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FOREWORD

The papers contained in this volume were presented in the Computer-Integrated Manufacturing and Robotics sessions of the Production Engineering Conference at the 1984 ASME Winter Annual Meeting. The authors discussed many important topics of computer automation, which include applications, modeling, and analysis of robots and robotic devices, flexible manufacturing systems, automation of machining and forming processes, and other applications of computers in manufacturing.

This is the second year that sessions on Computer-Integrated Manufacturing have been sponsored by the Production Engineering Division of ASME. Having been the session organizers for both years, we are extremely pleased to see the rapid growth of interest on this subject, which directly impacts the nation's productivity. The number of papers presented this year has reached 27, compared with last year's 18. Although this has also increased the amount of organizing work, we feel that our time and effort in this regard have been well spent. We hope this growth will continue and ASME will continue its leadership role in driving manufacturing automation.

We wish to extend our sincere thanks to the authors for preparing the papers and to the reviewers for providing constructive criticisms. Also acknowledged are the organizers and panelists for the two special sessions on solid geometric modeling and on automation of electronics manufacturing, which form an integral part of the program. Special thanks are due to Mrs. Judy Stage for secretarial assistance. Finally, the support of the Production Engineering Division's Executive Committee, especially Dr. Ranga Komanduri, is greatly appreciated.

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KINEMATIC ANALYSIS AND DESIGN OF TOOL GUIDE MECHANISMS FOR GRINDING ROBOTS

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ABSTRACT

A new method for increasing the stiffness of a machining robot is presented. Extending the application of robots to machining operations such as grinding requires that the robot be capable of achieving high accuracy under severe dynamic load conditions. This paper describes a device, called a "Jig Hand", which is used to couple the end of a robot to the work surface and which bears the vibratory interaction force during the machining operation. The kinematics and statics of the grinding operation are analysed and the condition for a jig hand not to cause conflict with the machining task is derived. It is shown that using a jig hand leads to an increase in the stiffness of the machining robot and a design example is worked out illustrating the advantage of this method.

INTRODUCTION

Industrial robots today are primarily used as a means for locating an end effector. Spray painting and arc welding robots simply move a spray gun or weld gun along preprogrammed trajectories. Spot welding robots position a welding fixture on a workpiece. The fixture then clamps itself to the workpiece and completes the weld. These are simple tasks and hence they were the ones for which robots were first used. To this day they remain the principal applications of industrial robots in the U.S.A.

In extending the application of robots to machining operations, robots require more skill and must be capable of achieving high accuracy under severe dynamic load conditions. Consider a few examples. For grinding of castings (1) a robot must locate the grinding tool accurately, apply the appropriate force to the work surface, and shape the raw mold to the desired form and a surface finish. The robot must be capable of performing this task stablely in the face of the large vibratory forces generated by the interaction between the grinding wheel and the workpiece. In drilling of aircraft parts, a robot must locate and hold a drill against a curved work surface whilst exerting a large force (2). Chipping operations cause large, impulsive forces which would break a robot if the chipping tool were directly coupled to the robot arm (3).

The critical difference between these machining tasks and the purely locational type of tasks to which robots are currently being applied is that machining tasks generate large interaction forces between the end effector and the workpiece. Robots used for machining therefore require a stiffer structure to bear this load.

Most robots available today have been designed primarily as locating devices and are inherently poor in mechanical stiffness. Their structure is generally that of an open kinematic chain. This structure is ideal for flexible motion with a large work space and a large number of degrees of freedom. It is not, however, a good structure for achieving high accuracy and high mechanical stiffness.

In this paper, a method is presented for increasing the mechanical stiffness of a robot without the sacrifice of work space or mobility. This approach can also be used to compensate for workpiece tolerance and for errors in the position of the robot and the workpiece.

