CALCULUS

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CALCULUS

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Preface

It is the purpose of this book to set forth in a systematic and thorough manner the fundamental principles, methods, and uses of calculus. The plan of the book is basically the same as it was in earlier editions. The changes arise largely from the experience of the authors, their colleagues, and other users of the book in the classroom. The aim of the changes is to make a clearer, more interesting, and more profitable book for the student, by incorporating various improvements in explanatory material, in diagrams, in arrangement, and in the sets of exercises.

The presentation is designed to give the student a sound understanding of the concepts of calculus, and of their applications, to make him aware of the logical structure of the subject, and to give him experience and training in the formulation and solution of problems. The illustrative examples and the exercises bear not only on mathematics, but on physics, chemistry, engineering, and to a lesser extent on economics and other disciplines.

In calculus the student has his first extensive acquaintance with limit processes. One of the foremost problems confronting the teacher of calculus is that of guiding his students toward an understanding of the basic definitions and theorems about limits. Such an understanding should lead the students to see the logical coherency in the structure of calculus, and develop in them the power to use limit concepts effectively in reasoning. It is not enough to rely entirely on a student's intuitive grasp of limit concepts. Intuitive understanding of limit processes, as they are met in every-day situations in geometry and physics, should be carefully cultivated. But the student should be taught to appreciate that mathematical reasoning in calculus, as in geometry and elsewhere, proceeds from fundamental definitions and assumptions, and systematizes its progress in theorems which can be accurately stated and proved. The student should be conscious of the definition of a derivative as an instance of a limit concept, and he should see that the development of the technique of differentiation is based largely on the theorems about limits of sums, products, and quotients. He should realize that the definite integral concept rests on a notion of limit distinct from that occurring in the definition of the derivative, and that it is by means of a chain of general theorems that we come to an understanding of how to compute the values of integrals by a process inverse to differentiation.

A feature of the present edition is the very early introduction of the inverse of differentiation, in Chapter II, §§**16, 17,** and **18.** The actual techniques here are limited to polynomials. There is a section on applications to problems of velocity and acceleration in rectilinear motion. This will be advantageous to students who take physics concurrently with the beginning of calculus. It is also possible to deal with simple area problems at an early stage, for Chapter I contains a discussion of the definition of area under a curve and a rudimentary definition of a definite integral (using subdivision of the interval into n equal parts), and Chapter II has a section on finding areas by antidifferentiation.

The full development of the relation between differentiation and integration comes in Chapter IX. The definite integral is defined as the limit of approximating sums, and the connection between differentiation and integration is worked out analytically. Not until this has been done is the word *integration* used in connection with the inverse differentiation. Adherence to this procedure in treating integration seems to us to be important if the concept of an integral is to be properly understood.

Many geometrical and physical quantities are defined by processes which closely resemble but do not exactly coincide with the definition of a definite integral as the limit of approximating sums. There is a systematic general argument which can usually be employed to show that the quantities in question actually are expressible as definite integrals, even though their definitions are not quite the same in form as the definition of a definite integral. The formalization of this general argument is what we call *Duhamel's principle*. The essential form of the principle, as stated in **§93,** is due to the late Professor W. F. Osgood. Since uniform continuity is the concept which lies back of this whole matter, no proof is presented for Duhamel's principle. The object is twofold: (1) to show that, when careful attention is paid to definitions, such topics as arc length, centroids of solids of revolution, and many others, call for something more than recognition of the definition of a definite integral, and (2) to supply a systematic procedure for attending to the situation.

The greatest changes have been made in Chapters I, II, XIV, XV, and XVI, and there are extensive changes in other chapters. The concept of a derivative has been introduced much sooner. Chapter I in its new form is less formidable for the beginner, and it makes possible more rapid progress into interesting applications of the fundamental concepts of calculus. The fundamental theorems about limits of sums, products and quotients are stated in Chapter I, but the proofs are not given until Chapter XIV. The fundamental limit theory relating to trigonometric functions has been moved from Chapter I to Chapter V. The discussion of e, both in Chapter V and the Appendix, is entirely revised. Improper integrals have been shifted from Chapter XI to Chapter XIV, and Chapter XIV contains a revised proof of l'Hospital's rule, including the $\frac{\infty}{\infty}$ case. Use of the rule is taken up earlier, in Chapter VI. The whole approach to Cauchy's principle of convergence and the convergence of bounded monotonic sequences has been rewritten. The treatment of infinite series and the expansion of functions in power series (Chapters XV and XVI) has been very much rearranged. The present arrangement is more flexible than the previous one if time does not permit the full coverage of both chapters in a single course. We have included the integral form of the remainder in Taylor's formula, and the discussion of operations with power series is somewhat fuller than it was.

