ELEMENTARY ANALYSIS L
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ELEMENTARY STRUCTURAL ANALYSIS

PREFACE

In this book the authors have covered that subject matter which they believe to be of importance in a well-organized sequence of undergraduate courses dealing with the theory of structures for civil engineers. Some material is also included which, because of restricted schedules, may have to be left out of formal assignments. Such portions of the book include the material in fine print throughout the text; Chap. 16, which deals with structures not directly related to civil engineering; and Chaps. 17–19, which treat the analysis of structures by means of models. Such material will, however, serve to develop the structural knowledge of the student whose interest has become genuinely aroused, and it will be valuable in connection with thesis work.

The material covered by this book has been confined almost entirely to methods of stress analysis. Design procedure, where mentioned at all, is covered only incidentally, since the length of the book as written indicates the desirability of a separate book dealing with design, and since our practice, which is also followed in a number of other schools, consists of teaching the theory of stress analysis and the principles of design as separate subjects.

The authors have attempted in this presentation to accomplish two results: (1) to tie in the various procedures of structural analysis with the principles of applied mechanics on which they are based, thus showing that the theory of structural analysis is but one phase of advanced applied mechanics, and (2) to show that the methods of analysis derived for civil engineering structures are applicable in principle to structures lying outside the field of practice of most civil engineers. With these thoughts in mind, they hope this book will prove to be of help both to students of structural engineering and to young practicing engineers.

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Both authors are also deeply grateful to those responsible for their training in structural engineering, particularly to Profs. Charles Milton Spofford and Charles Church More. They likewise acknowledge with appreciation the help they have received from their colleagues, Profs. W. M. Fife and Eugene Mirabelli, and the late Prof. J. D. Mitsch.

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CAMBRIDGE, MASS.
March, 1948

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CHAPTER 1

INTRODUCTION

1·1 Engineering Structures. The design of bridges, buildings, towers, and other fixed structures is very important to the civil engineer. Such structures are composed of interconnected members and are supported in a manner such that they are capable of holding applied external forces in static equilibrium. A structure must also hold in equilibrium the gravity forces that are applied as a consequence of its own weight. A transmission tower, for example, is acted upon by its own weight, by wind and ice loads applied directly to the tower, and by the forces applied to the tower by the cables that it supports. The members of the tower must be so arranged and designed that they will hold these forces in static equilibrium and thus transfer their effects to the foundations of the tower.

There are many kinds of structures in addition to those mentioned above. Dams, piers, pavement slabs for airports and highways, penstocks, pipe lines, standpipes, viaducts, and tanks are all typical *civil engineering* structures. Nor are structures of importance only to the *civil engineer.* The structural frame of an aircraft is important to the *aeronautical engineer;* the structure of a ship receives particular attention from the *naval architect;* the *chemical engineer* is concerned with the structural design of high-pressure vessels and other industrial equipment; the *mechanical engineer* must design machine parts and supports with due consideration of structural strength; and the *electrical engineer* is similarly concerned with electrical equipment and its housing.

The analysis of all these structures is based, however, on the same fundamental principles. In this book the illustrations used to demonstrate the application of these principles are drawn largely from civil engineering structures, but the methods of analysis described can be used for structures that are important in other branches of engineering.

1·2 General Discussion of Structural Design. A structure is designed to perform a certain function. To perform this function satisfactorily it must have sufficient strength and rigidity. Economy and good appearance are further objectives of major importance in structural design.

The complete design of a structure is likely to involve the following five stages:

1. Establishing the general layout to fit the functional requirements of the structure

2. Consideration of the several possible solutions that may satisfy the functional requirements
3. Preliminary structural design of the various possible solutions
4. Selection of the most satisfactory solution, considering an economic, functional, and aesthetic comparison of the various possible solutions
5. Detailed structural design of the most satisfactory solution

Both the preliminary designs of stage 3 and the final detailed design of stage 5 may be divided into three broad phases, although in practice these three phases are usually interrelated. First, the loads acting on the structure must be determined. Next, the maximum stresses in the members and connections of the structure must be analyzed. Finally, the members and connections of the structure must be dimensioned, *i.e.*, the make-up of each part of the structure must be determined.

