berkeley physics laboratory, 2d edition

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mathematics and statistics

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derivatives and integrals MS-1
trigonometric and exponential functions MS-2
loaded dice MS-3
probability distributions MS-4
binomial distribution MS-5
normal distribution MS-6

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preface

In developing this revised version of the *Berkeley Physics Laboratory*, we have tried to make the original material more useful for beginning students, many of whom will be taking their first college-level physics course concurrently with the use of this laboratory course. At the same time, we have tried very hard to preserve the essential characteristics and flavor of the original version, particularly with respect to its use of contemporary instrumentation and its frequent contact with current or recent research in physics. These qualities, we feel, were largely responsible for the fairly wide acceptance of the first edition, and we have tried to preserve them in this revision.

Most of the original experiments have been retained; the order and organization have been changed, and the discussions have all been completely rewritten with the aim of making them more readable and self-contained. In addition, a large number of completely new experiments have been developed, so that the total number of experiments is nearly double that of the original version.

Specifically, the experiments are organized into twelve units, with four to six experiments in each unit. Most units begin with rather elementary experiments and conclude with more challenging ones. Usually the same basic equipment is used for all experiments within a unit, with minor changes in accessories for the individual experiments. This scheme has the considerable pedagogical advantage that a student does not have to familiarize himself with a completely new setup for every experiment. Each experiment is subdivided into sections, each with a numbered paragraph of discussion. Thus, an instructor who wishes to assign only part of an experiment can refer to the sections by number.

Our hope is that this scheme will make a sufficiently flexible system so that instructors with various objectives can use this material as a basic resource to construct their own individualized courses, selecting those units, and those experiments or parts of experiments within units, which meet their needs. It is not essential to go straight through this course from beginning to end. Some experiments, however, do have desirable prerequisites. For example, a student should be familiar with the experiments on Electronic Instrumentation before continuing with Electric Circuits or Electrons and Fields.

In most cases the experiments have been designed so that they can be carried out reasonably thoroughly by an average student in one 3-hour laboratory period. In some cases it will be desirable to omit some sections of certain experiments or to allow more than one laboratory period. The organization of the material is, we feel, very suitable for an "open-ended" laboratory in which students work at their own pace, each according to his ability and motivation.

In the revised edition we have used the MKS system of units throughout, with occasional references to CGS or British units. Although the esthetic qualities of the MKS system can be debated, one overwhelming advantage of the MKS system is the universal use of this system in practical electrical measurements. In addition, most new elementary texts now use this system. A table of conversion factors is included for the benefit of those readers who were brought up on CGS units.

Finally, we wish to repeat the statement from the Preface to the first edition that this laboratory course may make greater demands on the average student than more conventional laboratory activities. We have tried very hard to avoid making the new material a "cookbook," and we are aware that as a result some students will have to struggle. This struggle is an essential part of the learning process, and from it will come greater strength.

Alan M. Portis
Hugh D. Young

units

In these discussions and laboratory activities, the MKSA system of units is used throughout, both for theoretical discussions and for actual measurements. In this system there are four basic units: the meter for length, the kilogram for mass, the second for time, and the ampere for electric current. All other mechanical and electrical quantities expressed in terms of these four units are given in the table of Units. We also list the corresponding CGS (Gaussian) units, where they have special names and conversion factors. In the CGS system there are only three fundamental units: the centimeter, the gram, and the second; the unit of electric charge, the statcoulomb, is expressible in terms of these.

Often it is convenient to use units related to these basic units by some power of 10. For example, we may measure length in meters, kilometers (10^3 meters), centimeters (10^{-2} meters), millimeters (10^{-3} meters), microns (10^{-6} meters), or angstroms (10^{-10} meters), depending on the scale of the corresponding physical situation. Ordinarily, with a few exceptions, related units are indicated by attaching a prefix to the basic unit. For example, kilo always means 10^3, and 1 kilometer = 10^3 meters. The prefixes in common use, with some examples of each are given in the table of Unit Prefixes.

