Oleg B. Lupanov
Oktay M. Kasim-Zade
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Third International Symposium, SAGA 2005
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Preface

This volume constitutes the proceedings of the 3rd Symposium on Stochastic Algorithms, Foundations and Applications (SAGA 2005), held in Moscow, Russia, at Moscow State University on October 20–22, 2005. The symposium was organized by the Department of Discrete Mathematics, Faculty of Mechanics and Mathematics of Moscow State University and was partially supported by the Russian Foundation for Basic Research under Project No. 05–01–10140–г. The SAGA symposium series is a biennial meeting which started in 2001 in Berlin, Germany (LNCS vol. 2264). The second symposium was held in September 2003 at the University of Hertfordshire, Hatfield, UK (LNCS vol. 2827).

Since the first symposium in Berlin in 2001, an increased interest in the SAGA series can be noticed. For SAGA 2005, we received submissions from China, the European Union, Iran, Japan, Korea, Russia, SAR Hong Kong, Taiwan, and USA, from which 14 papers were finally selected for publication after a thorough reviewing process.

The contributed papers included in this volume cover both theoretical as well as applied aspects of stochastic computations, which is one of the main aims of the SAGA series. Furthermore, five invited lectures were delivered at SAGA 2005: The talk by Alexander A. Sapozhenko (Moscow State University) summarizes results on the *container method*, a technique that is used to solve enumeration problems for various combinatorial structures and which has numerous applications in the design and analysis of stochastic algorithms. Christos D. Zaroliagis (University of Patras) presented recent advances in multiobjective optimization. Joachim Wegener (DaimlerChrysler AG, Research and Technology) introduced new search-based techniques for software testing, with particular emphasis on finding time-critical pathways in safety-relevant software components. Chrystopher L. Nehaniv (University of Hertfordshire) presented a comprehensive overview and the latest results on self-replication, evolvability and asynchronicity in stochastic worlds. The talk by Farid Ablayev (Kazan State University) analyzed from the communication point of view some proof techniques for obtaining lower complexity bounds in various classical models (deterministic, nondeterministic and randomized), and quantum models of branching programs.

We wish to thank all who supported SAGA 2005, all authors who submitted papers, all members of the Programme Committee and all reviewers for the great collective effort, all invited speakers, all members of the Organizing Committee, and the Russian Foundation for Basic Research for financial support.

October 2005

Oleg B. Lupanov, Oktay M. Kasim-Zade,
Alexander V. Chashkin, Kathleen Steinhöfel

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SAGA 2005 was organized by the Department of Discrete Mathematics, Faculty of Mechanics and Mathematics of Moscow State University, Russia.

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Table of Contents

Systems of Containers and Enumeration Problems

Alexander Sapozhenko*

Lomonosov University, Moscow

Abstract. We discuss a technique (named "the container method") for enumeration problems. It was applied for obtaining upper bounds and asymptotically sharp estimates for the number of independent sets, codes, antichains in posets, sum-free sets, monotone boolean functions and so on. The container method works even the appropriate recurrent equalities are absent and the traditional generating function method is not applicable. The idea of the method is to reduce a considered enumeration problem to evaluating the number of independent sets in the appropriate graph. We give some examples of such reduction and a survey of upper bounds for the number of independent sets in graphs. The method is usually successful if considered graphs are almost regular and expanders.

1 Introduction

We discuss a method for solving enumeration problems. The considered problems are not usually amenable to the traditional generating function method because of the absence of appropriate recurrent equations. The problems of estimating the number of independent sets in graphs, antichains in posets, sum-free sets in groups are among such problems. Our approach to solution of these problems based on the notion of covering system (or system of containers) for a family of sets. A family $\mathcal{B}$ of sets is called *covering* for a family $\mathcal{A}$, if for any $A \in \mathcal{A}$ there exists $B \in \mathcal{B}$ such that $A \subseteq B$. An element of the family $\mathcal{B}$ is called a *container* and $\mathcal{B}$ is called the *system of containers* for $\mathcal{A}$.

Suppose we want to know what is size of the family $\mathcal{A}$ of all sets with some given property Q. We can think of $\mathcal{A}$ as the family of subsets in the n-cube which are error correcting codes, sum-free sets in a finite group, vertex subsets of graph which are cliques and so on. The container method consists in finding a system of containers $\mathcal{B}$ with the special property. For example, it is good when there exists a subsystem $\mathcal{B}_1 \subseteq \mathcal{B}$ such that all (or almost all) $A \in \mathcal{B}_1$ possessing the property Q and simultaneously meeting the inequality

$$\sum_{A\in\mathcal{B}\setminus\mathcal{B}_1} 2^{|A|} \ll \sum_{B\in\mathcal{B}} 2^{|B|}. \tag{1}$$

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Then we know that $|\mathcal{A}| \approx \sum_{B \in \mathcal{B}} 2^{|B|}$. The proof of (1) is usually reduced to upper bounds for independent sets in appropriate graph.

