NUMERICAL OPTIMIZATION TECHNIQUES FOR ENGINEERING DESIGN
With Applications

Garret N. Vanderplaats

This book was set in Times Roman by Science Typographers, Inc.
The editors were Anne Murphy and Susan Hazlett;
the production supervisor was Diane Renda.
The drawings were done by ANCO/Boston.
Halliday Lithograph Corporation was printer and binder.

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1234567890 HALHAL 8987654

ISBN 0-07-066964-3

Library of Congress Cataloging in Publication Data

Vanderplaats, Garret N.
Numerical optimization techniques for engineering design.

(McGraw-Hill series in mechanical engineering)
Includes bibliographical references and indexes.
1. Engineering design. 2. Numerical calculations.
3. Mathematical optimization. I. Title. II. Series.
TA174.V36 1984 620'.00425 83-14938
ISBN 0-07-066964-3

McGraw-Hill Series in Mechanical Engineering

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PREFACE

The concept of optimization is intrinsically tied to humanity's desire to excel. Though we may not consciously recognize it and though the optimization process takes different forms in different fields of endeavor, this drive to do better than before consumes much of our energy, whether we are athletes, artists, businesspersons, or engineers. The field we are concerned with is that of engineering and, while the emphasis here is on mechanical, aeronautical, and civil engineering, this is by no means the limit of applicability; it simply reflects my own experience and interests.

We will consider the use of numerical optimization techniques to devise a rational, directed design procedure. Although these methods provide a computational tool for design, there is much more to be gained from this study. Indeed, numerical optimization provides us with a new design philosophy. It gives us an ordered approach to design decisions where before we relied heavily on intuition and experience. It is this order which makes the techniques presented here so very attractive, because it provides insight into the actual design process. However, this should not be construed to suggest that the design process can be reduced to a few computer runs or that our intuition and experience are unimportant. Rather, the computer can now be used to relieve us of the tedium of repetitive calculations, freeing us to spend time on the truly creative aspects of engineering design.

The methods presented here have gone through an extensive development period of approximately 20 years. While much research in automated design optimization remains to be done, the field has matured to the point where the techniques can be routinely applied by practicing engineers to a large percentage of design tasks. However, education in the concepts and applications of numerical optimization is not yet a principal part of most engineering curricula. Consequently, few practicing engineers are actually using this powerful tool in design. For this reason, there is strong motivation to provide a book which can be used for classroom education and also for self-study or as a continuing education aid. It is hoped that the concepts presented here are organized in a logical enough way and presented in plain enough language that many engineers can in some way use this information.

The purpose of this book is threefold. First, we wish to gain a basic knowledge of numerical optimization algorithms, together with their strengths and weaknesses. Second, the student is encouraged to gain computational experience by programming these algorithms for the computer, and finally, a variety of design applications are discussed to identify those design areas where numerical optimization techniques have been applied in the past and where they may be applied in the future.

This book is written to be a senior or graduate level textbook but is suited for self-study by practicing engineers. Also, the book will provide a ready reference for many of the best numerical optimization algorithms. The student should have a solid background in engineering fundamentals matrix algebra, and computer programming. Because the principal purpose here is to provide design tools for the engineer, we will avoid mathematical proofs and theoretical discussions of the various algorithms. Instead we will address such questions from a more pragmatic viewpoint of asking, does a given technique provide a good engineering answer in an efficient manner? To this end, we will emphasize those aspects of numerical optimization which attempt to model the thought processes of a good design engineer.

In the first chapter, we introduce the basic concepts of numerical optimization. We identify the characteristics of unconstrained and constrained optimization problems and offer the general mathematical problem statement. We describe the most common iterative approach to the solution of the optimization problem and very briefly discuss the necessary mathematical conditions for a solution to be the optimum among all possible solutions. These concepts provide the mathematical basis to understanding some of the more powerful optimization algorithms to be discussed in later chapters. Finally, we identify some of the advantages and limitations to the use of numerical optimization techniques in engineering design to provide a broad perspective on the application of this general design approach.

In Chap. 2, we begin to develop the numerical tools necessary for the creation of an efficient design capability. In this chapter we discuss the solution of optimization problems defined by only one variable Here we consider two of the most popular and powerful techniques for solving this problem, polynomial approximations and the golden section method. Methods are presented for finding either the minimum or the zero value of a function of one variable. Considerable generality is maintained throughout this discussion so that the student gains a broad capability for the solution of this problem. This chapter contains a discussion of minimizing a function of one variable subject to constraints on other variables. The chapter concludes with a recommended algorithm for minimizing functions of one variable. The techniques discussed here are important because these tools will be used throughout the remainder of the book when dealing with more general multivariable optimization problems.