UNIT PREFIXES

Power of ten	*Prefix*	*Abbreviation*	*Examples*
10^{12}	tera-	T	
10^9	giga-	G	gigahertz (GHz)
10^6	mega-	M	megahertz (MHz)
			megohm (MΩ), megawatt (MW)
10^3	kilo-	k	kilovolt (kV), kilowatt (kW)
10^{-2}	centi-	c	centimeter (cm)
10^{-3}	milli-	m	milliampere (mA), millihenry (mH)
10^{-6}	micro-	μ	microvolt (μV), microfarad (μF)
10^{-9}	nano-	n	nanosecond (nsec)
10^{-12}	pico-	p	picofarad (pF), picosecond (psec)

UNITS

Physical quantity	*MKSA unit*	*CGS Gaussian unit*
length	meter (m)	centimeter (cm) = 10^{-2} m
mass	kilogram (kg)	gram (g) = 10^{-3} kg
time	second (sec)	second (sec)
force	newton (N) = kg-m/sec^2	dyne = 10^{-5} N
energy	joule (J) = N-m	erg = 10^{-7} J
power	watt (W) = J/sec	erg/sec = 10^{-7} W
electric charge	coulomb (C)	statcoulomb = $\frac{10^{-9}}{2.998}$ C
electric current	ampere (A) = C/sec	abampere = 10 A
electric potential	volt (V) = J/C	statvolt = 2.998 × 10^2 V
electric field	volt/meter or newton/coulomb	
magnetic field (B)	Webers/meter^2 (Wb/m^2)	gauss = 10^{-4} Wb/m^2
resistance	ohm (Ω) = volt/ampere	
capacitance	farad (F) = coulomb/volt	
inductance	henry (H) = volt-second/ampere	

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constants

A list of physical constants which may be needed for your laboratory work is given in the table of Fundamental Physical Constants. The fundamental constants are given in MKSA units. In practical calculations, other units such as electron volts or atomic mass units are sometimes more convenient to use than the basic MKSA units. Some of the constants and combinations of constants that frequently occur are given with various units in the table of Other Useful Constants. A few commonly used conversion factors are also given.

FUNDAMENTAL PHYSICAL CONSTANTS

Name	*Symbol*	*Value*
Speed of light	c	2.998×10^{8} m/sec
Charge of electron	e	1.602×10^{-19} coul
Mass of electron	m	9.109×10^{-31} kg
Mass of neutron	m_n	1.675×10^{-27} kg
Mass of proton	m_p	1.672×10^{-27} kg
Planck's constant	h	6.626×10^{-34} joule sec
	$\hbar = h/2\pi$	1.054×10^{-34} joule sec
Permittivity of free space	ε_0	8.854×10^{-12} farad/m
	$1/4\pi\varepsilon_0$	8.988×10^{9} m/farad
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ weber/amp m
Boltzmann's constant	k	1.380×10^{-23} joule/K
Gas constant	R	8.314 joules/mole K
Avogadro's number	N_0	6.023×10^{23} molecules/mole
Mechanical equivalent of heat	J	4.186 joules/cal
Gravitational constant	G	6.67×10^{-11} N-m²/kg²

OTHER USEFUL CONSTANTS

Name	*Symbol*	*Value*
Planck's constant	h	4.136×10^{-15} eV sec
Boltzmann's constant	k	8.617×10^{-5} eV/K
Coulomb constant	$e^2/4\pi\varepsilon_0$	14.42 ev Å
Electron rest energy	mc^2	0.5110 MeV
Proton rest energy	M_pc^2	938.3 MeV
Energy equivalent of 1 amu	Mc^2	931.5 MeV
Electron magnetic moment	$\mu = e\hbar/2m$	0.9273×10^{-23} joule m²/weber
Bohr radius	$a = 4\pi\varepsilon_0\hbar^2/me^2$	0.5292×10^{-10} m
Electron Compton wavelength	$\lambda_c = h/mc$	2.426×10^{-12} m
Fine-structure constant	$\alpha = e^2/4\pi\varepsilon_0\hbar c$	1/137.0
Classical electron radius	$r_e = e^2/4\pi\varepsilon_0 mc^2$	2.818×10^{-15} m
Rydberg constant	R_∞	1.097×10^{-7} m

