

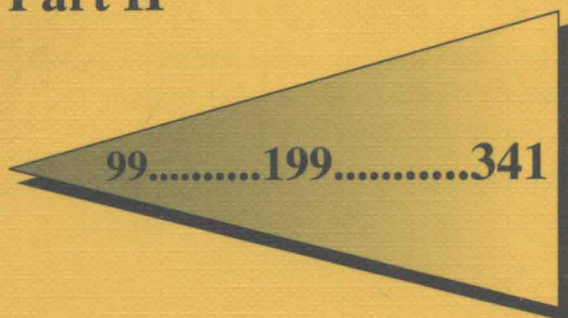
Lecture Notes in Mathematics

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V. P. Havin N. K. Nikolski (Eds.)

Linear and Complex Analysis Problem Book 3

Part II



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Linear and Complex Analysis Problem Book 3

Part II

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§ 1. Lecture Notes aim to report new developments - quickly, informally, and at a high level. The texts should be reasonably self-contained and rounded off. Thus they may, and often will, present not only results of the author but also related work by other people. Furthermore, the manuscripts should provide sufficient motivation, examples and applications. This clearly distinguishes Lecture Notes manuscripts from journal articles which normally are very concise. Articles intended for a journal but too long to be accepted by most journals, usually do not have this "lecture notes" character. For similar reasons it is unusual for Ph. D. theses to be accepted for the Lecture Notes series.

§ 2. Manuscripts or plans for Lecture Notes volumes should be submitted (preferably in duplicate) either to one of the series editors or to Springer-Verlag, Heidelberg. These proposals are then refereed. A final decision concerning publication can only be made on the basis of the complete manuscript, but a preliminary decision can often be based on partial information: a fairly detailed outline describing the planned contents of each chapter, and an indication of the estimated length, a bibliography, and one or two sample chapters - or a first draft of the manuscript. The editors will try to make the preliminary decision as definite as they can on the basis of the available information.

§ 3. Final manuscripts should preferably be in English. They should contain at least 100 pages of scientific text and should include

- a table of contents;
- an informative introduction, perhaps with some historical remarks: it should be accessible to a reader not particularly familiar with the topic treated;
- a subject index: as a rule this is genuinely helpful for the reader.

Further remarks and relevant addresses at the back of this book.

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PREFACE

In 1978 we published a book entitled "99 unsolved problems of Linear and Complex Analysis" (Volume 81 of "Zapiski nauchnyh seminarov LOMI"; English translation in Journal of Soviet Mathematics, **26** (1984), No. 5). It consisted of short problem articles sent by mathematicians of many countries in response to our invitation headed by the following lines:

'Which problems of Linear and Complex Analysis would you propose to your numerous colleagues if you had a possibility to address them all simultaneously?'

The editorial board of "Investigations in Linear Operators and Function Theory" edited by the Leningrad Branch of the V. A. Steklov Mathematical Institute of the Academy of Sciences of the USSR (LOMI) has decided to put this question to a hundred specialists joined in an invisible collective working on the common circle of problems and to publish their answers as "Collection of unsolved Problems of Linear and Complex Analysis". Such "Collection ..." may be useful not only to its authors but to their colleagues including the beginner analysts.'

It seems we were right. In 1984 the second edition appeared. Instead of 99, its title mentioned 199 problems.* Both editions have interested many colleagues. A big part of problems is now solved, but time has brought with it more new problems and questions. That is why one more (yet again enlarged) publication of the Collection seemed desirable. Its third version reproduces a large part of the second with addition of new problems and of information concerning the old ones. Our motives, the style and the general direction of the book were described in detail in the preface to the 2nd edition. Excerpts from that preface are reproduced below. There is not much to add. We only make several remarks on some *new* moments.

The first is the increase of the size. Instead of 13, the number of chapters is now 20; the total number of problems is 341 (they were 199 in 1983). The book consists of two volumes (both preceding editions were one-volume books). This growth can be explained by the abundance of new results and ideas in Spectral Operator-and-Function Theory. Our purely operator-theoretic chapters are now six (namely, Chapters 4-9); they were only two in the second edition. This fact bears witness to the intense activity of operator theorists gaining new areas and discovering new connections. So much for Operator Theory, a key subject underlying and unifying the whole book; the word "Linear" in the title refers mainly to this theme. As to the word "Complex", this part of the book is also enriched by the inclusion of new chapters 13, 18, 19, not to mention new problems gathered under the "old" titles.