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Chapter 3 extends the discussion to the minimization of multivariable unconstrained functions. This is a natural progression because this is the simplest multivariable problem to understand and yet the methods developed here are later used to solve constrained multivariable optimization problems. Unconstrained minimization techniques are separated into zero-, first-, and second-order methods. It is seen that each level of algorithmic sophistication has a place in automated design, depending on the nature of the problem being solved.

Chapter 4 introduces the most extensively developed of the multivariable constrained optimization techniques. This is the solution of strictly linear analysis and design problems. The basic algorithm presented here is the standard simplex method. While this is not the most efficient of the simplex methods, it is most easily developed and understood and therefore serves best to introduce the concepts. Linear problems are encountered in such diverse areas as limit analysis and design of structures and industrial resource allocation and, as such, are valuable in their own right. Also, linear programming techniques will be used later in solving nonlinear constrained minimization problems. While the student is encouraged to program the standard simplex algorithm to gain a proper understanding of the concepts, it is noted that a very efficient, thoroughly debugged, and well-documented linear programming code probably already exists on the local computer system or is readily available from other sources. Therefore, if extensive use of linear programming techniques is anticipated, it is best to obtain an available code elsewhere and spend time on more fruitful endeavors.

Chapters 5, 6, and 7 provide the fundamental capability we wish to develop for general engineering design. This gives the student the ability to solve multivariable nonlinear constrained optimization problems of the type most often encountered in practice.

Chapter 5 develops a technique for the solution of the constrained optimization problem by conversion to an equivalent unconstrained problem. The basic concept here is to convert the original constrained optimization problem to a sequence of unconstrained problems through the use of penalties for constraints. Three techniques are presented, the exterior, interior, and extended interior penalty functions. Each approach has its own attractions as well as drawbacks, and a good understanding of these techniques aids in determining which is most useful for solving specific design problems. A powerful sequential unconstrained minimization technique known as the augmented lagrange multiplier method is presented in this chapter. This method is capable of solving nonlinear programming problems subject to nonlinear equality as well as inequality constraints. Furthermore, it avoids many of the numerical difficulties associated with the more traditional sequential unconstrained minimization techniques. Each of the methods developed in this chapter makes use of the algorithms developed in Chaps. 2 and 3 for unconstrained minimization.

Chapter 6 is devoted to so-called direct methods. These methods attempt to incorporate information about the constraints directly into the optimization problem rather than converting the problem to an equivalent unconstrained one as was done in Chap. 5. Chapter 6 includes sequential linear programming, the method of feasible directions and the generalized reduced gradient method as well as a robust feasible directions method containing desirable features from the other techniques. The final section of Chap. 6 presents a recent sequential quadratic programming technique which is found to be a particularly powerful method.

Chapter 7 addresses many of the practical aspects of using numerical optimization in engineering design. These include design variable linking where one design variable in the optimization process controls more than one variable in the actual engineering system. The concept of a reduced basis for design is shown to be a generalization of design variable linking and is particularly powerful for including practical constraints into the mathematical design problem. The use of approximation techniques in design optimization is discussed in detail. The principal motivation here is to develop methods which use whatever detailed analysis techniques we choose but formally approximate this problem in such a way that the efficiency of the optimization process is greatly improved while retaining all of the features of the original problem. Chapter 7 concludes with the presentation of techniques whereby the sensitivity of the optimum design with respect to various changes in the problem statement is obtained. This provides us with the ability to predict the effect of changes in material properties and loading or constraint limits on the design, even after the optimization process has been completed.

Chapter 8 introduces advanced optimization techniques, specifically the concept of duality. Duality is not directly useful in all design tasks, but for those where it applies, duality offers yet another dimension to our design optimization capability In developing these methods, the concept of convexity is first presented, followed by the concepts of a saddle point and mathematical separability. It is then shown that if a design problem naturally exhibits the separability property or can be reasonably approximated as a separable problem, significant efficiency improvements in optimization can be achieved. There is an added benefit to primal-dual methods; that is, that we can now design using discrete values for the independent design variables. This includes the number of plies in a composite material or selection of the design from a table of available structural sections as examples. Finally, primal-dual methods incorporate many of the concepts developed in previous chapters in such a way as to provide fundamental insight into the concepts of engineering design optimization and lay the groundwork for much future development in this exciting field.

