STATISTICAL MODEL OF THERMAL
STRESS FRACTURE OF CERAMICS

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Statistical Model of Thermal Stress Fracture of Ceramics

by

Wayne Phillip Rogers

A dissertation submitted in partial fulfillment
of the requirements for the degree of

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1987

Approved by Ashley F. Emery
Professor Ashley F. Emery
Chairman of the Supervisory Committee

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Abstract

Statistical Model of Thermal Stress Fracture of Ceramics

by Wayne Phillip Rogers

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A general model of fracture of ceramic materials under thermal stress conditions is presented. The model is a hybrid of the Weibull-Batdorf statistical theory of fracture and Stanley's modified approach to strength of brittle materials under transient stress conditions. Both analytical and numerical solution techniques for temperature, stress and probability of failure are presented. A procedure is described for incorporating the statistical theory of fracture into a finite analysis of heat transfer and stress. The probability of failure during a thermal stress cycle is predicted based on thermal and elastic constants, thermal and mechanical boundary conditions, and mechanical strength parameters. A technique is described for obtaining test-independent statistical strength parameters from mechanical tests. Strength was measured at room temperature and at high temperature in three point bending. These results were compared to tests of disk specimens in concentric ring flexure.

Theoretical predictions of failure probability as a function of thermal shock severity and time were compared to experimental results from two thermal shock tests. Four ceramic materials were studied in the water quench test. The model accurately predicted the statistical distributions of critical temperature difference for all four materials. It was found that boiling phenomena lead to a highly temperature dependent convection coefficient which strongly affects the temperature distribution, stress, and probability of failure.

A second thermal shock test, the brass rod contact test, was developed as a means of producing well defined thermal boundary condition, high rates of heat transfer, and a uniform equibiaxial stress state in disk shaped specimens. The specimen temperature was

monitored in order to calibrate the convective heat transfer coefficient directly. The time to failure during the thermal shock test was measured acoustically with a digital data acquisition apparatus. A finite element analysis of the brass rod contact test, combined with the statistical model of fracture, accurately described the experimental results for the probability of failure as a function of initial specimen temperature and time.

List of Symbols

a - cylinder radius, disk radius
Bi - Biot modulus
c - heat capacity
d - disk thickness
E - Young's modulus
G - shear modulus, function of principal stresses
h - convection coefficient
J - Bessel function, Jacobian
k - thermal conductivity
m - Weibull modulus
n - flaw density function
N - number of specimens
l - outer span in bending
L - cylinder length
P - probability of failure
P_s - probability of survival
r - radial position
R - surface resistance
s - unit surface area
t - time
t_f - time to failure
t_{max} - time to max. surface stress
S - strength
T - temperature
T_o - initial temperature
T_∞ - bath temperature
u_i - displacements
v - unit volume
x_i - cartesian coordinates
X_i - body force

α - coefficient of thermal expansion
β - zero of Bessel function
δ - Kroneker delta
Δ - temperature difference
ε - strain
θ - angle
κ - thermal diffusivity
ν - Poisson's ratio
ξ,η - local element coordinates
ρ - density
σ - stress
σ_b - mean strength in bending
σ_{mx} - maximum bending stress
σ_s - mean strength of unit surface area
σ_v - mean strength of unit volume
σ_0 - Weibull scaling parameter
$\sigma_1,\sigma_2,\sigma_3$ - principal stresses
τ - Fourier modulus
Ω - solid angle

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The author wishes to express his sincere appreciation for the guidance and support provided by his advisor, Professor Ashley Emery, during the course of this research. When the path was uncertain Dr. Emery's lightning quick insight frequently showed the way. It has also been a privilege to be associated with the members of the Supervisory Committee from the Department of Materials Science and Engineering, Professors Richard Bradt and William Scott , who contributed in many ways at various stages. This work was made possible by the National Aeronautics and Space Administration under Grant No. NAGW 199.

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