DESIGN CONCEPTS

Robots With Jig Hands

The poor mechanical stiffness of present day robots is a result of their open kinematic chain structure. The load is supported at the end of what is essentially a cantilevered beam. If the arm were additionally supported by some means at a point closer to the end effector, forming a closed kinematic chain, then its mechanical stiffness could be significantly increased. Figure 1 shows an arm with a closed loop structure of the type discussed in this paper. This robot has a device that couples the end of the arm to the work surface and helps to support the load generated at the end effector. The load transmitted to the main arm is reduced and consequently the effective stiffness of the robot is increased. The coupling device can also be used to measure the position of the end effector relative to the work surface - an essential requirement for accurate machining. This is particularly important when the location of the work part is not accurately known due to work part tolerance or errors in set up position. The coupling device is referred to as a "Jig Hand" in this paper because it has the same purpose as a jig. It is used for locating the end effector and bearing the interaction force during machining operations.

There are a variety of ways to design a jig hand. The end of the jig hand may have a clamping device to fix it to the work part or to a fixture in the work space as in Figure 1. Alternatively it can be coupled to the work part simply by pressing the tip against the surface, Figure 2. In this case the jig hand is not fixed but can slide across the workpiece. The jig hand may be solid, Figure 2, or articulated as in Figure 1. An articulated jig hand may have actuators to actively control the position and the force it applies to the main arm or it may be a purely passive mechanism with free joints. A jig hand can also be equipped with sensors to monitor the tool performance and to provide information to aid in control of the task.

Clearly there are trade-offs to be made in designing a jig hand. The advantage of a sliding jig hand is that no special fixturing is required to attach the hand to the workpiece; contact is maintained through a preloading force. Jig hands that are coupled in this way cannot in general reduce the static load borne by the main arm. Unless the load at the end effector is in the direction of the work surface the static load in the main arm is increased in preloading the jig hand. A sliding jig hand can, however, be used to increase the stiffness of the robot to dynamic loads. A jig hand that is fixed to the workpiece can always reduce both the static and dynamic load borne by the main arm but the attachment is more complex. Both types of jig hand are useful for obtaining part referenced manipulation information.

Task Analysis And Jig Hand Design

The jig hand approach can be applied to many machining operations, however, for the purpose of this paper we have chosen grinding as a representative application for more detailed analysis. The grinding task is characterized by large vibratory forces generated between the end effector and the workpiece and is therefore a suitable application for the sliding type of jig hand.

Using a jig hand to couple a robot arm to the work surface reduces the vibratory load borne by that arm. However, the jig hand also constrains the motion of the tool. This constraint will, in general, reduce the degree of freedom and degrade the mobility of the robot arm. If the desired tool motions are constrained by a jig hand then the robot cannot perform its task. In this section we discuss the basic principles in jig hand design that lead to structures which do not cause conflict with the desired motions of the tool.

Figure 3(a) shows a simple example of a planar robot which moves a hemispherical grinding tool in the x-y plane. A robot with three degrees of freedom is needed in order to locate the tool at an arbitrary position and with an arbitrary orientation in the plane. The motion of the tool relative to the work surface can be described using a coordinate frame fixed to the workpiece. The contact point of the grinding wheel is denoted by p_x and p_y and the orientation of the tool to the work surface is denoted by angle α_z. In order to perform the grinding operation, the robot must be able to move the tool along the p_x and p_y axes accurately. The tool orientation, however, does not have to be accurately controlled. It is only required that the orientation be maintained within a certain range, ± 20 degrees say. As long as the orientation remains within this range the grinding operation can be performed adequately. This observation suggests that there are two types of variables involved in the tool motion: one is a set of variables which must be controlled actively and precisely to specific values and the other is a set of variables which are only required to be held within certain bounds. The former type of variables represent the essential motions of the tool which significantly affect the task performance and are referred to as essential variables. The latter type of variables have no significant impact on the task; they are arbitrary within certain limits and are called arbitrary variables.