The teaching of calculus is not static, fortunately, for both teachers and students. Our notions of what is best, and of what goals are attainable, undergo some changes with the years. Also, we like to try new approaches. Our colleagues have been cooperative and kind, and our students have continued to teach us both our strengths and our weaknesses. We are grateful to them all.

G.E.F.S.
A.E.T.

Foreword to the Student

You are no doubt wondering what calculus is about. We shall try to give you some idea of the nature of the subject and its importance before you set out to study it in detail.

The most important applications of calculus are to the physical sciences and to geometry. It enables us to study effectively many of the phenomena of physics: the velocity, acceleration, and general character of the motion of objects acted upon by known forces; the work done by known forces under given circumstances; the force exerted by an impounded fluid upon the walls of its container; the gravitational attraction due to material objects of various shapes and composition. It enables us to compute or measure many important things: areas; volumes; the masses of bodies of variable density; the location of the center of gravity of a body; the moment of inertia of a body when it is revolved about an axis. The laws of electricity and magnetism and all of the modern developments of atomic physics require calculus for their elaboration.

There are applications of calculus to chemistry, biology, and others of the natural sciences. Calculus is also being used in increasing measure in the study of economics.

What are the methods of calculus? It draws freely upon the subjects with which you are already familiar: algebra, trigonometry, and analytic geometry. The new concept is that of *limiting processes.* The notion of a limit is the fundamental underlying idea of calculus. It is what gives the subject its power. Accordingly, the first chapter of this book deals with the limit idea. Again, in Chapter XIV, a further development of the notion is set forth. You will not learn about limits from these chapters alone, however; the concept of a limit must be clothed with meaning as you go along, observing its occurrence in each new situation.

To study calculus, or any mathematical discipline, effectively, you must develop proper habits of study. Always read the text with a pencil and paper at hand, ready to elaborate for yourself any step in the reasoning which is not clear. It is especially desirable that you work out each illustrative example yourself, supplying any details that may have been omitted from the presentation in the book. When doing the exercises, remember that you develop your own power over the subject, and implant the method more firmly in mind, if you get along with a minimum of help from the text. You will also, in this way, discover those things that are causing you trouble.

G.E.F.S.
A.E.T.

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Greek Alphabet; Formulas and Tables for Reference

I. The Greek alphabet

Letters		*Names*	*Letters*		*Names*	*Letters*		*Names*
Α	α	alpha	Ι	ι	iota	Ρ	ρ	rho
Β	β	beta	Κ	κ	kappa	Σ	σ ς	sigma
Γ	γ	gamma	Λ	λ	lambda	Τ	τ	tau
Δ	δ	delta	Μ	μ	mu	Υ	υ	upsilon
Ε	ϵ	epsilon	Ν	ν	nu	Φ	ϕ	phi
Ζ	ζ	zeta	Ξ	ξ	xi	Χ	χ	chi
Η	η	eta	Ο	ο	omicron	Ψ	ψ	psi
Θ	θ	theta	Π	π	pi	Ω	ω	omega

II. Elementary geometry

1. Let r denote radius, h height, B area of base, θ central angle in radians, s length of arc subtended by θ, and l slant height.

Circle. $s = r\theta$. Circumference $= 2\pi r$. Area $= \pi r^2$. Area of sector $= \frac{1}{2}r^2\theta$. Area of segment $= \frac{1}{2}r^2(\theta - \sin\theta)$.

Sphere. Volume $= \frac{4}{3}\pi r^3$. Surface $= 4\pi r^2$.

Segment of sphere. Volume $= \pi h^2\left(r - \frac{h}{3}\right)$. Surface $= 2\pi rh$.

Prism. Volume $= Bh$.

Pyramid. Volume $= \frac{1}{3}Bh$.

Right circular cylinder. Volume $= \pi r^2 h$. Lateral surface $= 2\pi rh$.

Right circular cone. Volume $= \frac{1}{3}\pi r^2 h$. Lateral surface $= \pi rl$.

2. *Triangle.* Area $= \frac{1}{2}ab\sin C$, where a and b are two sides and C the included angle.

3. *Parallelogram.* Area $= ab\sin C$, where a and b are two adjacent sides and C the included angle.

4. *Frustum of pyramid or cone.* Volume $= \frac{1}{3}(B_1 + B_2 + \sqrt{B_1 \cdot B_2})h$, where h is the height and B_1 and B_2 the areas of the parallel bases.

III. Elementary algebra

1. *Quadratic equations.* If $ax^2 + bx + c = 0$, $a \neq 0$,

then
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. *General equation of degree n.* The equation $a_0x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n = 0$, $a_0 \neq 0$, n a positive integer, coefficients a_i real, has exactly n roots, multiple roots being counted to degree of multiplicity, and imaginary roots occurring in conjugate pairs.