That these three steps are interrelated may be seen from considerations such as the following: The weight of the structure itself is one of the loads that a structure must carry, and this weight is not definitely known until the structure is fully designed; in a statically indeterminate structure, the stresses depend on the elastic properties of the members, which are not known until the main members are designed. Thus, in a sense, the design of any structure proceeds by successive approximations. For example, it is necessary to assume the weights of members in order that they may be properly designed. After the structure is designed, the true weights may be computed; and unless the true weights correspond closely to those assumed, the process must be repeated.

In designing a structure, it is important to realize that each part must have sufficient strength to withstand the maximum stress to which it can be subjected. To compute such maximum stresses, it is necessary to know, not only *what* loads may act, but the exact *position* of these loads on the structure that will cause the stress under consideration to have its maximum value.

Thus, when a railroad locomotive crosses a bridge, a given portion of the bridge receives its maximum stress with the locomotive at a given position on the bridge. A second part of the structure may be subjected to its maximum stress with the locomotive in another position.

In this book, the emphasis is placed on the stress analysis of structures. But, in order to discuss stress analysis satisfactorily, it is desirable to give some attention to the loads acting on a structure and to the design of members and connections.

1·3 Dead Loads. The dead load acting on a structure consists of the weight of the structure itself and of any other immovable loads that are constant in magnitude and permanently attached to the structure. Thus, for a highway bridge, the dead load consists of the main supporting trusses or girders, the floor beams and stringers of the floor system, the roadway slabs, the curbs, sidewalks, fences or railings, lampposts, and other miscellaneous equipment.

Since the dead load acting on a member must be assumed before the member is designed, one should design the members of a structure in such a sequence that, to as great an extent as is practicable, the weight of each member being designed is a portion of the dead load carried by the next member to be designed. Thus, for a highway bridge, one would first design the road slab, then the stringers that carry the slab loads to the floor beams, then the floor beams that carry the stringer loads to the main girders or trusses, and finally the main girders or trusses.

In designing a member such as a floor slab, stresses due to dead loads are likely to be only a small percentage of the total stress in a member, so that, even if dead loads are not very accurately estimated, the total stress can be predicted with fair accuracy and hence the first design be quite satisfactory. For main trusses and girders, however, the dead loads constitute a greater portion of the total load to be carried, so that it is more important to make a reasonably accurate first estimate of dead weights. Often data concerning the dead weights of other similar structures will serve as a guide to the designer. Many investigations have been carried out with the purpose of presenting such data in a convenient form.[1] It should be emphasized, however, that the original dead-weight estimate is tentative, whatever the source of the data may be. After a

Table 1·1

Material	Weight, lb per cu ft
Steel or cast steel	490
Cast iron	450
Aluminum alloys	175
Timber (treated or untreated)	50
Concrete (plain or reinforced)	150
Compacted sand, earth, gravel, or ballast	120
Loose sand, earth, and gravel	100
Macadam or gravel, rolled	140
Cinder filling	60

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[1] The student is referred to p. 72 of "Structural Theory" (John Wiley & Sons, Inc., New York, 1942) by H. SUTHERLAND and H. L. BOWMAN for tables giving the weights of roof trusses and to p. 84 of the same book for an excellent summary of formulas giving the weights of bridges. Charts dealing with the weights of railroad bridges, highway bridges, and signal bridges are given in Chap. I of C. M. Spofford's "Theory of Structures," 4th ed., McGraw-Hill Book Company, Inc., New York, 1939.

structure is designed, its actual dead weight should be accurately computed and the stress analysis and design revised as necessary. This is necessary for safety and desirable for economy.

If the dimensions of a structure are known, dead loads may be computed on the basis of unit weights of the materials involved. Unit weights for some of the materials commonly used in engineering structures are given in Table 1·1.

Unit weights for other materials are readily available in many books and handbooks.[1]

1·4 Live Loads—General. As contrasted to dead loads, which remain fixed in both magnitude and location, it is usually necessary to consider live loads, *i.e.*, loads that vary in position. It is sometimes convenient to classify live loads into movable loads and moving loads. Movable loads are those which may be moved from one position to another on a structure, such as the contents of a storage building. They are usually applied gradually and without impact. Moving loads are those which move under their own power, such as a railroad train or a series of trucks. They are usually applied rather rapidly and therefore exert an impact effect on the structure.