From this analysis of tool motion it follows that, although a jig hand reduces the degree of freedom of an arm, it does not necessarily conflict with the tool motion. In the case of the planar robot in Figure 3(a), a simple jig hand with a sliding contact can be attached near the end point as shown in Figure 3(b). During the grinding operation, the jig hand allows the tool to slide along the surface. Although the degree of freedom of the tool has been reduced from three to two, the two essential variables, p_x and p_y can still be varied independently. The tool angle α_z can no longer be controlled independently of the other two variables, but it is an arbitrary variable not critical to the performance of the grinding task. The net effect of the jig hand is to increase the stiffness of the robot to dynamic loads.

The jig hand approach depends upon the assumption that there exist some degrees of freedom that do not have to be controlled independently during the machining operation. For many manufacturing operations the number of essential variables in local coordinates with reference to the work surfaces is less than the number of global degrees of freedom necessary for locating the tool in space.

KINEMATICS

Kinematic Modelling

In this section, we develop a more detailed kinematic model for a grinding robot with six degrees of freedom and derive the condition for a jig hand not to cause conflict with the essential variables of the grinding process. Then, in section 4, we show how the mechanical stiffness of an arm is improved by using a jig hand.

Grinding is performed by moving a grinding wheel over a work surface and applying an appropriate force to the surface. Figure 4(a) shows a grinding tool attached to the end of a robot arm. A coordinate frame O_t - x_t y_t z_t is fixed to the grinding tool with origin at the tool center whilst the frame O_o - x_o y_o z_o is fixed to the ground. The position of the tool center is represented by the vector $\mathbf{x}_t$ which is the origin of frame O_t - x_t y_t z_t with respect to O_o - x_o y_o z_o. Let us denote small translational displacements of the tool center by the vector $d\mathbf{x}_t$ and rotational displacements by the vector $d\phi_t$, where the elemtents of $d\phi_t$ are the small angles of rotations about the x_o, y_o and z_o axes.

The area of contact between the grinding wheel and the work surface is shown by the shaded circle and the center of the area of contact is represented by the point O_c. Figure 4(b) shows a magnified view of the grinding operation. The grinding operation is performed by moving the contact point along the surface and controlling the depth of cut into the surface. Let $\mathbf{e}_z$ be a unit vector perpendicular to the work surface at O_c, $\mathbf{e}_y$ a unit vector tangential to the work surface at O_c and perpendicular to the axis of rotation of the grinding wheel, and $\mathbf{e}_x$ a unit vector also tangential to the work surface which completes a right handed coordinate frame. Assuming the work surface is flat[1] in the vicinity of O_c then the motion of the contact point can be expressed as a vector $\mathbf{p}$ in this coordinate system. p_x and p_y representing the motion over the surface and p_z the depth of cut. If the orientation of the tool is kept constant the motion of the tool center is the same as the motion of the contact point and

$$d\mathbf{x}_t = \mathbf{e}_x p_x + \mathbf{e}_y p_y + \mathbf{e}_z p_z \tag{1}$$

p_x, p_y and p_z are the three essential variables for the grinding task.

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[1]It is possible to extend this analysis to a curved work surface, in which case the translational displacement of contact point O_c depends upon the curvature of the work surface.

In addition to the three essential variables, there are an additional three arbitrary variables to complete the description of the contact between the grinding tool and the work surface. The shape of the wheel surface may be described by the cross sections in the e_y-e_z and e_x-e_z planes as shown in figure 4(c). The radii of curvature of the wheel surface in these planes are r_x and r_y and the centers of curvature are located at C_x and C_y respectively. Let e_x' and e_y' be unit vectors through the centers of curvature parallel to e_x and e_y, then small rotations α_x and α_y about these axes do not affect the grinding performance. The other arbitrary variable is the angle of rotation about the normal to the surface at O_c denoted by α_z.

Let us find the relationship between the tool's motion in terms of dx_t and $d\phi_t$ and that of the arbitrary variables α_x, α_y, α_z.

