APPLIED SUPERCONDUCTIVITY

Edited by

VERNON L. NEWHOUSE

Volume I

APPLIED SUPERCONDUCTIVITY

Edited by
VERNON L. NEWHOUSE

School of Electrical Engineering
Purdue University
Lafayette, Indiana

Volume I

ACADEMIC PRESS New York San Francisco London 1975
A Subsidiary of Harcourt Brace Jovanovich, Publishers

ACADEMIC PRESS, INC.
111 Fifth Avenue, New York, New York 10003

United Kingdom Edition published by
ACADEMIC PRESS, INC. (LONDON) LTD.
24/28 Oval Road, London NW1

Library of Congress Cataloging in Publication Data

Newhouse, Vernon L
Applied superconductivity.

Includes bibliographies.
1. Superconductors. I. Title.
TK7872.S8N42 621.39 74-1633
ISBN 0-12-517701-1

PRINTED IN THE UNITED STATES OF AMERICA

List of Contributors

Numbers in parentheses indicate the pages on which the authors' contributions begin.

C. L. BERTIN (267), IBM System Development Division, Manassas, Virginia

L. S. COSENTINO (209), RCA Laboratories, Princeton, New Jersey

WILLIAM S. GOREE (113), Superconducting Technology, Inc., Mountain View, California

VICTOR W. HESTERMAN (113), Superconducting Technology, Inc., Mountain View, California

WALTER H. HOGAN (309), Cryogenic Technology, Inc., Waltham, Massachusetts

R. M. KATZ (267), MITRE Corporation, Westgate Research Park, McLean, Virginia

K. ROSE (267), Electrophysics and Electronic Engineering Division, Rensselaer Polytechnic Institute, Troy, New York

A. R. SASS (209),* Quasar Electronics Corporation, Franklin Park, Illinois

ARNOLD H. SILVER (1), Electronics Research Laboratory, The Aerospace Corporation, El Segundo, California

W. C. STEWART (209), RCA Laboratories, Princeton, New Jersey

JAMES E. ZIMMERMAN (1), Institute for Basic Standards, National Bureau of Standards, Boulder, Colorado

* Formerly of IBM System Products Division, East Fishkill, New York.

Preface

Attempts to exploit the fascinating properties of superconductors started shortly after the original discovery of the effect by Onnes when he tried to produce dissipation-free high-field electromagnets. As is well known, these attempts failed owing to the relatively low critical field of the superconductors known at that time. Although later efforts to use superconductors were more successful, e.g., the use of magnetic-field-controlled switches by Casimir–Jonkers and de Haas in the early 1930s, applied superconductivity can only be said to have come of age in the 1960s when high-field superconducting magnets began to be used widely, even for experiments at room temperature. This very important application provides a good example of the impact of basic research on applied science, since the high critical field of Nb_3Sn, which made these magnets possible, was discovered in the course of fundamental researches aimed at elucidating the origins of superconductivity itself. An example of how applied research leads to improved fundamental understanding is exemplified by the fact that research on high-field superconductors led to the rediscovery of Abrikosov's work on class-II superconductors, which might otherwise have continued to be ignored for many years.

Since superconductors exhibit zero resistance at low frequencies, they are already important in the production of large magnetic fields and show promise in the production and transport of large quantities of electric power. Since they operate close to zero temperature where Johnson noise becomes small, they either already are or promise soon to become the most sensitive detectors of magnetic fields and of radiation at all frequencies. Furthermore since superconductive circuits can easily store single-flux quanta, they promise to become the most compact and, therefore, the fastest means of handling and storing information.

The current interest in pure superconductivity is proved by the award, in the last few years, of separate Nobel prizes for the theory of superconductivity as well as for superconductive and semiconductor tunneling. The speed of progress in the field of applied superconductivity is exemplified by the fact that high-field superconductors were unknown in 1962 when John Bremer published "Superconductive Devices," the initial work in this field, and that the Josephson effect, which is the basis of most of the radiation detection and magnetic-field measurement devices mentioned above, had not yet been discovered at the time of publication of this editor's book, "Applied Superconductivity," in 1964!

Since the subject of applied superconductivity has now grown to an extent where it can no longer be covered exhaustively by a single author, this treatise is divided into chapters on the various areas, each written by one or more authorities on the subject in question. The work is divided into two volumes, the first of which deals with electronic applications and radiation detection and contains a chapter on liquid helium refrigeration.

The second volume discusses magnets, electromechanical applications, accelerators, microwave and R. F. devices, and ends with a chapter on future prospects in applied superconductivity.

A corollary to being an authority on a subject is the many demands made upon one's time. Thus a deadline must often be secondary to other responsibilities. For reasons of this sort, the original versions of some chapters in this treatise were completed before others, and we were never able to obtain a chapter on high-power rotating machinery.