The aim of this paper to give examples where these simple considerations are successful. Except for the simplest cases, we does not present the complete proofs but only some sketches. We give references on the paper with the proofs. At the end, we consider the application of the container technique to estimating the complexity of some algorithms correctly working almost always.

2 Definitions

All graphs under consideration are finite, undirected, and simple. The vertices are considered as numbered. Denote the degree of a vertex v by $\sigma(v)$.

A subset of vertices of a graph G is called *independent* if the subgraph of G induced by A does not contain an edge. The family of all independent sets of G will be denoted by $\mathcal{I}(G)$. We put $I(G) = |\mathcal{I}(G)|$. Let $G = (V; E)$ be a graph with the vertex set V and the edge set E, and $v \in V$. We call the set $\partial v = \{u : (u, v) \in E\}$ the *boundary* of v. It is clear that $\sigma(v) = |\partial v|$. The *boundary* of $A \subseteq V$ in a graph $G = (V; E)$ is the set $\partial A = \left(\bigcup_{v \in A} \partial v\right) \setminus A$. Suppose $0 \leq \delta < 1$. A graph G is called *δ-expander*, if $|A| \leq |\partial A|(1 - \delta)$ for any independent set A.

3 Reduction

In this section we discuss ways of reducing various enumeration problems to that of counting independent sets.

The simplest for reductions is the problem of enumerating cliques. We just use the fact that a clique in graph G corresponds to the independent set in the complement of G.

The enumeration of error-correcting codes with distance r is equivalent to that of independent sets in graph on the set of vertices of the n-cube, where an edge is an arbitrary pair (u,v) with Hamming distance less than r.

The number of sum-free sets in groups is also estimated above by the number of independent sets in the Cayley graph (see, for example, N.Alon [1]). Recall that a set A is sum-free in an additive group G if A does not contain triples a, b, c satisfying the equality $a + b = c$. For a group G and sets A, V the Cayley graph $\mathcal{C}_A(V)$ is defined as the graph on vertex set V s.t. (u, v) is an edge if $u + v$, $u - v$ or $v - u$ are contained in A. If $B \subseteq G$ is sum-free then B is independent in the Cayley graph $\mathcal{C}_A(G)$ for any $A \subseteq B$.

4 On the Number of Independent Sets

Here we obtain upper bounds for the number of independent sets in graphs by means of container method. We start with a simple lemma for regular graphs. A regular graph of degree k on n vertices will be called (n, k)-graph.

Lemma 1. *Let $G = (V; E)$ be a $(n,k)-$graph, $k > 1$, and $A \subseteq V$ be independent. Suppose $0 < \varphi < k$. Then there exists $T \subseteq A$ such that*

$$|T| \le |\partial A|/\varphi \ , \tag{2}$$

$$A \subseteq D, \quad \textit{where} \quad D = D(T,\varphi) = \{v \in V \setminus \partial T : |\partial v \setminus \partial T| < \varphi\} \ , \tag{3}$$

and

$$|D| \le |\partial T| \frac{k}{k-\varphi} \ . \tag{4}$$

Proof. The set T can be constructed by the following step-by-step procedure.

Step 1. Let u_1 be an arbitrary vertex from A. Set $T_1 = \{u_1\}$.
Suppose m steps have already been done and the set $T_m = \{u_1, \dots, u_m\}$ has been constructed.

Step $m+1$. If there exists $u_{m+1} \in A$ such that $|\partial u_{m+1} \setminus \partial T_m| \ge \varphi$, we put $T_{m+1} = T_m \cup \{u_{m+1}\}$. Otherwise, the procedure is complete and the result is $T = T_m$. The inequality (2) and the inclusion (3) evidently hold for the set T constructed as above. The inequality (4) follows from the facts that $|\partial v \cap \partial T| \ge k - \varphi$ for every $v \in D$ and $|\partial u| \le k$ for every $u \in \partial T$. □

Denote by $\mathcal{I}(G)$ the family of independent sets of a graph G and set $I(G) = |\mathcal{I}(G)|$.