The tool's rotational motion is given by

$$d\phi_t = e_x\,\alpha_x + e_y\,\alpha_y + e_z\,\alpha_z \tag{2}$$

Rotations at the contact point also cause translational motion at the tool center. Let x^o_{tc} be the position vector from O_t to O_c with reference to the O_o - x_o y_o z_o frame, then the translational motion due to α_x, α_y and α_z is given by

$$dx_t = (x^o_{tc} + r_x e_z) \times \alpha_x e_x + (x^o_{tc} + r_y e_z) \times \alpha_y e_y + x^o_{tc} \times \alpha_z e_z \tag{3}$$

Where x represents a vector product. The resultant translational motion of the tool center is therefore given by.

$$dx_t = E\,p + A\,\alpha \tag{4}$$

and the rotational motion given by

$$d\phi_t = E\,\alpha \tag{5}$$

where

$$E = [e_x : e_y : e_z]$$

$$A = [x^o_{tc} \times e_x + r_x e_y : x^o_{tc} \times e_y - r_y e_x : x^o_{tc} \times e_z]$$

$$p = [p_x : p_y : p_z]^T$$

$$\alpha = [\alpha_x : \alpha_y : \alpha_z]^T$$

the superscript ^{T} indicates the transpose of a matrix

The three essential variables, p_x, p_y and p_z, and the three arbitrary variables, α_x, α_y and α_y form a linearly independent set spanning the position and orientation space of the tool.

This analysis can be extended to general manufacturing operations. Combining translations and rotations let us represent the small motion of the tool center by a six dimensional vector $\mathbf{dq} = [dx_t\ dy_t\ dz_t\ d\phi_{t,x}\ d\phi_{t,y}\ d\phi_{t,z}]^T$. The essential degrees of freedom are denoted by p_1 through p_m and remaining (6-m) degrees of freedom are classified as arbitrary variables and denoted by α_1 through α_{6-m}. The direction of motion corresponding to each essential variable, p_i, is given by a six-dimensional vector, $\mathbf{e}_i^*$, and the direction of motion corresponding to each arbitrary variable α_j is given by a six-dimensional vector $\mathbf{a}_j^*$. The tool motion can thus be represented in terms of the essential and arbitrary variables as

$$\mathbf{dq} = \mathbf{E}^* \mathbf{p} + \mathbf{A}^* \alpha \qquad (6)$$

where $\mathbf{E}^*$ and $\mathbf{A}^*$ are 6xm and 6x(6-m) matrices given by

$$\mathbf{E}^* = [\mathbf{e}_1^*, \ldots \mathbf{e}_m^*]$$

$$\mathbf{A}^* = [\mathbf{a}_1^*, \ldots \mathbf{a}_{6-m}^*]$$

Admissible Constraints

Since a jig hand mechanically couples the arm tip to the work surface, it constrains the motion of a tool attached to the arm and reduces its degree of freedom. In order to perform the grinding task, however, the tool must retain its freedom to move in the directions corresponding to the essential variables. The freedoms that are lost must be in the directions corresponding to the arbitrary variables. In this section the condition is derived for determining whether or not a jig hand conflicts with the tool motion necessary for performing a task.

There are many types of jig hand and a few of the different design options have been mentioned in section 2. In this paper the focus is on a jig hand that is a link mechanism with passive joints with the end of the jig hand fixed to the work surface. Let the joint angles of the jig hand be denoted by θ_1 through θ_n, then the allowable motion of the tool is given by

$$\mathbf{dq} = \mathbf{J}_j\, \mathbf{d\theta} \qquad (7)$$

where $\mathbf{J}_j$ is the Jacobian matrix associated with the link mechanism (the transformation from joint displacements to tool motion), and $\mathbf{d\theta}$ is an arbitrary variation in the vector of joint angles $[d\theta_1, \ldots d\theta_n]^T$. If there is to be no conflict between the jig hand and the necessary motion of the tool, then it must be possible to move the tool in all the directions corresponding to the essential variables. Equating equations (6) and (7) yields

$$\mathbf{J}_j\, \mathbf{d\theta} = \mathbf{E}^* \mathbf{p} + \mathbf{A}^* \alpha \qquad (8)$$