Each chapter in these two volumes can be read independently, and most assume very little or no background in the physics of superconductivity. The topics treated do not require the use of advanced quantum mechanics; thus the books should be accessible to students or research workers in any branch of engineering or physics. They are intended to serve both as a source of reference material to existing techniques and as a guide to future research. For those wishing to extend their background in the physics of superconductivity, some recent books on the subject, selected from a larger list kindly compiled by Arthur J. Bond, are given in the following Bibliography.

Bibliography

The books listed on the following page are those written recently from the experimentalists' point of view and can be recommended for those wishing to expand their background in the physics of superconductivity.

De Gennes, P. G., (1966). "Superconductivity of Metals and Alloys." Benjamin, New York.
Kulik, I. O. and Yanson, I. K. (1972). "Josephson Effect on Superconducting Tunneling Structures." Halsted, New York.
Kuper, C. G. (1968). "An Introduction to the Theory of Superconductivity." Oxford Univ. Press, London and New York.
Lynton, E. A. (1972). "Superconductivity." Halsted, New York.
Parks, R. D. (1972). "Superconductivity," Vols. 1, and 2. Dekker, New York.
Saint-James, D. (1969). "Type II Superconductivity." Pergamon, Oxford.
Savitsky, *et al.* (1973). "Superconducting Materials." Plenum, New York.
Solymar, L. (1972). "Superconductivity Tunneling and Applications." Wiley, New York.
Williams, J. E. C. (1970). "Superconductivity and Its Applications." Pion, New York.

Contents of Volume II

Contents

4. Radiation Detectors

K. Rose, C. L. Bertin, and R. M. Katz

5. Refrigerators and Cryostats for Superconducting Devices

Walter H. Hogan

Chapter 1

Josephson Weak-Link Devices

ARNOLD H. SILVER

Electronics Research Laboratory
The Aerospace Corporation
El Segundo, California

JAMES E. ZIMMERMAN

Institute for Basic Standards
National Bureau of Standards
Boulder, Colorado

I. Introduction

Superconductivity affords us a unique opportunity to observe and utilize the fundamental quantum-mechanical nature of matter, for in superconductors this quantization is on a truly macroscopic scale and is synonymous with the electronic coherence found otherwise only in atomic systems. Josephson weak-link devices are superconducting elements which exhibit an intrinsic quantum electronic behavior. Devices based on such quantum effects exhibit a periodic parametric behavior with respect to very low applied fields, and therefore are highly nonlinear and sensitive at low power levels.

Probably the most significant discoveries precipitating the development of these devices were those of flux quantization and the Josephson effect. Following London's ideas (1950) of more than one decade earlier, Deaver and Fairbank (1961) and Doll and Näbauer (1961) experimentally discovered that the magnetic flux threading a superconducting circuit is quantized in integral multiples of $h/2e$. This unit is equal to 2.07×10^{-15} W (2.07×10^{-7} G cm^2). Josephson (1962) predicted that there should be a quantum-phase-dependent zero-voltage tunneling current between two superconductors separated by a very thin insulating barrier and that in the presence of a nonzero voltage an alternating tunneling current will flow. Attempts to verify Josephson's predictions coupled with renewed interest in the quantum properties of superconductors have spurred the development of these devices.

In this chapter we review a number of devices and device possibilities based on the coherent precession of the quantum phase across weakly superconducting connections such as thin-film tunneling junctions, point contacts, and thin-film bridges. Particular emphasis is placed on low-inductance radio-frequency-biased loop devices since these are particularly amenable to detailed analysis and have certain attractive attributes as practical devices. A weak link shunted by a sufficiently low-inductance loop becomes voltage biased to the extent that the theoretical calculations of the device response are relatively straightforward and in direct quantitative agreement with experiment. Furthermore, in contrast with high-impedance current-biased weak links, these devices are uncluttered by nonsuperconducting or "non-Josephson" effects, which themselves are poorly defined and exceedingly complex. Several reviews in recent years have discussed the Josephson effect (Anderson, 1964). We take as our point of primary emphasis multiply-connected circuits incorporating one, or more, weak links and direct our attention toward device characterization and experimental situations.

Section II discusses the quantum electronics of superconductors, emphasizing the close relation between the Josephson effects and fluxoid quantization. A simple mechanical analog of weak-link phenomena is described and invoked to understand the nature of certain quantum transitions.

In Section III we characterize various device topologies and discuss some of the experimental results. Some of the device construction techniques are reviewed in Section IV, and an array of applications, attempted and proposed, are discussed in Section V.