Chapter 9 is the first applications chapter and here the specific topic of structural optimization is addressed. A concerted effort is made to identify efficient means of approaching the problem and understanding lessons learned from past experience. It is a clear understanding of how numerical optimization has been applied in the past that gives us insight into how it may be applied in the future.

Chapter 10 concludes the book with a discussion of a variety of optimization applications. The purpose here, as in Chap. 9, is to identify problems where optimization has been used effectively. While a comprehensive list is not possible, this discussion does identify typical applications and provides further insight into problem formulation and possible future applications.

Throughout the book, the practicalities of making engineering design decisions on the computer are stressed. Theoretical detail is limited to that necessary to understand the concepts. Most of all, it is hoped that the practicing engineer of the future will be more completely equipped to use advanced optimization techniques to improve the quality of life for all.

Garret N. Vanderplaats

CONTENTS

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CHAPTER
ONE
BASIC CONCEPTS

1-1 INTRODUCTION

The concept of optimization is basic to much of what we do in our daily lives. The desire to run a faster race, win a debate, or increase corporate profit implies a desire to do or be the best in some sense. In engineering, we wish to produce the "best quality of life possible with the resources available." Thus in "designing" new products, we must use design tools which provide the desired results in a timely and economical fashion. Numerical optimization is one of the tools at our disposal.

In studying design optimization, it is important to distinguish between analysis and design. Analysis is the process of determining the response of a specified system to its environment. For example, the calculation of stresses in a structure that result from applied loads is referred to here as analysis. Design, on the other hand, is used to mean the actual process of defining the system. For example, structural design entails defining the sizes and locations of members necessary to support a prescribed set of loads. Clearly, analysis is a subproblem in the design process because this is how we evaluate the adequacy of the design.

Much of the design task in engineering is quantifiable, and so we are able to use the computer to analyze alternative designs rapidly. The purpose of numerical optimization is to aid us in rationally searching for the best design to meet our needs.

While the emphasis here is on design, it should be noted that these methods can often be used for analysis as well. Nonlinear structural analysis is an example where optimization can be used to solve a nonlinear energy minimization problem.

Although we may not always think of it this way, design can be defined as the process of finding the minimum or maximum of some parameter which may be called the objective function. For the design to be acceptable, it must also satisfy a certain set of specified requirements called constraints. That is, we wish to find the constrained minimum or maximum of the objective function. For example, assume we wish to design an internal-combustion engine. The design objective could be to maximize combustion efficiency. The engine may be required to provide a specified power output with an upper limit on the amount of harmful pollutants which can be emitted into the atmosphere. The power requirements and pollution restrictions are therefore constraints on the design.

Various methods can be used to achieve the design goal. One approach might be through experimentation where many engines are built and tested. The engine providing maximum economy while satisfying th constraints on the design would then be chosen for production. Clearly this is a very expensive approach with little assurance of obtaining a true optimum design. A second approach might be to define the design process analytically and then to obtain the solution using differential calculus or the calculus of variations. While this is certainly an attractive procedure, it is seldom possible in practical applications to obtain a direct analytical solution because of the complexities of the design and analysis problem.

Most design organizations now have computer codes capable of analyzing a design which the engineer considers reasonable. For example, the engineer may have a computer code which, given the compression ratio, air-fuel mixture ratio, bore and stroke, and other basic design parameters, can analyze the internal-combustion engine to predict its efficiency, power output, and pollution emissions. The engineer could then change these design variables and rerun the program until an acceptable design is obtained. In other words, the physical experimentation approach where engines are built and tested is replaced by numerical experimentation, recognizing that the final step will still be the construction of one or more prototypes to verify our numerical results.

With the availability of computer codes to analyze the proposed design, the next logical step is to automate the design process. In its most basic form, design automation may consist of a series of loops in the computer code which cycle through many combinations of design variables. The combination which provides the best design satisfying the constraints is then termed optimum. This approach has been used with some success and may be quite adequate if the analysis program uses a small amount of computer time. However, the cost of this technique increases dramatically as the number of design variables to be changed increases and as the computer time for a single analysis increases.

Consider, for example, a design problem described by three variables. Assume we wish to investigate the designs for 10 values of each variable. Assume also that any proposed design can be analyzed in one-tenth of a central processing unit (CPU) second on a digital computer. There are then 10^3 combinations of design variables to be investigated, each requiring one-tenth second for a total of 100 CPU seconds to obtain the desired optimum design. This would probably be considered an economical solution in most design situations. However, now consider a more realistic design problem where 10 variables describe the design. Again, we wish to investigate 10 values of each variable. Also now assume that the analysis of a proposed design requires 10 CPU seconds on the computer. The total CPU time now required to obtain the optimum design is 10^{11} seconds, or roughly 3200 years of computer time! Clearly, for most practical design problems, a more rational approach to design automation is needed.