When live loads are involved, attention must be given to the placing of such loads on a structure so that the stress in the structural member or connection under consideration will have its maximum possible value. Thus, while we speak of dead stresses due to dead loads, we refer to maximum live stresses due to live loads.

1·5 Live Loads for Highway Bridges. The live load for highway bridges consists of the weight of the applied moving load of vehicles and pedestrians. The live load for each lane of the roadway consists of a train of heavy trucks following each other closely. The weight and weight distribution of each truck vary with the specification under which one designs, but a typical example is afforded by the H-series trucks specified by the American Association of State Highway Officials (AASHO).

These H-series trucks are illustrated in Fig. 1·1. They are designated H, followed by a number indicating the gross weight in tons for the standard truck. The choice as to which of the H-series trucks shall be used for the design of a given structure depends on circumstances such as the importance of the bridge and the expected traffic. Actually, the traffic over a highway bridge will consist of a multitude of different types of vehicles. It is designed, however, for a train of standard trucks, so chosen that the bridge will prove safe and economical in its actual performance.

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[1] The student is referred, for example, to the section on Weights and Specific Gravities in "Steel Construction," American Institute of Steel Construction, New York.

It is seen that the loading per lane of roadway consists of a series of concentrated wheel loads. The stress analysis involved in computing maximum live stresses due to a series of concentrated live loads may become rather complicated. Under some conditions it is permissible to substitute for purposes of stress analysis an equivalent loading consisting of a uniform load per foot of lane, plus a single concentrated load. Thus, for the H-20 loading, the equivalent live load consists of a uniform load of 640 lb per lin ft of lane, plus a concentrated load of either 18,000 lb or 26,000 lb, depending on whether live moments or shears, respectively, are being computed. This equivalent live load is not exactly equivalent to the series of concentrated wheel loads, but it permits a simpler computation of maximum stresses that correspond closely enough to those which would be computed from the actual loads to be used for design purposes.

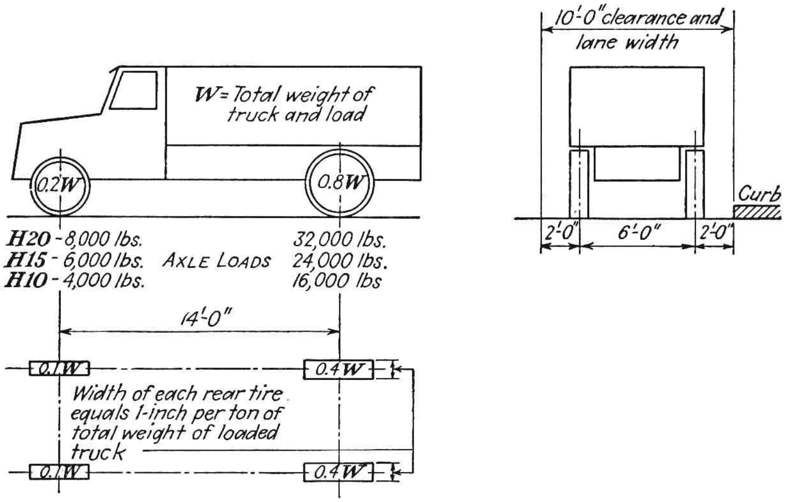
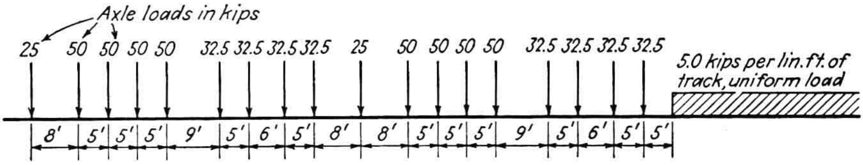


FIG. 1·1

It is often necessary to design a highway bridge to carry electric-railway cars. Specifications define the wheel loads and spacings to be used for this purpose.