II. Quantum Electronics of Weak-Link Devices

A. Theoretical Background

The theoretical model which is useful in understanding and postulating weak-link devices can be presented in phenomenological and analog forms. We take as our point of departure an elementary description of the super-conducting state in terms of a single complex order parameter Ψ as in Ginzburg–Landau theory. This order parameter can be considered an effective wave function and satisfies an equation similar to the Schrödinger wave equation. The complex function

$$\Psi = |\Psi| e^{i\phi} \tag{1}$$

can be generally dependent on both space and time in both its amplitude and phase. Our major concern will be with phase-dependent effects. Thus we restrict the variations of Ψ to that introduced by ϕ. In standard form then

$$\hbar\phi = \int \mathbf{p} \cdot dl - 2e \int \mu \, dt \tag{2}$$

where $\mathbf{p}$ and μ are the canonical momentum and chemical potential of the electron pairs of the superconductor, respectively.

The current density associated with the order parameter is given from the standard form by

$$\mathbf{j} = (eh/m)|\Psi|^2(\nabla\phi - (2e/\hbar)\mathbf{A}) \tag{3}$$

where $\mathbf{A}$ is the magnetic vector potential and we have introduced the double mass and charge of the electron pairs. For a London superconductor $\nabla\phi = 0$ and we have

$$\mathbf{j} = -(2e^2/m)|\Psi|^2\mathbf{A} \tag{4}$$

which is London's equation for diamagnetism if we associate $2|\Psi|^2$ with n, the density of superelectrons in the two-fluid theory. Thus we note in passing that London theory (1950) corresponds to the real-order-parameter limit of Ginzburg–Landau theory. We interpret $(\hbar/2m)[\nabla\phi - (2e/\hbar)\mathbf{A}]$ as the velocity of the electron pairs. Differentiation of Eq. (3) with time gives

$$\frac{d\mathbf{j}}{dt} = -\frac{2e^2}{m}|\Psi|^2\left[\nabla V + \frac{\partial\mathbf{A}}{\partial t}\right] \tag{5}$$

where we have taken μ equal to the electric potential V. Equation (5) is London's second equation relating the electric field to the time derivative of the current,

$$\frac{d\mathbf{j}}{dt} = \frac{ne^2}{m}\mathbf{E} \tag{6}$$

Thus, starting from a complex order parameter and a quantum-mechanical formalism one can derive London's classical electrodynamic equations.

The conservation, and more specifically the quantization, of magnetic flux in superconductors is readily evident from this approach. The uniqueness of Ψ imposes a condition on ϕ such that for all $(\mathbf{x},t)$

$$\oint \mathbf{p}\cdot dl - 2e\oint \mu\, dt = kh \tag{7}$$

where k is any integer. The stationary behavior then reduces to

$$\oint 2m\mathbf{v}\cdot dl + \oint 2e\mathbf{A}\cdot dl = kh \tag{8}$$

In conventional notation the electric current density j and magnetic flux Φ are

$$\mathbf{j} = ne\mathbf{v} \tag{9}$$

and

$$\Phi = \oint \mathbf{A} \cdot dl \tag{10}$$

which gives

$$\oint (m/ne^2)\mathbf{j} \cdot dl + \Phi = k\Phi_0 \tag{11}$$

where

$$\Phi_0 = h/2e = 2.07 \times 10^{-15}\ \text{Wb} = (2.07 \times 10^{-7}\ \text{G cm}^2) \tag{12}$$

is called the flux quantum. Equation (11) reduces to London's fluxoid conservation relation in the special case $k = 0$. Specifically this predicts quantization of the magnetic flux whenever one has a sufficiently thick superconductor that $j = 0$ along an entire circuit. This condition is satisfied for thicknesses much larger than a penetration depth λ.

B. Linear Weak-Link Approximation

Devices of interest in this chapter involve systems where $\oint \mathbf{j} \cdot dl \neq 0$. In particular, we are interested in systems where j approaches its maximum supercurrent value j_c and where $\oint (m/ne^2)\mathbf{j} \cdot dl$ becomes $\gtrsim \Phi_0/4$. Several obvious ways to generate such systems are:

1. utilize surface effects on bulk superconductors;
2. use bulk Type-II superconductors in the vortex flow state;
3. constrict the region of supercurrent flow small compared to a penetration length;
4. produce a region of very low superelectron density and hence make the penetration length larger than the region of current flow.

Those systems which fall in the first two classes are not of interest here. Classes three and four are grouped together under the name Josephson weak-link devices. Generally speaking, class 3 is a superconducting metallic bridge of small lateral cross section while class 4 is a tunneling section between two superconductors, called a Josephson tunneling junction. Many actual devices are really hybrids of these two models.