Conversion factors

1 eV = 1.602×10^{-19} joule
1 Å = 10^{-10} m
1 amu = 1.661×10^{-27} kg ↔ 931.5 MeV

mathematics and statistics

INTRODUCTION The first two experiments in this unit review some of the mathematics that you will use in your introductory physics course. This review will take the form of laboratory activities in which you will develop certain relations empirically. We begin with the calculus and introduce differentiation and integration in an operational way. Next we consider several special functions which are particularly useful in physics—trigonometric and exponential functions.

In the remaining experiments in this unit you will learn a few basic concepts in probability and statistics, and you will see some applications of these concepts to physical measurements. (These experiments need not be performed at the beginning of the laboratory sequence but can be introduced at any time.) Because of the central role of measurements in all of science, these concepts are of great importance. In all branches of science we deal with numbers which originate in experimental observations. In fact, the very essence of science is discovering and using correlations among quantitative observations of physical phenomena.

Statistical considerations are important for two reasons. First, measurements are never exact; the numbers which result are of very little value unless we have some idea of the extent of their inaccuracy. If several numbers are used to compute a result, we need to know how the inaccuracies of the individual numbers influence the inaccuracy of the final result. In comparing a theoretical prediction with an experimental result, we need to know something about the accuracies of both if anything intelligent is to be said about whether or not they agree. By considering the statistical behavior of errors of observation we can deal with these problems systematically to obtain results which are as precise as possible and whose remaining uncertainties are known.

A second reason for the importance of statistical concepts is that some physical laws are intrinsically statistical in nature. A familiar example is the radioactive decay of unstable nuclei. In a sample of a given unstable element, we have no way of predicting when any individual nucleus will decay, but we can describe in statistical terms how many are likely to decay in a given time interval, how many will probably be left after a certain time, and so forth. Thus, in this case, we deal not with precise predictions of events but with *probabilities* of various combinations of events. In the development of quantum mechanics, probability theory is of even more fundamental importance.

experiment **MS-1**

derivatives and integrals

introduction

Although the ideas of the calculus can be introduced without reference to any particular physical situation, we prefer to show the physical usefulness of the basic concepts by discussing a particular laboratory situation.

experiment

We shall consider the motion of a cart along a straight track. The position of the cart is described at any instant by giving its distance from some reference point on the track. We call this distance x; clearly, it varies with time (t) when the cart moves, so x is a *function* of t.

We now tilt the track slightly and release the cart at time $t = 0$ from the reference point $x = 0$, which we take near the top of the track. Then we measure the position at a succession of times, using a multiple-flash photograph, a spark timer, or some other means. The spark timer, to be discussed in more detail in Experiment M-1, uses a high-voltage pulse which causes a spark to jump from cart to track at a succession of equally spaced time intervals. The spark positions are recorded as holes in a strip of paper laid along the track, thus providing a permanent record of the successive positions.

In a certain experiment, the data obtained were as shown in Table 1. This table also includes additional columns for calculations to be described later.

Plot the data given in Table 1 on a sheet of graph paper. (K & E 46–1320, which has 10 × 10 divisions to the half inch, is suitable.) Plot time along the long direction and displacement along the short direction. Draw a smooth curve through the data points.

AVERAGE VELOCITY The average velocity during a time interval between t_1 and t_2 in which the displacement has changed from x_1 to x_2 is *defined* as

$$\bar{v} \equiv \frac{x_2 - x_1}{t_2 - t_1} \tag{1}$$

From the data given in Table 1 find the average velocity during the first second; during the first 10-sec interval; during the first 20-sec interval; during the second 10-sec interval.