3. *Logarithms.* $\log_b N = x$ means $b^x = N$. $\log_b b = 1$. $\log 1 = 0$.

$\log xy = \log x + \log y$. $\log \frac{x}{y} = \log x - \log y$. $\log x^n = n \log x$.

$\log_b x = \frac{\log_c x}{\log_c b}$, $b \neq 1$, $c \neq 1$.

4. *Binomial theorem.* (n a positive integer)

$$(a + b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \cdots + \frac{n(n-1)(n-2)\cdots(n-r+1)}{r!}a^{n-r}b^r + \cdots + nab^{n-1} + b^n,$$

where

$$\frac{n(n-1)(n-2)\cdots(n-r+1)}{r!} = {}_nC_r,$$

$n! = \underline{|n} = 1 \cdot 2 \cdot 3 \cdots \cdot (n-1) \cdot n$, and $0! = 1$.

5. *Arithmetic progression.* $a, a + d, a + 2d, \cdots$ where a is the first term, d the common difference, n the number of terms, S the sum of n terms, and l the last or nth term, $l = a + (n - 1)d$, $S = \frac{n}{2}(a + l)$; $A = \frac{a + b}{2}$, where A is the *arithmetic mean* of a and b.

6. *Geometric progression.* $a, ar, ar^2, \cdots$, where a is the first term, r the common ratio, n the number of terms, S_n the sum of n terms, and l the last or nth term, $l = ar^{n-1}$, $S_n = \frac{a(1 - r^n)}{1 - r}$; $G = \sqrt{ab}$, where G is the *geometric mean* of a and b (a and b positive).

$$S_\infty = \lim_{n \to \infty} S_n = \frac{a}{1 - r}$$

if and only if $r^2 < 1$.

7. *Harmonic progression.* $\frac{1}{a}, \frac{1}{a + d}, \frac{1}{a + 2d}, \cdots$; $H = \frac{2ab}{a + b}$, where H is the *harmonic mean* of a and b. $G^2 = AH$.

8. *Permutations and combinations.* For a set of n things taken r at a time,

$${}_nP_r = n(n-1)\cdots(n-r+1) = \frac{n!}{(n-r)!}. \quad 0! = 1.$$

$${}_nC_r = \frac{{}_nP_r}{r!} = \frac{n(n-1)\cdots(n-r+1)}{r(r-1)\cdots 1} = \frac{n!}{r!(n-r)!} = {}_nC_{n-r}.$$

9. *Determinants.* The determinant of order n,

$$D \equiv \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & \cdots & \cdots & a_{nn} \end{vmatrix},$$

has n^2 elements and $n!$ terms in its expansion.

$$D = \Sigma(\pm) a_{1i} a_{2j} a_{3k} \cdots a_{nl},$$

the plus or minus sign being taken according as the number of inversions of the positive integers 1, 2, 3, $\cdots$ in the sequence $i, j, k, \cdots, l$ is even or odd. The *minor* M_{ij} of the element a_{ij} is the determinant of order $n - 1$ which remains after deleting the ith row and jth column of D. The *cofactor* of the element a_{ij} is $A_{ij} = (-1)^{i+j} M_{ij}$. $D = \sum_{i=1}^{n} a_{ij} A_{ij}$, $j = 1, 2, \cdots, n$. $0 = \sum_{i=1}^{n} a_{ij} A_{ik}$ if $j \neq k$ and $j, k = 1, 2, \cdots, n$.

Properties of determinants. (1) If the corresponding rows and columns of D be interchanged, D is unchanged; (2) if any two rows (or columns) of D be interchanged, D becomes $-D$; (3) if any two rows (or columns) be identical, $D = 0$; (4) if each element of a row (or column) of D be multiplied by k, D becomes kD; (5) if to each element of a row (or column) be added k times the corresponding element of another row (or column), D is unchanged.

Application to the solution of a simultaneous system of linear equations.

$$\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = k_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = k_2 \\ \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = k_n \end{array}$$

(1) k_i not all zero: equations are nonhomogeneous and solution is unique if $D \neq 0$. If K_i is the determinant which results when the elements of the ith column of D are replaced by $k_1, k_2 \cdots k_n$, the solution is $Dx_i = K_i$, $i = 1, 2, \cdots, n$.

(2) k_i all zero: equations are homogeneous and solutions other than the obvious $x_i = 0$ exist if and only if $D = 0$.

IV. Plane trigonometry

An angle is positive if the rotation in its description is counterclockwise.

The plane angle about a point is 360 degrees or 2π radians.

In addition to the ordinary six trigonometric functions of an angle, viz. sine, cosine, tangent, cotangent, secant, and cosecant, the following

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