If we rewrite Eq. (11) as

$$(2\pi/\Phi_0) \oint (m/ne^2)\mathbf{j} \cdot dl + (2\pi\Phi/\Phi_0) = 2\pi k \tag{13}$$

we note that the first term represents the phase angle associated with the current and the second term represents the angle generated by the magnetic flux. Further, restricting the path to $\mathbf{j} = 0$ everywhere except at the weak link we may say

$$\theta + 2\pi\Phi/\Phi_0 = 2\pi k \tag{14}$$

where

$$\theta = (2\pi/\Phi_0) \int_{\text{w.l.}} (m/ne^2)\mathbf{j}\cdot dl \tag{15}$$

As a simplifying assumption, let the weak link have an effective length l_0 and cross section σ small enough that j is uniform. If the total current is denoted by i, the change in phase across the weak link is

$$\theta = (2\pi/\Phi_0)(m/ne^2)(l_0/\sigma)i, \qquad i \leq i_c \tag{16}$$

When this weak link is incorporated in a bulk superconducting ring of inductance L, we have

$$2\pi\gamma Li/\Phi_0 + 2\pi\Phi/\Phi_0 = 2\pi k \tag{17}$$

where

$$\gamma = (m/ne^2)(l_0/\sigma L) \tag{18}$$

is a characteristic parameter of the system.

Equation (16) is a linear current–phase relation, applicable only for systems of uniform j, hence for $\sigma \lesssim \lambda^2$. A reasonable calculation for small superconductors (dimensions comparable to or less than a penetration length) has shown that the maximum supercurrent i_c is $G\Delta$, where Δ is the energy gap in volts and G is the normal-state conductance of the junction (Zimmerman and Silver, 1966a). With reasonable values for the variables involved, the quantum phase across such links can be shown to be of the order of 2π. In fact, it has been experimentally demonstrated that such phase changes are generally limited to $\pi/2$ and do not exceed π (Zimmerman and Silver, 1966b).

Intriguing and novel characteristics result from the nature of the quantum states and transitions between states for these devices (Silver and Zimmerman, 1967b). This is illustrated by the superconducting ring with a weak link described by Eq. (17). In the presence of an applied magnetic field the ring will intercept a portion of the applied flux Φ_x. The total magnetic flux in the ring differs from Φ_x by Li, the flux generated by the circulating current in the ring inductance L,

$$\Phi = \Phi_x + Li \tag{19}$$

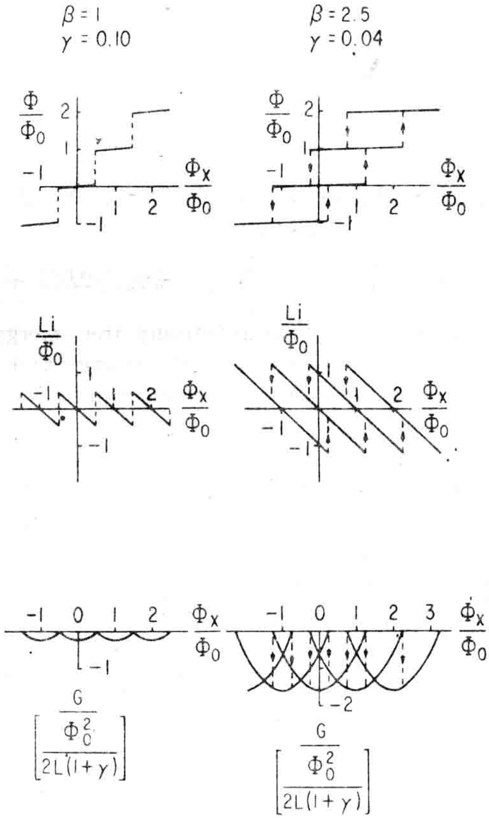

FIG. 1. Theoretical solutions of the quantum states of a weakly connected superconducting ring as a function of the applied magnetic field Φ_x for a linear current–phase-shift relation. β is defined in Eq. (22), γ by Eq. (18), and G is the Gibbs free energy given in Eq. (24).

Solutions of Eqs. (17) and (19) for Φ and Li are

$$Li = -(\Phi_x - k\Phi_0)/(1 + \gamma), \qquad i \leq i_c \tag{20}$$

$$\Phi = (k\Phi_0 + \gamma\Phi_x)/(1 + \gamma) \tag{21}$$

where experimentally γ is usually a small number. Figure 1 shows graphs of these solutions for several values of i_c. We recognize that the important parameter in the problem has been reduced to the ratio $(1 + \gamma)Li_c/\Phi_0$, essentially the ratio of the maximum screened (or trapped) flux to the flux quantum. If we define the dimensionless parameter β as

$$\beta \equiv 2(1 + \gamma)Li_c/\Phi_0, \tag{22}$$