INSTANTANEOUS VELOCITY The instantaneous velocity may be thought of as the value of the average velocity when the time interval becomes extremely short. As an example let us

TABLE 1

Time t, sec	Displacement x, m	Δx, m	Velocity v, m/sec	Acceleration a, m/sec^2
0.000	0.0000			
1.000	0.0064			
2.000	0.0249			
3.000	0.0544			
4.000	0.0937			
5.000	0.1420			
6.000	0.1984			
7.000	0.2621			
8.000	0.3324			
9.000	0.4088			
10.000	0.4905			
11.000	0.5772			
12.000	0.6683			
13.000	0.7633			
14.000	0.8621			
15.000	0.9641			
16.000	1.0680			
17.000	1.1769			
18.000	1.2871			
19.000	1.3994			
20.000	1.5137			

attempt from the data given in Table 1 to find the instantaneous velocity at $t = 10$ sec.

We use the shorthand notation $\Delta x \equiv x_2 - x_1$ and $\Delta t \equiv t_2 - t_1$, where the symbol Δ is the Greek letter delta. The composite symbol Δx can be called "change in x"; it is *not* the product of Δ and x! Fill in Table 2 for $\bar{v}$.

Make a plot of the average velocity $\bar{v}$ as a function of the time interval Δt. Extrapolate your data to $\Delta t = 0$. What is your estimate of the instantaneous velocity at $t = 10$ sec? What we have done graphically is to find the value which $\bar{v}$ approaches as Δt approaches zero; this is called the *limit* of $\bar{v}$ as Δt

TABLE 2

t_1	t_2	Δt	Δx	$\bar{v}$	$\bar{a}$
0	20				
5	15				
8	12				
9	11				

approaches zero and is the mathematical definition of instantaneous velocity. This defines the instantaneous velocity:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \tag{2}$$

This expression is also called the *derivative* of x with respect to t.

It may seem odd and even inconsistent that we have used velocities over definite time intervals to define instantaneous velocity at a single point, where no time interval is involved. Yet we know intuitively that instantaneous velocity at a point is a sensible concept. The concept of derivative which we have just discussed provides a sound mathematical basis for the idea of an instantaneous velocity (or any other instantaneous rate of change), and this is its most fundamental significance.

Because the velocity is changing slowly, the average velocity for an interval $\Delta t = 1$ sec and the instantaneous velocity at the center of the interval should be reasonably close. Using $\Delta t = 1$ sec as a time interval, fill in the velocity column in Table 1 from $t = 0.500$ to 19.500 sec.

With reference to your plot of Table 1 the average velocity between 0 and 20 sec is just the slope of the *chord* drawn through the displacement data points at $t = 0$ and 20 sec. Note that the *slope* of a line is not equal to the tangent of the angle the line makes with the horizontal, as it would be if the vertical and horizontal scales were the same. Here the scales are different, and have different units; to find the slope of the line we choose two points, find the differences $x_2 - x_1$ and $t_2 - t_1$, and take their quotient. Draw this chord. Also draw chords for the other intervals given in Table 2. Draw the tangent to your curve at $t = 10$ sec. Compute the slope of the tangent and compare with your extrapolated value of average velocity in the limit Δt goes to zero. What is the relation between instantaneous velocity and the slope of the tangent?

ACCELERATION

Using the same graph on which you plotted the displacement data of Table 1, also plot the velocity data, using a new coordinate scale on the right side of the paper. What can you say about the velocity as a function of time? The rate of change of velocity is called the acceleration. The average acceleration during a time interval $\Delta t = t_2 - t_1$, during which the velocity changes by an amount $\Delta v = v_2 - v_1$, is defined as

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} \tag{3}$$

What is the average acceleration in the interval between $t = 0$ and 20 sec? Fill in Table 2 for $\bar{a}$. The instantaneous acceleration is defined as the limit of the average acceleration as the time interval Δt approaches zero:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \tag{4}$$

Assuming that an interval of 1 sec is sufficiently short to give a good approximation of the instantaneous acceleration a, complete Table 1 from $t = 1.000$ to 19.000 sec. Note that the *average* acceleration between $t = 0$ and 20 sec is just the slope of the chord drawn through these velocity data points. Draw chords through your velocity data for the other time intervals in Table 2. Note that as the time interval becomes shorter the slope of the chord approaches the slope of the velocity curve. What is the relation between the slope of the tangent and the instantaneous acceleration?

