HADAMARD TRANSFORM OPTICS

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PREFACE

Over the past ten years a new technique known as *Hadamard transform optics* has been developed in spectroscopy and imaging. The basic idea is as follows. In order to determine the spectrum of a beam of light, instead of measuring the intensity at each wavelength separately, the spectral components are combined in groups and the total intensity of each group is measured. Thus the different wavelength components are *multiplexed*. As a result the spectrum is determined much more accurately. The best multiplexing schemes are based on Hadamard matrices, and in effect measure the Hadamard transform of the spectrum. In the most favorable cases, the mean square error per frequency component is reduced by a factor proportional to n, if there are n frequency components to be measured.

Exactly the same technique may be used to increase the efficiency of *imagers* i.e., devices for reconstructing an image or picture. Again the best multiplexing schemes are based on Hadamard matrices, and so we use the name Hadamard transform optics to describe all these instruments.

Our purpose in writing this book has been to collect in one place much of the work on Hadamard encoded optical instruments that has appeared in the literature. Our aim, in the first place, is to provide a comprehensive description of the techniques developed to date, together with a unified mathematical treatment that should facilitate comparisons between different classes of instruments. With this approach, we are able to treat singly encoded Hadamard transform spectrometers in very much the same way as encoded imaging devices, and are able to make comparisons between the advantages offered by singly and multiply encoded instruments designed for a wide variety of purposes.

Hadamard encoded instruments are discretely multiplexed devices. The traditional mathematical treatment uses matrix equations and has differed substantially from the integral equation approach used in analyzing Fourier transform spectrometers. This difference in approach has made it difficult to compare Fourier and Hadamard transform spectrometers, or Fourier transform instruments and grill spectrometers of the type developed by Girard and others during the 1960s. In our approach we have found the matrix formalism capable of treating accurately not only Hadamard encoded instruments, but also Michelson interferometric spectrometers and Girard grill instruments. A side product of our analysis is a general method for evaluating the relative merits of different classes of monochromators and multiplexing spectrometers under a variety of different operating conditions. In a number of places in the book, these comparisons are put into an absolute perspective by means of optimality theorems. Some of these theorems are new; others have appeared in a variety of papers published in the optical or mathematical literature.

Chapter 1 outlines the basic ideas of Hadamard encoded optical instruments and shows how the problem of designing these instruments is related to the theory of weighing designs. Chapter 2 gives a description of other methods of multiplexing, and Chapter 3 shows how all such instruments can be described by a uniform treatment based on, but also extending, the theory of weighing designs. In Chapter 4 we make use of the results derived up to this point to compare the use of different types of spectrometers or imaging instruments under a variety of operating conditions. This allows us to decide when it makes sense to use multiplexing techniques and when they are to be avoided.

In Chapters 5 and 6 we examine imperfect instruments. Chapter 5 concentrates on imperfections arising from the optical components—diffraction, aberrations, distortion of images, and other purely optical defects. Chapter 6 augments this listing of deficiencies with a set of difficulties that can arise from imperfections in the construction of the encoding mask or in its motion during a set of measurements. In both chapters we not only identify the difficulties but, wherever possible, suggest means for dealing with them to remove at least some of the problems encountered. Many of these remedial steps have not yet been tried and amount to recipes based on the predictions of our theoretical approach. Other procedures have in fact been tried out, and in those cases we present the results obtained.

Chapter 7 completes the main text of the book with a series of past and potential applications in chemistry, astronomy, medicine, and other areas. We feel that Hadamard transform techniques have valuable contributions to make in a variety of applications. That does not mean, however, that Hadamard encoded instruments will always afford advantages. Chapter 4 is intended to dispel this notion not only for Hadamard techniques, but also for Fourier transform instruments. It is always necessary to weigh the advantages of multiplexing against the disadvantages, and one of the main purposes of this book is to provide methods for rationally arriving at a correct decision. Chapter 7 applies the methods developed in preceding chapters to concrete situations, and shows where Hadamard encoded instruments are likely to prove useful.

The appendix describes a number of mathematical concepts encountered in the use of discretely encoded instruments. Several methods of constructing Hadamard codes are described, and the fast Hadamard and Fourier transforms are explained. An extension of the fast Hadamard transform to the fast transformation of data encoded by means of an S-matrix—the code most often employed in the instruments described in this book—is also given. An extensive bibliography concludes the book.

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We are grateful to the editors of *Applied Optics* and to the Optical Society of America for permission to make use of a large number of figures that previously appeared in that journal. Chapter 6 quotes extensively from an article by M.-H. Tai and the authors which also appeared in *Applied Optics*. We should like to thank the editors of John Wiley and Sons for permission to reproduce Figs. 2.11

[1] See Mathlab Group (1977).
[2] See Fox (1977).

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CONTENTS

Chapter 1

An Introduction to Optical Multiplexing Techniques

This chapter begins with a description of multiplexing methods, shows the connections with weighing designs, and describes the best masks for use in optical instruments and the improvements produced by them.

1.1. Introduction

Color photography is so much a part of everyday life that we tend to forget how unique a color picture really is. There are no direct counterparts of a color print in X-ray optics, nor is there any similar device in the infrared or radio domain.