1·6 Live Loads for Railroad Bridges. The live load for railroad bridges consists of the locomotives and cars that cross them. The live load for each track is usually taken as that corresponding to two locomotives followed by a uniform load which represents the weight of the cars. To standardize such loadings a series of E-loadings was devised by Theodore Cooper. These loadings are designated by the letter E, followed by a number indicating the load in kips[1] on the driving axle. The loads on other axles always bear the same ratio to the load on the driving axle. The uniform load following the two locomotives always has an intensity per foot of track equal to one-tenth the load on the driving axle. The wheel spacings do not vary with the Cooper's rating. Figure 1·2 illustrates a Cooper's E-50 loading. Modern railroad bridges are designed for at least an E-60 loading and often for an E-70 or even a heavier loading. It should be noted that the wheel loads for these heavier Cooper's loadings can be obtained from Fig. 1·2 by direct proportion.



COOPERS E-50 LOADING

FIG. 1·2

Simplified equivalent loadings are sometimes used in place of actual wheel loadings to represent live loads on railroad bridges.

1·7 Live Loads for Buildings. Live loads for buildings are usually considered as movable loads of uniform intensities. The intensity of the floor loads to be used depends on the purpose for which the building is designed, as indicated in Table 1·2.[2]

Table 1·2

	Minimum Live Load, lb per sq ft
Human occupancy:	
Private dwellings, apartment houses, etc.	40
Rooms of offices, schools, etc.	50
Aisles, corridors, lobbies, etc., of public buildings	100
Industrial or commercial occupancy:	
Storage purposes (general)	250
Manufacturing (light)	75
Printing plants	100
Wholesale stores (light merchandise)	100
Retail salesrooms (light merchandise)	75
Garages	
All types of vehicles	100
Passenger cars only	80
Sidewalks	250 lb per sq ft or 8,000 lb concentrated, whichever gives the larger moment or shear

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[1] To facilitate computations, loads are usually given in units of kips, 1 kip being equal to 1,000 lb.

[2] "Steel Construction," *op. cit.*

When floors are to carry special live loads of known intensities greater than those suggested above, these special loads should of course be used in design.

1·8 Impact.

Unless live load is applied gradually, the distortion of the structure to which the live load is applied is greater than it would be if the live load were considered as a static load. Since the distortion is greater, the stresses in the structure are higher. The increase in stress due to live load over and above the value that this stress would have if the live load were applied gradually is known as impact stress. Impact stresses are usually associated with moving live loads. For purposes of structural design, impact stresses are usually obtained by multiplying the live-load stresses by a fraction called the impact fraction, which is specified rather empirically. The determination of a wholly rational fraction for this purpose would be very complicated, since it depends on the time function with which the live load is applied, the portion of the structure over which the live load is applied, and the elastic and inertia properties of the structure itself.

For highway bridges, the impact fraction I is given in the specifications of the AASHO by

$$I = \frac{50}{L + 125} \qquad \text{but not to exceed } 0.300 \qquad (1\cdot 1)$$

in which L is the length in feet of the portion of the span loaded to produce the maximum stress in the member considered. For example, suppose that the maximum live positive shear at the center of a 100-ft longitudinal girder of a highway bridge equals 1,000,000 lb and occurs with the live load extending over half the 100-ft span. Then the loaded length L is 50 ft; the impact fraction $I = 50/(50 + 125) = 0.286$; the impact shear is obtained by multiplying the live shear by the impact fraction and therefore equals $1{,}000{,}000 \times 0.286 = 286{,}000$ lb. The total effect of the live load, *i.e.*, live shear plus impact shear, is equal to 1,000,000 lb plus 286,000 lb, or 1,286,000 lb.

The Specifications for the Design and Construction of Steel Railway Bridges, published by the American Railway Engineering Association (AREA), treat impact as follows (note that impact percentage discussed equals 100 times the impact fraction as previously defined):

"To the maximum computed static live-load stresses, there shall be added the impact, consisting of

a. The lurching effect:

A percentage of the static live-load stress equal to . . . . $\frac{100}{S}$

S = spacing, in feet, between centers of longitudinal girders, stringers, or trusses; or length, in feet, of floor beams or transverse girders.

b. The direct vertical effect:

With steam locomotives (hammer blow, track irregularities, and car impact) a percentage of the live-load stress equal to