The second moment is the new technique of preparation of the text. Both preceding versions of the book (as a whole) were prepared by its editors (though assisted by a collective of collaborators). This time every chapter had its own editor (or editors). The

* *Linear and complex Analysis Problem Book. 199 Research Problems.* Lect. Notes Math. **1043**, Springer-Verlag, 1984

PREFACE

initiative and organization and coordination problems were ours, a difficult task, to say the least (see also the explanations in *Acknowledgements* below). Almost all chapters are provided with introductions by the chapter editors. In these introductions they try to help the reader to grasp the general direction of the chapter, to record additional bibliography, and sometimes also to explain their point of view on the subject or to make historical comments.

Chapters are divided into sections. They total 341 (in 1984 and 1978 there were 199 and 99 respectively). We treat the words “section” and “problem” as synonymous for the purposes of classification (though a section may contain more than one problem). “Problem 1.25” means the 25-th section of the first chapter; “Problem 1.26 old” (“Problem 1.26 v. old”) mean that Problem 1.26 is reproduced from the 1984 edition (1978 edition, respectively) and has not been completely solved (as far as we know); “Problem S.1.27” means the 27-th section of Chapter 1 representing a solution of a problem from the previous edition. Some notation (used sometimes without further explanations) is indicated at the end of the book. A subject index and an author index are provided. We took the liberty to modify the section titles in “Contents” to make it shorter.

And **the third moment** in which this edition differs from its predecessors is the unfavorable situation in former Soviet mathematics caused by the well-known events that interfered brutally with our project just when it was started and could not be stopped. As we already mentioned, both preceding versions were prepared by “an informal editorial board” consisting for the most part of the members of the (then) Leningrad Seminar of the Spectral Function-and-Operator Theory. It was a numerous and energetic group of enthusiasts whose participation ensured the success of the undertaking. In 1990, proposing the project of the 3rd edition to our colleagues throughout the world, we hoped that we still could rely upon the same group. We also reckoned with the technical group of LOMI (now POMI), the Leningrad Branch of the Steklov Institute, remembering our experience of 1978 and 1983. But when our project was really started the situation changed dramatically. Our group melted away and soon became unable to achieve a joint effort, and POMI couldn’t support us anymore (such things as, say, keyboarding, paper and so on, are now a big problem in Russia). In fact, the project turned out to be a purely private enterprise of the editors.

But now, after all, thanks to generous help of our friends and colleagues (see *Acknowledgements* below) this book lies before its reader. We hope that it will serve “the invisible community” of analysts working in Linear and Complex Analysis and will help them in solving and discovering many new and exciting problems.

FROM THE PREFACE TO THE PREVIOUS EDITION

This volume offers a collection of problems concerning analytic functions, linear function spaces and linear operators.

The most exciting challenge to a mathematician is usually not what he understands, but what still eludes him. This book reports what eluded a rather large group of analysts in 1983 whose interests have a large overlap with those of our Seminar.* Consequently,

*i.e., the Seminar on Spectral Theory and Complex Analysis consisting principally of mathematicians working in the Leningrad Branch of the V. A. Steklov Mathematical Institute (LOMI) and in Leningrad University.

PREFACE

therefore, the materials contained herein are chosen for some sort of mild homogeneity, and are not at all encyclopaedic. Thus, this volume differs markedly from some well-known publications which aim at universality. We confine ourselves to the (not very wide) area of Analysis in which we work, and try—within this framework—to make our collection as representative as possible. However, we confess to obeying the Bradford law (the exponential increase of difficulties in obtaining complete information). One of our purposes is to publish these problems promptly, before they lose the flavour of topicality or are solved by their proposers or other colleagues.