Even the simpler black-and-white photos have no direct analog in the far infrared or radio regions. To obtain such a picture we may scan an image of the scene with a single detector or receiver, whose output is then used to reconstruct the scene. In the near infrared, images can sometimes be obtained using arrays of detectors, or a vidicon or charge injection device. But these sensors operate only at certain wavelengths (where appropriate detector materials are available), and in any case are expensive.

These limitations can be overcome, and black-and-white pictures obtained, by using a technique known as *multiplexing*. In this technique the incident radiation is first separated into distinct bundles of rays, corresponding to different portions of the scene. Then certain combinations of these bundles are allowed to fall on the detector and the total intensity is recorded. After a series of say *n* suitably chosen combinations have been recorded, the individual intensities of *n* different bundles can be calculated, and a black-and-white picture obtained.

Multiplexing is also useful in spectroscopy. A conventional spectrometer sorts electromagnetic radiation into distinct bundles of rays, corresponding to different colors. Thus each bundle is labeled by the appropriate frequency, wavelength or wavenumber. The spectrum of the radiation is found by measuring the intensity of each bundle. Alternatively, the bundles can be multiplexed: instead of measuring the intensity of each bundle separately, we can measure the total intensity of various combinations of bundles. After measuring *n* suitably chosen combinations, the individual intensities of *n* different bundles can be calculated, and the spectrum obtained.

Finally, by combining these two forms of multiplexing — multiplexing radiation from different parts of a picture and from different frequency bands — it is possible to reconstruct a color picture of the scene.

What are the advantages of such a complicated technique? Why multiplex? To answer this we must discuss the role of noise in optical measurements. Any detector is a source of noise, no matter how carefully it is constructed. (Even when no radiation is present, a detector will produce spurious signals, which are often indistinguishable from the signals produced when light does fall on the detector.) An experimenter's task is to minimize the effects of this noise, and to reconstruct with as much fidelity as possible the intensity of radiation incident on the detector.

If the noise is independent of the strength of the incident signal, it may be advantageous to combine the radiation from a large number of bundles of rays, because then the total intensity of the light may provide a signal considerably larger than the detector noise. Thus the primary purpose of multiplexing is to maximize the radiant flux incident on the detector in order to improve the signal-to-noise ratio of the final intensity display.

The final display may be a spectrum, a black-and-white picture, or a color picture. Although the optical apparatus needed to separate spectral components is different from that needed to separate spatial components, the principle is the same in the three cases.

Two quite different multiplexing techniques are available. These can be loosely described by saying that the first uses *interference* *techniques* and *Fourier transforms,* while the second uses *masks* and *discrete* (often *Hadamard)* transforms. Examples of the first technique are (i) the *Michelson interferometric spectrometer* (see Chapter 2), in which an interferometer is used to modulate the intensities of different transmitted wavelengths, (ii) the method of *aperture synthesis* in radio astronomy, in which spatial maps of the sky are obtained by measuring the interference patterns between radio waves reaching two or more antennas from cosmic sources (see for example Steinberg and Lequeux (1963, Ch. 4)), and (iii) the technique of *holography,* in which an interference pattern is stored and then used to reconstruct a spatial, even three-dimensional, image (see for example DeVelis and Reynolds (1967)).

The underlying principle in the second technique is the use of masks which either block or transmit light. As we shall see, the best masks for these instruments are constructed from matrices named after the French mathematician Jacques Hadamard (cf. Hadamard (1893)). We have therefore called this general class of instruments ***Hadamard Transform Optics.*** These instruments are capable of obtaining color pictures at any wavelength and over a wide range of spectral and spatial resolutions. Figure 1.1 gives a simple view of how one of those instruments works.

The basic Hadamard transform instrument consists of four essential components: an optical separator, an encoding mask, a detector and a processor (Fig. 1.2). More complex instruments may make use of additional components. In some devices, for example, two masks are used.

The separator may be nothing more than a lens which produces a focused image at the mask, and separates light arriving from different spatial elements of a scene. Or the separator may be a dispersing system (a prism or grating) which separates different frequency components of a beam and focuses them onto different locations on the mask.

In the instrument shown in Fig. 1.2 the mask is made up of three types of elements. A particular location on the mask either transmits light to the main detector, absorbs the light, or reflects it towards a reference detector. In this way the corresponding element of the separated beam is modulated. If we record the difference between the reading of the main detector and the reference detector, the intensity of this element of the beam has been multiplied by $+1$, 0 or -1 respectively.