Corollary 1. *Let G be a (n,k)-graph, $1 \le \varphi < k$. Then there exists the system of containers $\mathcal{F}$ for the family $\mathcal{I}(G)$ with the following properties:*

$$|\mathcal{F}| \le \sum_{i \le n/\varphi} \binom{n}{i}, \tag{5}$$

$$\textit{for any } D \in F \quad |D| \le nk/(2k-\varphi). \tag{6}$$

Proof. By Lemma 1, every independent set A contains a subset T of size not exceeding $|\partial A|/\varphi$ provided that $A \subseteq D(T,\varphi)$. Note that $D(T,\varphi)$ is uniquely defined by T if φ is fixed. Thus the number of containers does not exceed the number of vertex subsets of size $|\partial A|/\varphi \le n/\varphi$. From this (5) follows.

The inequality (6) follows from (4) in view of $|D| \le n - \partial T$. □

The following theorem improves the error term in Alon's upper bound [1]. It also gives an example of application of the container method.

Theorem 1. *For any (n,k)-graph*

$$I(G) \le 2^{\frac{n}{2}\left(1+O\left(\sqrt{(\log k)/k}\right)\right)} \ . \tag{7}$$

Proof. The inequality (7) immediately follows from the corollary. All independent sets can be enumerated in the following way. Fix $\varphi = \sqrt{k \log k}$. Choose some $T \subseteq V$ and construct $D = D(T,\varphi)$. Then choose $A \subseteq D$. Note that

$$|D| \le nk/(2k-\varphi) \ , \tag{8}$$

in view of (6). Hence, under $\varphi = \sqrt{k \log k}$, we have

$$I(G) \le \sum_{T \subseteq V, |T| \le n/\varphi} 2^{|D(T,\varphi)|} \le \sum_{i \le n/\varphi} \binom{n}{i} 2^{nk/(2k-\varphi)}$$

$$\le 2^{\frac{n}{2}\left(1+O\left(\sqrt{(\log k)/k}\right)\right)} . \qquad (9)$$

□

Theorem 1 was extended to the case of irregular graphs (see[17]). The need of extensions caused by applications to enumeration problems from algebra and number theory (see[3], [5], [11], [18], [19]).

G is called *(n,k,θ)–graph,* if it has n vertices and $k \le \sigma(v) \le k+\theta$ for any vertex v. By definition, $(n,k,0)$-graph is regular of degree k.

Theorem 2. *For any (n,k,θ)-graph Γ*

$$I(\Gamma) \le 2^{\frac{n}{2}\left(1+O\left(\theta/k+\sqrt{(\log k)/k}\right)\right)} . \qquad (10)$$

This theorem extends the previous one to "almost" regular graphs.
Denote by $I_\beta(\Gamma)$ the number of subsets $A \in \mathcal{I}(\Gamma)$ meeting the condition

$$||A| - n/4| \ge \beta n/4 .$$

Theorem 3. *Let $\Gamma = (V;E)$ be (n,k,θ)-graph and $0 < \beta < 1$. Then*

$$I_\beta(\Gamma) \le 2^{\frac{n}{2}\left(1-\frac{\beta^2}{2\ln 2}+O\left(\frac{\theta}{k}+\sqrt{\frac{\log k}{k}}\right)\right)} . \qquad (11)$$

Theorem 4. *Let (n,k,θ)-graph $\Gamma = (V;\ E)$ be δ-expander for some $0 \le \delta < 1$. Then*

$$I(\Gamma) \le 2^{\frac{n}{2}\left(1-\delta/7+O\left(\theta/k+\sqrt{(\log k)/k}\right)\right)} . \qquad (12)$$

Theorems 3 and 4 allow to obtain the upper bounds of the form $I(G) \le 2^{n(1/2-c)}$, where $c > 0$ is some constant. It is very helpful when proving assertions that some subsystem of containers gives small contribution (see, for example, [18]).

The further similar extensions were proved in [21]. Suppose l, k, θ, n meet the inequalities $l \le k-\theta \le k+\theta \le n$. A graph on n vertices is called a *$(n,l,k,m,\delta,\varepsilon,\theta)$-graph* if it meets the following condition: the minimal vertex degree is at least l, the maximal vertex degree is at most m, the fraction of vertices with a degree exceeding $k+\theta$ is not more than ε, and the fraction of vertices with a degree less than $k-\theta$ is not more than δ. In [19] the following statement is proved.