This Problem Book evolved from the earlier version published as volume 81 of “Zapiski Nauchnyh Seminarov LOMI” in 1978 (by the way, much of the work arising from the above mentioned Seminar is regularly published in this journal). It is now twice the size, reflecting the current interests of a far wider circle of mathematicians. For five years now the field of interests of the “invisible community” of analysts we belong to has enlarged and these interests have drifted towards a more intense mixing of Spectral Theory with Function Theory. And the volume as a whole is rather accurate reflection of this process.

We are pleased that almost a half of the problems recorded in the first edition, 50 of 99, have been solved, partly or completely. The problems of 1978 (we call them “old” problems) are sometimes accompanied with commentary reporting what progress towards their solution has come to our attention. Moreover, those “old” problems which have been almost completely solved are assembled under the title “SOLUTIONS” at the end of each chapter (including information as to how and by whom they have been solved).

When we decided to prepare this new edition, we solicited the cooperation of many colleagues throughout the world. Some two hundred responded with ample and helpful materials, doubling the number of collaborators of the first edition. Their contributions ranged from carefully composed articles (not always short) to brief remarks. This flow it was our task to organize and to compress into the confines of a single volume. To effectuate this we saw no alternative to making extensive revisions (more exactly, abbreviations) in the texts supplied. We hope that we have succeeded in preserving the essential features of all contributions and have done no injustice to any.

At first sight the problems may appear very heterogeneous. But they display a certain intrinsic unity, and their approximate classification (i.e. division into chapters) did not give us much trouble. We say “approximate” because every real manifestation of life resists systematization. Some problems did not fit into our initial outline and so some very interesting ones are collected under the title “Miscellaneous Problems” . . .

EDITORS

PREFACE

ACKNOWLEDGEMENTS

The publication of these volumes would have been impossible without the generous and self-denying help of our colleagues. To explain this, we start by describing some of the obstacles we had to overcome.

Our instructions, sent to all chapter editors, were very thorough and detailed. They contained a lot of technical explanations, and \TeX -macros* (prepared by A. V. SUDAKOV). Unfortunately, they were largely ignored or neglected (except by those editors who worked in LOMI). We got a huge collection of texts in disorder; they required enormous work to coordinate and unify them (which could have been dispensed with almost completely if our instructions had been followed). We had to typeset hundreds of pages anew; many solved problems had to be detected and separated from the unsolved ones. Innumerable instances “to appear” from the preceding edition had to be replaced by correct bibliographical data (actually, some 1000 new references have been added!). Dozens of new commentaries had to be written (some chapter editors practically didn’t revise “old” problems). And we had no technical staff necessary to turn a motley set of chapters into a book.

Meanwhile, as a result of the deteriorating situation and the decay of all structures in the former USSR (a malignant process whose rapidity we underestimated starting the project), the collective we could rely upon had disappeared and its members dispersed throughout the world. The e-mail became the only way of communication between them (including the authors of these lines). The situation looked desperate and the project could not have been rescued without assistance of our colleagues. This assistance was really invaluable to us. These skilled mathematicians in their most active years put their research aside and did a huge amount of purely technical work, making it possible for this book, to see the light of day.

The job has been done by three consecutive “technical teams”. The first was headed by A. A. BORICHEV, the second by V. V. KAPUSTIN, the third by V. I. VASYUNIN, whose contribution to the project was especially great. The teams spent much time and energy retyping the text, tidying it up, hunting out inconsistencies and omissions to make the book a well-organized and handy tool for the user. We are not sure the debugging process has been completed and all defects have been discovered and removed: we apologize to the reader for remaining flaws. Our possible excuse is the fact that collectives capable of doing gratuitous work of such proportions existed only in the USSR, the country where the project was conceived, but which disappeared just at the final (and the hardest) stage of editing.

We are happy to thank the Mathematical Department of the University Bordeaux-I for its financial support. Its Graduate School (then headed by J.-L. JOLY and P. FABRIE) made possible the work of V. Vasyunin, putting at his disposal all necessary technical facilities.

We hope that this introduction is sufficient to explain why our gratitude to all who contributed to the book is especially deep and sincere. It is our duty and pleasure to name the following colleagues.

*This book was typeset using $\mathcal{A}\mathcal{M}\mathcal{S}$ - \TeX macro package.

PREFACE

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We are indebted very much to all of them as well as to the Mathematical Editorial Board of Springer-Verlag for patience and support of our project.

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