Plot your acceleration data on the same sheet of graph paper, showing a new scale for acceleration.

DIFFERENTIATION The limit indicated in Eq. (2) is called the *derivative of x with respect to time*. The process of determining the instantaneous velocity if x is known as a function of time for all times is called *differentiation*. This operation is written symbolically as

$$\frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = v \tag{5}$$

Similarly, the instantaneous acceleration is expressed as

$$\frac{dv}{dt} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = a \tag{6}$$

ACCELERATION DATA We introduce the process of *integration* by considering again the cart on an inclined track. Let us imagine that this cart has mounted on it an accelerometer* and that we are able to obtain directly instantaneous values of the acceleration of the cart. We shall now see how it is possible from the acceleration alone to determine the velocity as a function of time (knowing that the cart started from rest) and the position data (knowing that the cart was at $x = 0$ at $t = 0$). The accelerometer data for the cart are given in Table 3. This table also includes additional columns for calculations to be described below. Using a new sheet of graph paper of the same kind as used earlier, plot the data for acceleration as a function of time.

VELOCITY We may use Eq. (3) to find the change in velocity during any time interval:

$$v_2 = v_1 + \bar{a}(t_2 - t_1) \tag{7}$$

That is, the velocity v_2 at the end of a time interval $(t_2 - t_1)$ is equal to the velocity v_1 at the beginning of the interval plus the average acceleration $\bar{a}$ over

* A device for measuring instantaneous acceleration.

TABLE 3

Time t, sec	*Acceleration a*, m/sec^2	$a\,\Delta t$, m/sec	*Velocity v*, m/sec	*Displacement x*, m
0.000	0.01333			0.00000
1.000	0.01206			
2.000	0.01192			
3.000	0.00988			
4.000	0.00894			
5.000	0.00809			
6.000	0.00732			
7.000	0.00662			
8.000	0.00599			
9.000	0.00542			
10.000	0.00490			
11.000	0.00444			
12.000	0.00402			
13.000	0.00363			
14.000	0.00329			
15.000	0.00298			
16.000	0.00268			
17.000	0.00244			
18.000	0.00220			
19.000	0.00200			
20.000	0.00180			

the interval multiplied by the time interval $(t_2 - t_1)$. How can we use the data of Table 3 to determine velocity? Note that this table gives the *instantaneous* acceleration and not the average acceleration. However, as we have seen, if the time interval is short enough the *average* acceleration and the *instantaneous* acceleration at the center of the interval are approximately equal.

As an example, we find the velocity change in the interval from $t = 0.500$ to 1.500 sec. We take the average acceleration $\bar{a}$ in this interval to be the instantaneous acceleration at the center ($t = 1.000$ sec), which is 0.01206 m/sec^2. Thus, according to Eq. (7), the velocity change during this interval is just 0.01206 m/sec. Similarly, the velocity change in the interval from 1.500 to 2.500 sec is 0.01192 m/sec, and so on.