As $\mathbf{p}$ varies while performing the task, there must exist $\mathbf{d\theta}$ and α that satisfy the above equation. Otherwise, the necessary tool motion, $\mathbf{p}$, is not allowed by the constraint. The necessary and sufficient condition for the linear simultaneous equation (8) to have a solution for arbitrary $\mathbf{p}$ is given by

$$rank\ [J_j : A^*] = rank\ [J_j : A^* : E^*] \quad (9)$$

In the grinding example described in section 3.1 the three essential variables are purely translational. By partitioning the Jacobian matrix, J_j, into 3 x 3 matrices associated with translations, $J_{j,x}$, and with rotations, $J_{j,\phi}$, equation (8) can be written in terms of equations (4) and (5) as

$$J_{j,x}\ d\theta = E\ p + A\ \alpha \quad (10)$$

$$J_{j,\phi}\ d\theta = E\ \alpha \quad (11)$$

Since **E** is an orthonormal matrix,

$$\alpha = E^T\ J_{j,\phi}\ d\theta \quad (12)$$

Substituting equation (12) into equation (10) yields

$$p = E^T\ [J_{j,x} - A\ E^T\ J_{j,\phi}]\ d\theta \quad (13)$$

In order to vary **p** arbitrarily, the rank of matrix $[J_{j,x} - A\ E^T\ J_{j,\phi}]$ must equal 3, which is the number of essential degrees of freedom. The minimum number of degrees of freedom of the jig hand is accordingly 3 and the condition for admissible constraint in such a case is given by

$$det[J_{j,x} - A\ E^T\ J_{j,\phi}] \neq 0 \quad (14)$$

STIFFNESS

In this section we analyse the stiffness of the end point of the arm at the tool center point. Since the jig hand bears most of the load acting on the tool, the force transmitted to the main arm is reduced. Therefore the deflection of the main arm is reduced and the resultant stiffness of the end point is increased.

Denoting the external force and moment at the tool center by f_{ext} and the stiffness of the main arm by a 6 x 6 matrix K_m then, without a jig hand, the deflection at the tool center, **q**, is given by

$$q = K_m^{-1}\ f_{ext} \quad (15)$$

Note that the stiffness, K_m, depends upon both the servo stiffness of each active joint and the stiffness of the arm's links. Let K_s be the servo stiffness of the active joints with respect to joint coordinates and let J_m be the Jacobian matrix associated with the transformation from the joint coordinates of the main arm to the end point position and orientation. Then the deflection at the end point due to compliance in the joint servos, q_s, is given by

$$q_s = J_m\ K_s^{-1}\ J_m^T\ f_{ext} \quad (16)$$

Let K_l be the link stiffness then the deflection at the end point due to compliance of the links, q_l, is given by

$$q_l = K_l^{-1} f_{ext} \tag{17}$$

For small deflections the two sources of compliance may be added. The total deflection of the main arm is therefore given by

$$q = K_m^{-1} f_{ext} = (J_m K_s^{-1} J_m^T + K_l^{-1}) f_{ext} \tag{18}$$

Now let us look at the stiffness of the jig hand. For a jig hand with passive joints the servo stiffness is zero, therefore the end point load that the jig hand can support, f_j, must satisfy

$$J_j^T f_j = 0 \tag{19}$$

The condition on the stiffness matrix of the jig hand, K_j, is that

$$J_j^T K_j = 0 \tag{20}$$

This condition is an expression of the fact that, for a jig hand with passive joints, the directions in which the end of the hand can move are perpendicular to the directions in which the jig hand can bear load. The Jacobian is therefore orthogonal to the stiffness matrix. The stiffness matrix for an arm with passive joints can be conveniently obtained by first locking the passive joints and measuring the resultant stiffness matrix, K_j'. K_j, the stiffness matrix with the passive joints released, is related to K_j' by

$$K_j = (I - J J^{\#}) K_j' \tag{21}$$

Where $J^{\#}$ is the Moore-Penrose generalized inverse[2] of J .