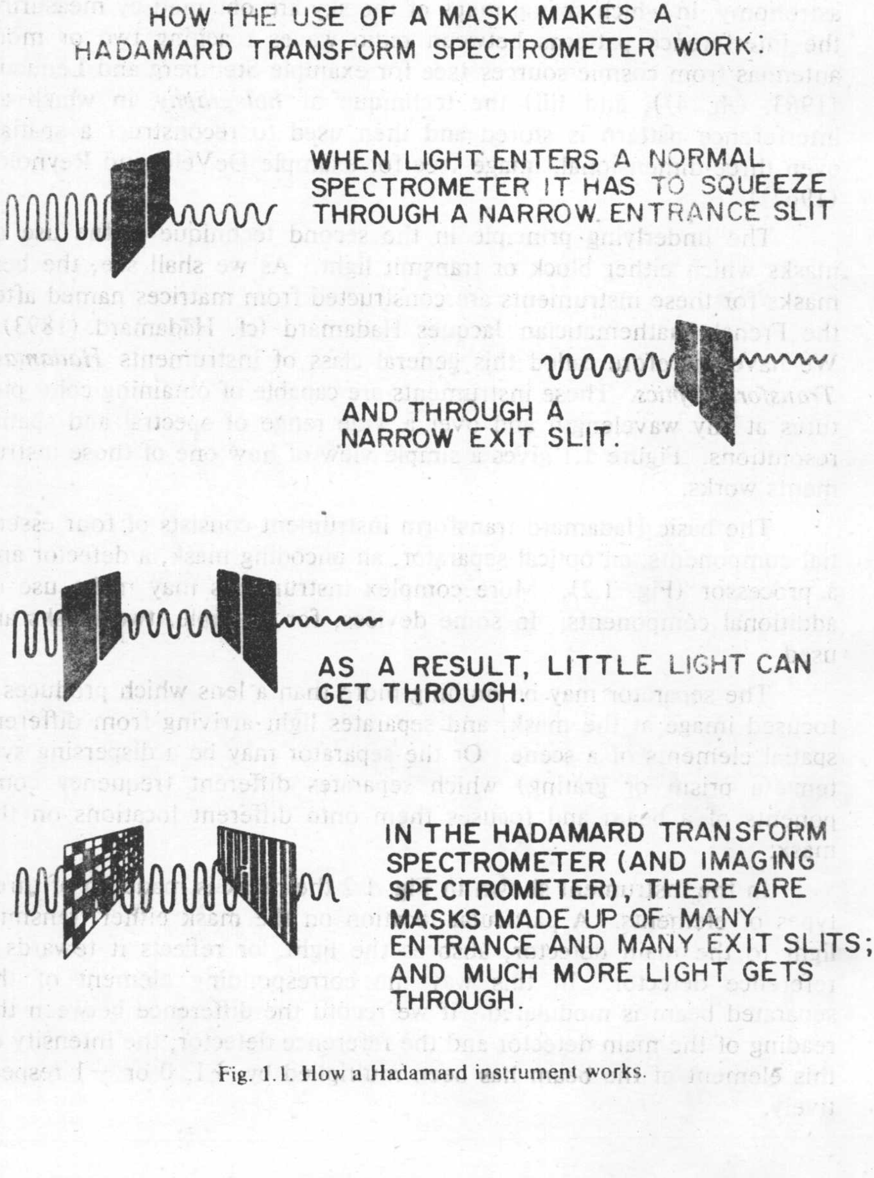

Fig. 1.1. How a Hadamard instrument works.

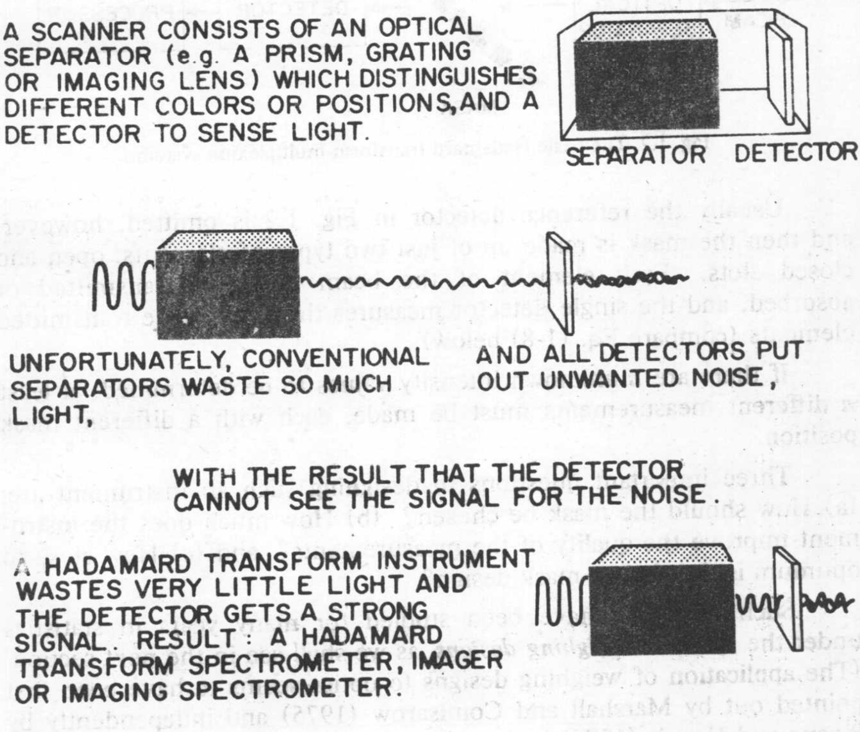

Fig. 1.1 (Continued)