Numerical optimization techniques offer a logical approach to design automation, and many algorithms have been proposed in recent years. Some of these techniques, such as linear, quadratic, dynamic, and geometric programming algorithms, have been developed to deal with specific classes of optimization problems. A more general category of algorithms referred to as nonlinear programming has evolved for the solution of general optimization problems. Methods for numerical optimization are referred to collectively as mathematical programming techniques.

Though the history of mathematical programming is relatively short, roughly 30 years, there has been an almost bewildering number of algorithms published for the solution of numerical optimization problems. The author of each algorithm usually has numerical examples which demonstrate the efficiency and accuracy of the method, and the unsuspecting practitioner will often invest a great deal of time and effort in programming an algorithm, only to find that it will not in fact solve the particular optimization problem being attempted. This often leads to disenchantment with these techniques which can be avoided if the user is knowledgeable in the basic concepts of numerical optimization. There is an obvious need, therefore, for a unified, nontheoretical presentation of optimization concepts.

The purpose here is to attempt to bridge the gap between optimization theory and its practical applications. The remainder of this chapter will be devoted to a discussion of the basic concepts of numerical optimization. We will consider the general statement of the nonlinear constrained optimization problem and some (slightly) theoretical aspects regarding the existence and uniqueness of the solution to the optimization problem. Finally, we will consider some practical advantages and limitations to the use of these methods.

Numerical optimization has traditionally been developed in the operations research community. The use of these techniques in engineering design was popularized in 1960 when Schmit [1] applied nonlinear optimization techniques to structural design and coined the phrase "structural synthesis." While the work of Ref. 1 was restricted to structural optimization, the concepts presented there offered a fundamentally new approach to engineering design which is applicable to a wide spectrum of design problems. The basic concept is that the purpose of design is the allocation of scarce resources [2]. The purpose of numerical optimization is to provide a computer tool to aid the designer in this task.

1-2 OPTIMIZATION CONCEPTS

Here we will briefly describe the basic concepts of optimization by means of two examples.

Example 1-1 Unconstrained function minimization
Assume we wish to find the minimum value of the following simple algebraic function.

$$F(\mathbf{X}) = 10X_1^4 - 20X_1^2X_2 + 10X_2^2 + X_1^2 - 2X_1 + 5 \tag{1-1}$$

$F(\mathbf{X})$ is referred to as the objective function which is to be minimized, and we wish to determine the combination of the variables X_1 and X_2 which will achieve this goal. The vector **X** contains X_1 and X_2, and we call them the design, or decision, variables. No limits are imposed on the values of X_1 and X_2 and no additional conditions must be met for the "design" to be acceptable. Therefore, $F(\mathbf{X})$ is said to be unconstrained. Figure 1-1 is a graphical representation of the function, where lines of constant value of $F(\mathbf{X})$ are drawn. This function is often referred to as the *banana function* because of its distinctive geometry. Figure 1-1 is referred to as a two-variable design space, where the design variables X_1 and X_2 correspond to the coordinate axes. In general, a design space will be n dimensional, where n is the number of design variables of which the objective is a function. The two-variable design space will be used throughout our discussion of optimization techniques to help visualize the various concepts.

From Fig. 1-1 we can estimate that the minimum value of $F(\mathbf{X})$ will occur at $X_1^* = 1$ and $X_2^* = 1$. We know also from basic calculus that at the optimum, or minimum, of $F(\mathbf{X})$, the partial derivatives with respect to X_1 and X_2 must vanish. That is

$$\partial F(\mathbf{X})/\partial X_1 = 40X_1^3 - 40X_1X_2 + 2X_1 - 2 = 0 \tag{1-2}$$

$$\partial F(\mathbf{X})/\partial X_2 = -20X_1^2 + 20X_2 = 0 \tag{1-3}$$

Solving for X_1 and X_2, we find that indeed $X_1^* = 1$ and $X_2^* = 1$. We will see later that the vanishing gradient is a necessary but not sufficient condition for finding the minimum.

In this example, we were able to obtain the optimum both graphically and analytically. However, this example is of little engineering value, except for demonstration purposes. In most practical engineering problems the minimum of a function cannot be determined analytically. The problem is further complicated if the decision variables are restricted to values within a specified range or if other

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