Theorem 5. *Let $G = (V,E)$ be a $(n,l,k,m,\delta,\varepsilon,\theta)$-graph. Then there exists a system $\mathcal{B}$ of containers for $\mathcal{I}(G)$ meeting the following two conditions:*

1. *For any* $B \in \mathcal{B}$

$$|B| \leq n \frac{k + \delta(k-l) + \varepsilon(m-k) + \theta}{2k - \sqrt{k \log k}} \; ; \tag{13}$$

2. *For* $k > 3$ *and large enough* n

$$|\mathcal{B}| \leq 2^{n \sqrt{\frac{\log k}{k}}} \; . \tag{14}$$

Furthermore, for large enough n

$$I(G) \leq 2^{\frac{n}{2}\left(1+\delta\left(1-\frac{l}{k}\right)+\varepsilon\left(\frac{m}{k}-1\right)+O\left(\frac{\theta}{k}+\sqrt{\frac{\log k}{k}}\right)\right)} \; . \tag{15}$$

The following two theorems proved in ([21]) are extensions of theorems 3 and 4.

Theorem 6. *Let* G *be a* $(n, l, k, m, \delta, \Delta, \theta)$*-graph and*

$$\gamma = \delta\,(1 - l/k) + \Delta\,(m/k - 1) + O\left((\theta + \sqrt{k \log k})/k\right) \; .$$

Then for large enough n

$$I_\beta(G) \leq 2^{\frac{n}{2}\left(1-(\beta-\gamma)^2/(2(1+\gamma)\ln 2)\right)} \; . \tag{16}$$

Theorem 7. *Let* $(n, l, k, m, \delta, \Delta, \theta)$*-graph* G *be* ϵ*-expander. Then for some absolute constant* $c > 0$ *and large enough* n

$$I(G) \leq 2^{\frac{n}{2}\left(1-c\epsilon+\delta(1-l/k)+\Delta(m/k-1)+O\left((\theta+\sqrt{k \log k})/k\right)\right)} \; . \tag{17}$$

Theorems 5 – 7 sufficiently extend abilities of the container method. In particular, Theorem 5 is applied for proving the Cameron-Erdös conjecture in [19].

5 Bipartite Graphs

In the case of bipartite graphs the container method allows to get asymptotically ultimate result if the graph is an expander. A bipartite graph $G = (X, Z; E)$ with the vertex partition sets X, Z and the edge set E will be called an *two-sided* (ϵ, δ)*-expander* if $|A| \leq |\partial(A)|(1-\delta)$ for any $A \subseteq X$ such that $|A| \leq \epsilon\,|X|$ and for any $A \subseteq Z$ such that $|A| \leq \epsilon\,|Z|$. The first asymptotical result for the number of the number of independent sets in regular bipartite graphs obtained with the help of container idea concerned the n-cube. Let B^n be the n-cube, which is n-regular graphs on $N = 2^n$ vertices. In [10] the following assertion was proved

Theorem 8.

$$I(B^n) \sim 2\sqrt{e}2^{2^{n-1}} = 2\sqrt{e}2^{N/2} \; . \tag{18}$$

In [16] the following theorem was proved.

Theorem 9. *Let a bipartite (n,k,θ)-graph $G=(X,Z;E)$ be a two-sided$(1/2,\delta)$-expander and z be maximum of solutions of the equation $x = \log_2(2ex/c\delta)$. Then for large enough n and k*

$$2^{|X|}+2^{|Z|}-1 \le I(G) \le \left(2^{|X|}+2^{|Z|}\right)\left(1+2^{-k\delta/z+O\left(\sqrt{k}\log k+\theta\right)}\right) \ . \qquad (19)$$

The proof of the upper bound in (19) is based on the idea that X and Z are "main" containers covering almost all independent sets of G. The number of remaining independent sets does not exceed

$$\left(2^{|X|}+2^{|Z|}\right)2^{-k\delta/z+O\left(\sqrt{k}\log k+\theta\right)}.$$

This follows from smallness of size and number of containers covering these sets.

Corollary 2. *Let $G_n = (X_n,Z_n;E_n)$ be a sequence of bipartite $(n,k(n),\theta_n)$-graphs which are simultaneously $(1/2,\delta_n)$-expanders. Suppose*

$$k(n)\delta_n \to \infty \quad \text{and} \quad \theta_n/k(n)+k(n)^{-1/2}\log k(n) \to 0 \qquad (20)$$

as $n\to\infty$. Then

$$I(G_n) \sim 2^{|X_n|}+2^{|Z_n|} \ . \qquad (21)$$

Note that the corollary 2 means that the following statement holds: If a sequence $G_n = (X_n,Z_n;E_n)$ satisfies the condition of 2, then the portion of independent sets of G_n intersecting both X_n and Z_n tends to 0 as $n\to\infty$. The similar assertions plays a part in describing the phase transition processes (see, for example, citeBB and citeST).

A bipartite graph $G=(X,Z;E)$ is called *boundary $(\epsilon, delta)$-expander*, if

$$|A| \le |\partial A|(1-\delta) \qquad (22)$$

for all $A\subseteq X$ with $|\partial A| \le \lceil \epsilon|Z|\rceil$.