The first interval (t = 0.000 to 0.500 sec) requires special treatment, being only half as long as the others. The instantaneous acceleration at t = 0.500 sec is approximately equal to the average of the values at 0.000 and 1.000 sec, which is $\frac{1}{2}(0.01333\ \text{m/sec}^2 + 0.01206\ \text{m/sec}^2)$. Then the instantaneous acceleration at the center of the interval (i.e., t = 0.250 sec) is approximately the average of this value for t = 0.0500 sec and the value for t = 0.000 sec. The end result of all this is that for the first interval,

$$\bar{a} \cong (\tfrac{3}{4})0.01333\ \text{m/sec}^2 + (\tfrac{1}{4})0.0126\ \text{m/sec}^2 = 0.01302\ \text{m/sec}^2$$

This is equivalent to taking a weighted average of the values of a at 0.000 and 1.000 sec, weighting the former three times as much as the latter because the center of the interval (0.250 sec) is "three times as close" to 0.000 as to 1.000 sec.

Thus we find that the velocity change from 0.000 to 0.500 sec is 0.00651 m/sec. Since $v = 0$ at $t = 0$, this is also the actual velocity at 0.500 sec. Now, using the successive changes in v, we can compute the actual values at t = 1.500 sec, 2.500 sec, and so on. The interval between 19.500 and 20.000 sec is handled the same way as for the first interval. Compute these velocities and record the results in Table 3. Plot your velocity data on the same sheet of graph paper used for the accelerometer data, and compare this plot with your earlier plot of velocity as determined from the displacement data. Note that the velocity of the cart at any time t is just the *area* under the acceleration curve from $t = 0$ to the final time t. If the initial velocity at time t = 0.000 is not zero but some initial value v_0, this area still gives the total *change* during the interval, and the final velocity is then the sum of v_0 and this change.

DISPLACEMENT

From Eq. (1) we have for the displacement

$$x_2 = x_1 + \bar{v}(t_2 - t_1) \tag{8}$$

This equation is analogous to Eq. (7) and states that the displacement at the end of an interval is equal to the displacement at the beginning of the interval plus the product of the average velocity over the interval and the duration of the interval. If the interval is sufficiently short, the average velocity should be just the velocity at the midpoint. As an example we compute the displacement at t = 1.000 sec. We take the average velocity between t = 0.000 and 1.000 sec to be the value computed at t = 0.500 sec, namely, v = 0.00651 m/sec. Then the displacement at t = 1.000 sec will be 0.00651 m. Similarly, the displacement at t = 2.000 sec may be computed to be

$$x = 0.00651 + 0.01857\ \Delta t = 0.02508\ \text{m}$$

In this way determine the displacement as a function of time. Note that the displacement at time t is the *area* under the velocity curve from $t = 0$ to time t. Compare your calculated displacement data with the direct data given in Table 1.

From the above discussion we see that the total velocity change in any time interval can be obtained by dividing this interval into many smaller intervals, which we may call Δt_i, multiplying each interval by the average value of a in that interval, denoted by $\bar{a}_i$, and summing these products. Symbolically,

$$v_t = v_0 + \sum_{i=1}^{N} \bar{a}_i \, \Delta t_i \tag{9}$$

If the acceleration is known continuously for every instant of time, the time intervals can be made arbitrarily small, and we speak of the *limit* of this expression as all the $\Delta t_i \to 0$ and $N \to \infty$. The usual notation is

$$\lim_{\substack{\Delta t_i \to 0 \\ N \to \infty}} \sum_{i=1}^{N} \bar{a}_i \, \Delta t_i = \int_0^t a \, dt \tag{10}$$

and the expression is called the *integral* of a. Thus we have

$$v_t = v_0 + \int_0^t a \, dt \tag{11}$$

Similarly,

$$x_t = x_0 + \int_0^t v \, dt$$

questions

1 Although the displacement data appeared to lie on a smooth curve and the computed velocity data appeared to lie on a smooth curve, some scatter appears in the acceleration data. Explain the origin of this scatter.

2 What would happen to the computed velocity data if larger time intervals were taken? What would happen to the computed accelerations?

3 In determining velocity from acceleration, what would be the deviation of the computed velocity if larger intervals were taken? What would be the deviation of the computed displacement? Does this explain any discrepancy that you obtain between direct displacement data and the displacement as computed from accelerometer data?