problems of this type (consciously or unconsciously) every day of our lives.

Part 3

(Chapters 9 through 12). Growth and Symmetry. This part deals with nontraditional geometric ideas. How do sunflowers and seashells grow? How do animal populations grow? What are the symmetries of a snowflake? What is the symmetry type of a wallpaper pattern? What is the geometry of a mountain range? What kind of symmetry lies hidden in our circulatory system?

Part 4

(Chapters 13 through 16). Statistics. In one way or another, statistics affects all of our lives. Government policy, insurance rates, our health, our diet, and public opinion are all governed by statistical laws. This part deals with some of the most basic aspects of statistics. How should statistical data be collected? How is data summarized so that it is intelligible? How should statistical data be interpreted? How can we measure the inherent uncertainty built into statistical data? How can we draw meaningful conclusions from statistical information? How can we use statistical knowledge to predict patterns in future events?

Exercises

We have endeavored to write a book that is flexible enough to appeal to a wide range of readers in a variety of settings. The exercises, in particular, have been designed to convey the depth of the subject matter by addressing a broad spectrum of levels of difficulty—from the routine drill to the ultimate challenge. For convenience (but with some trepidation) we have classified them into three levels of difficulty:

1 Walking.

These exercises are meant to test a basic understanding of the main concepts, and they are intended to be within the capabilities of students at all levels.

2 Jogging.

These are exercises that can no longer be considered as routine—either because they use basic concepts at a higher level of complexity, or they require slightly higher order critical thinking skills, or both.

3 Running.

This is an umbrella category for problems that range from slightly unusual or slightly above average in difficulty to problems that can be a real challenge to even the most talented of students. This category also includes an occasional open-ended problem suitable for a project.

Teaching Extras Available with the Third Edition

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THE THIRD EDITION

This third edition of *Excursions in Modern Mathematics* retains the topics and organization of previous editions, in a more attractive and hopefully more user friendly package. Most chapters have been rewritten, and new examples and applications have been added throughout. In addition, some chapters have undergone substantive changes in organization (Chapters 5, 6, and 7), or coverage (Chapters 11, 13, 15, and 16). New topics not in previous editions are: the classification of finite shapes by their symmetry types (Chapter 11); the capture-recapture method for estimating the size of a population (Chapter 13); permutations and combinations (Chapter 15); and an introduction to statistical inference (Chapter 16).

A FINAL WORD

This book grew out of the conviction that a liberal arts mathematics course should teach students more than just a collection of facts and procedures. The ultimate purpose of this book is to instill in the reader an overall appreciation of mathematics as a discipline and an exposure to the subtlety and variety of its many facets: problems, ideas, methods, and solutions. Last, but not least, we have tried to show that mathematics can be fun.

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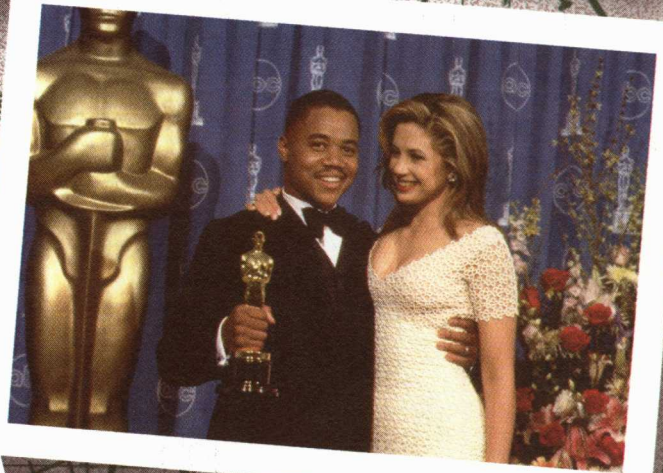
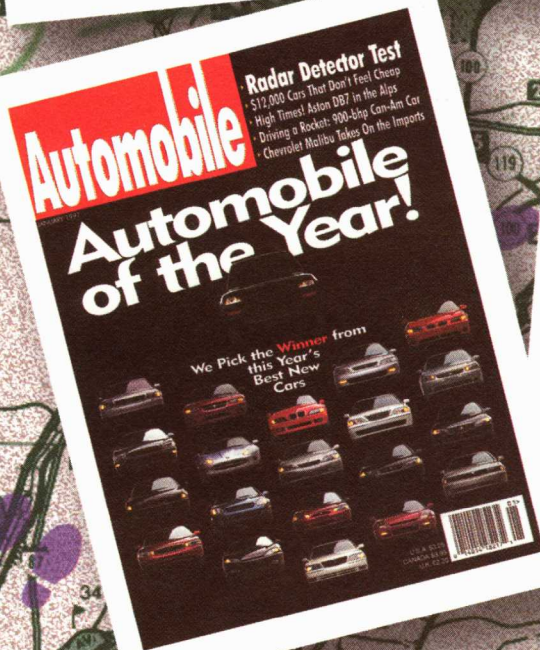
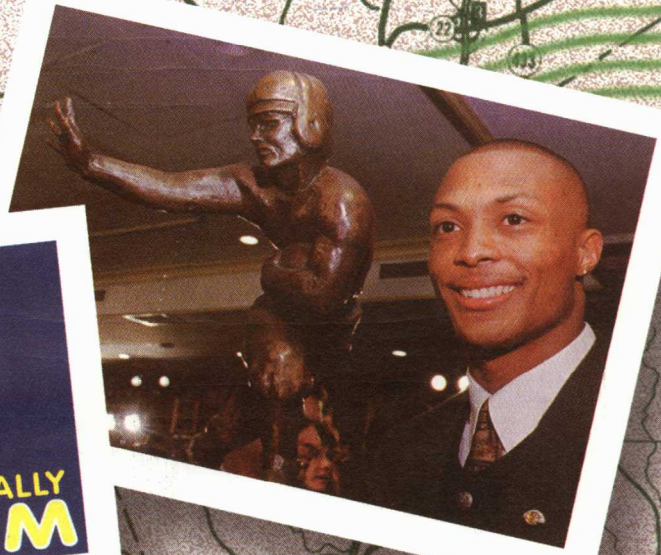
PART 1

***The Mathematics
of Social Choice***



COUNTY
SCHOOL
BOARD

SALLY
TANNENBAUM



*It's not the voting that's
democracy; it's the counting.*

Tom Stoppard