Let (κ,ν,q,p,λ) be a tuple of numbers with κ, ν, λ integers, and q, p real. A bipartite graph $\Gamma=(X,Z;E)$ such that

$$(1)\quad \min_{v\in X}\sigma(v)=\kappa, \qquad (2)\quad \max_{v\in Z}\sigma(v)=\nu, \qquad (3)\quad \min_{v\in Z}\sigma(v)=\lambda,$$

will be called the (κ,ν,q,p,λ)*–graph*, if the following inequalities hold

$$(4)\quad \max_{v\in X\cup Z}\sigma(v)\le\kappa^p; \qquad (5)\quad \max_{\substack{u,v\in X\\ v\ne u}}\left|\partial u\cap\partial v\right|\le q.$$

If some of these conditions fails or are not sufficient, the corresponding coordinates in the tuple will be replaced by "-". For example, if the properties 1, 5 hold and the remaining maybe are not, we say about a $(\kappa,-,q,-,-)$-graph.

A bipartite graph $G=(X,Z;E)$ is called *one-sided (ε,δ)– expander* if

$$|A| \le |\partial A|(1-\delta)$$

for any subset $A \subseteq X$ such that

$$|A| \leq \lceil \varepsilon |X| \rceil.$$

A bipartite graph $G = (X, Z; E)$ is called *one-sided boundary* (ε, δ)*– expander* if

$$|A| \leq |\partial A|(1 - \delta)$$

for any $A \subseteq X$ such that

$$|\partial A| \leq \lceil \varepsilon |Z| \rceil.$$

Let $G = (X, Z; E)$ be a (Δ, κ, q, p)–graph. Set

$$r_0 = r_0(G) = \lceil \kappa / q \rceil, \qquad g_1 = g_1(G) = \min_{A \subseteq X : |A| > r_0} |\partial A|,$$

$$g_2 = g_2(G) = \kappa^3 \log^{-7} \kappa, \qquad \varepsilon_1 = \varepsilon_1(G) = g_2 / |Z|,$$
$$\delta_1 = \delta_1(G) = \kappa^{-1} \log^2 \kappa, \qquad \delta_0 = \delta_0(G) = \kappa^{-2} \log^9 \kappa.$$

A $(\kappa, -, q, p, -)$–graph $G = (X, Z; E)$ will be called a (Δ, κ, q, p)–expander, if it is a boundary $(\varepsilon_1, \delta_1)$–expander, a one-sided $(1, \delta_0)$–expander and the following inequalities hold:

$$\kappa / \sqrt{\log \kappa} \leq \min_{v \in Z} \sigma(v) \leq \max_{v \in Z} \sigma(v) \leq \kappa, \tag{23}$$

$$|X| \leq 2^{(\Delta+1)\kappa - 2\log^2 \kappa}. \tag{24}$$

A sequence $(v_1, v_2, ..., v_k)$ of vertices in a graph $G = (V, E)$ is called a *d–chain* if the distance between v_i and v_{i+1} in G does not exceed d for $1 \leq i < k$. A set $A \subset V$ will be called *connected with distance* d, if any two vertices of A are connected by d-chain. A subset $B \subseteq A$ is called d-component of A, if B is d–connected but $B \bigcup \{v\}$ is not connected for any $v \in A \setminus B$. Denote by $I_\Delta(G)$ the number of independent sets A in a bipartite graph $G = (X, Z; E)$ with the property that the size of each 2-component of $A \bigcap X$ does not exceed Δ. In [15] the following statement is proved

Theorem 10. *Let $G = (X, Z; E)$ be total (Δ, κ, q, p)-expander. Then*

$$I_\Delta(G) \leq I(G) \leq \left(1 + O\big(2^{-\frac{3}{2}\log^2 \kappa}\big)\right) I_\Delta(G). \tag{25}$$

Theorem 11. *Let $G = (X, Z; E)$ be a total $(1, \kappa, q, p)$-expander. Then*

$$I(G) = \left(1 + O\big(2^{-\frac{3}{2}\log^2 \kappa}\big)\right) 2^{|Z|} \exp\Big\{ \sum_{v \in X} 2^{-|\partial\{v\}|} \Big\}. \tag{26}$$

If $G = (X, Z; E)$ be a total $(2, \kappa, q, p)$-expander. Then

$$I(G) = 2^{|Z|} \exp\Big\{ \big(1 + O(\kappa^2 2^{-\kappa})\big) \sum_{v \in X} 2^{-|\partial\{v\}|} \Big\}. \tag{27}$$

Theorem 11 gives the asymptotically sharp estimate for many important cases.

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