For L less than 100 ft. . . . . $100 - 0.60L$

For L 100 ft or more . . . . $\frac{1{,}800}{L - 40} + 10$

With electric locomotives (track irregularities and car impact), a percentage of static live-load stress equal to . . . . $\frac{360}{L} + 12.5$

L = length, ft, center to center of supports for stringers, longitudinal girders, and trusses (chords and main members)

or L = length of floor beams or transverse girders, ft, for floor beams, floor-beam hangers, subdiagonals of trusses, transverse girders, and supports for transverse girders"

To illustrate the application of the foregoing impact specification, let us assume that the longitudinal girder described in the previous example is one of the two main girders of a steam-railroad bridge and that these two girders are spaced at 18 ft center to center. Then, for the lurching effect, since $S = 18$, the percentage impact equals $^{100}/_{18} = 5.5\%$; for the direct vertical effect, since $L = 100$, the percentage impact equals $1{,}800/(100 - 40) + 10 = 40.0\%$ (note that in this case $L = 100$ because this is the span of the girder, whereas in the previous example we used $L = 50$ because the loaded length of the girder was 50 ft); thus the total percentage impact equals $5.5 + 40.0 = 45.5\%$; the impact shear equals $1{,}000{,}000 \times 0.455 = 455{,}000$ lb; the total effect of the load, *i.e.*, live shear plus impact shear $= 1{,}000{,}000 + 455{,}000 = 1{,}455{,}000$ lb.

Other specifications give still other rules for determining impact, but the two methods discussed are perhaps the most important of those in common use. They illustrate, moreover, the type of impact equations specified elsewhere.

It is usually unnecessary to consider impact stresses in designing for movable live loads such as the live loads for buildings. Moreover, when a structure is designed of timber, impact is often ignored. This is largely because timber, as a material, is much stronger in resisting loads of short duration than in resisting permanently applied loads, and it therefore can use this reserve of strength to carry impact loads.

1·9 Snow and Ice Loads. Snow loads are often of importance, particularly in the design of roofs. Snow should be considered as a movable load, for it will not necessarily cover the entire roof, and some of the members supporting the roof may receive maximum stresses with the snow covering only a portion of the roof. The density of snow, of course, will vary greatly, as will the fall of snow to be expected in different regions. In a given locality, the depth of snow that will gather on a given roof will depend on the slope of the roof and on the roughness of the roof surface. On flat roofs in areas subjected to heavy snowfalls, snow load may be as large as 45 lb per sq ft. Whether or not snow and wind loads should be assumed to act simultaneously on a roof is problematical, since a high wind is likely to remove much of the snow.

Ice loads may also be of importance, as, for example, in designing a tower built up of relatively small members which have proportionately large areas on which ice may gather. Ice having a density equal approximately to that of water may build up to a thickness of 2 or more inches on such members. It may also build up to much greater thicknesses, but when it does it is apt to contain snow or rime and hence have a lower density. When ice builds up on a member, it alters the shape and the projected area of the member. This should be considered in computing wind loads acting on members covered with ice.

1·10 Lateral Loads—General. The loadings previously discussed usually act vertically, although it is not necessary that live loads and their associated impact loads shall act in that direction. In addition, there are certain loads that are almost always applied horizontally, and these must often be considered in structural design. Such loads are called lateral loads. We shall now consider some of the more important kinds of lateral loads.

Wind loads, soil pressures, hydrostatic pressures, forces due to earthquakes, centrifugal forces, and longitudinal forces usually come under this classification.

1·11 Wind Loads. Wind loads are of importance, particularly in the design of large structures, such as tall buildings, radio towers, and long-span bridges, and for structures, such as mill buildings and hangars, having large open interiors and walls in which large openings may occur. The wind velocity that should be considered in the design of a structure depends on the geographical location and on the exposure of the structure. For most locations in the United States, a design to withstand a wind velocity of 100 mph is satisfactory.

The AASHO specifies that the wind force on a highway bridge shall be assumed as a movable horizontal load equal to 30 lb per sq ft acting on 1½ times the area of the structure as seen in elevation, including the floor system and railings, and on one-half of the area of all trusses or girders in excess of two in the span. This amounts to specifying 30 lb for each square foot of projected area on the windward truss or girder but only half that amount for other trusses or girders, since they are partly shielded from the wind by the windward portion of the structure.