By attaching a jig hand at the end of the main arm, the externally applied load is shared between the main arm and the jig hand so that the load seen by the main arm, f_m, is reduced to

$$f_m = f_{ext} - f_j \tag{22}$$

and the deflection at the tool center is given by

$$q = (K_m + K_j)^{-1} f_{ext} \tag{23}$$

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[2] If A is a non-square matrix then the least square solution to $Ax=b$ is given by u where $u = A^{\#}b$ and $A^{\#}$ is the Moore-Penrose inverse of A (4)

High stiffness is required particularly in the directions of the essential variables at the grinding wheel contact point. From equations (4) and (5) the deflection in the direction of the essential variables at the contact point, **p**, is related to the deflection at the tool center, **q**, by

$$\mathbf{p} = \mathbf{B}\, d\mathbf{q} \tag{24}$$

where

$$\mathbf{B} = [\mathbf{E}^T \,\vdots\, -\mathbf{E}^T \mathbf{A}\, \mathbf{E}^T]$$

If a 3 x 1 linear force vector, $\mathbf{f}_c$, acts at the contact point, then the equivalent force and moment at the tool center, $\mathbf{f}_{ext}$, is given by

$$\mathbf{f}_{ext} = \mathbf{B}^T \mathbf{f}_c \tag{25}$$

If we presume that the grinding tool itself is infinitely stiff then the resultant deflection of the essential variables at the contact point can be obtained from equations (23), (24) and (25)

$$\mathbf{p} = \mathbf{B}\,(\mathbf{K}_m + \mathbf{K}_j)^{-1}\, \mathbf{B}^T \mathbf{f}_c \tag{26}$$

The goal in designing a jig hand is to minimize the deflection at the contact point whilst satisfying the condition of admissible constraint described in section 3.2.

DESIGN EXAMPLE

In this section a conceptual design example is presented for demonstrating the jig hand approach. The task is to grind the surface of a large curved work part such as an aircraft wing or a large turbine blade. The main robot arm has 6 degrees of freedom to locate the tool on the work surface at an appropriate po ition and orientation.

As described in section 3.1, the essential requirements of the grinding process are that the tool contact point can be moved along the vectors, $\mathbf{e}_x$, $\mathbf{e}_y$ and $\mathbf{e}_z$ indicated in Figure 4(b). The reaction force, on the other hand, lies mostly in the $\mathbf{e}_y$-$\mathbf{e}_z$ plane. It is the stiffness in this plane that is required to be high.

The jig hand selected for this example is shown in Figure 5 and 6. It has three revolute joints and a contact disk. The first joint of the jig hand, θ_1, is a revolute joint about an axis normal to the work surface, the second and third joints, θ_2 and θ_3, are about two horizontal axes which are parallel to each other. In advance of a grinding operation the robot locates the contact disk at an appropriate point close to the area to be machined and then applies a preloading force through the jig hand to firmly anchor the disk to the work surface.

At the instance shown in Figure 5 with $\theta_1 = 0$ and $\theta_2 = \theta_3 = \pi$, the Jacobian matrix of the jig hand with respect to the motion of the tool center, O_t, is given by

$$\mathbf{J}_{j.x} = \begin{bmatrix} 0 & l_2 & l_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{J}_{j.\phi} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \tag{27}$$

and the vector connecting the tool center to the contact point, $\mathbf{x}_{tc}$, is given by $[x^o_{tc}\ 0\ z^o_{tc}]^T$. Matrix E is the 3x3 identity matrix, because $\mathbf{e}_x$, $\mathbf{e}_y$ and $\mathbf{e}_z$ are parallel to the x_t, y_t and z_t axes and matrix A is obtained from equation (3) as

$$\mathbf{A} = \begin{bmatrix} 0 & -z^o_{tc}-r_y & 0 \\ z^o_{tc}+r_x & 0 & -x^o_{tc} \\ 0 & x^o_{tc} & 0 \end{bmatrix} \tag{28}$$

Substituting the above equations into equation (14) yields

$$\det\,[\mathbf{J}_{j.x} - \mathbf{A}\,\mathbf{E}^T\,\mathbf{J}_{j.\phi}] = \mathit{det} \begin{bmatrix} 0 & l_2+z^o_{tc}+r_y & l_3+z^o_{tc}+r_y \\ x^o_{tc} & 0 & 0 \\ 0 & -x^o_{tc} & -x^o_{tc} \end{bmatrix} \neq 0 \tag{29}$$

Therefore the jig hand provides an admissible constraint; one that does not conflict with the required motion of the tool.

In order to demonstrate the effectiveness of the jig hand we have to show that whilst allowing motion in the direction of the essential variables, it can still improve the stiffness of the robot to forces in those directions. For the sake of simplicity let us assume that the stiffness matrix of the robot at point O_t is diagonal and given by

$$\mathbf{K}_m = \begin{bmatrix} k_{m,x} & & & & & \\ & k_{m,y} & & & & \\ & & k_{m,z} & & & \\ & & & g_{m,x} & & \\ & & & & g_{m,y} & \\ & & & & & g_{m,z} \end{bmatrix} \tag{30}$$

Let us also assume that the stiffness matrix for the jig hand, $\mathbf{K}_j$, is diagonal and given by

$$K_j = \begin{bmatrix} 0 & & & & & \\ & k_{j,y} & & & & \\ & & k_{j,z} & & & \\ & & & g_{j,x} & & \\ & & & & 0 & \\ & & & & & 0 \end{bmatrix} \tag{31}$$

Note that this matrix satisfies the free joint condition of equation (20). In general, the stiffness elements, $k_{j,y}$, $k_{j,z}$ and $g_{j,x}$ are much higher than those of the main arm, because the link lengths are shorter.

Substituting equations (28), (29), (30) and (31) into equation (26) gives the deflection of the tool contact point in the directions of the essential variables due to a force, f_c, at that point.

$$\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \begin{bmatrix} f_{c,x} \\ f_{c,y} \\ f_{c,z} \end{bmatrix} \tag{32}$$

where the compliance elements c_{mn} are given by

$$c_{12} = c_{21} = c_{23} = c_{32} = 0$$

$$c_{11} = \left\{ \frac{1}{k_{m,x}} + \frac{(z^o{}_{tc}+r_y)^2}{g_{m,y}} \right\}$$

$$c_{13} = c_{31} = \frac{-x^o{}_{tc}\ (z^o{}_{tc}+r_y)}{g_{m,y}}$$

$$c_{22} = \left\{ \frac{1}{k_{m,y}+k_{j,y}} + \frac{(z^o{}_{tc}+r_x)^2}{g_{m,x}+g_{j,x}} + \frac{(x^o{}_{tc})^2}{g_{m,z}} \right\}$$

$$c_{33} = \left\{ \frac{1}{k_{m,z}+k_{j,z}} + \frac{(x^o{}_{tc})^2}{g_{m,y}} \right\}$$

Without the jig hand the compliance elements c_{22} and c_{33} are increased to

$$c_{22} = \left\{ \frac{1}{k_{m,y}} + \frac{(z^o{}_{tc}+r_x)^2}{g_{m,x}} + \frac{(x^o{}_{tc})^2}{g_{m,z}} \right\}$$