

STATISTICAL MODEL OF THERMAL
STRESS FRACTURE OF CERAMICS

ROGERS, WAYNE PHILLIP
DEGREE DATE: 1987

U·M·I Dissertation
Information Service

This is an authorized facsimile, made from the microfilm master copy of the original dissertation or masters thesis published by UMI.

The bibliographic information for this thesis is contained in UMI's Dissertation Abstracts database, the only central source for accessing almost every doctoral dissertation accepted in North America since 1861.

江苏工业学院图书馆
藏书章

U·M·I Dissertation
Information Service

University Microfilms International
A Bell & Howell Information Company
300 N. Zeeb Road, Ann Arbor, Michigan 48106
800-521-0600 OR 313/761-4700

Printed in 1989 by xerographic process
on acid-free paper

Order Number 8810586

Statistical model of thermal stress fracture of ceramics

Rogers, Wayne Phillip, Ph.D.

University of Washington, 1987

Copyright ©1987 by Rogers, Wayne Phillip. All rights reserved.

U·M·I

300 N. Zeeb Rd.
Ann Arbor, MI 48106

Order Number 8810586

Statistical model of thermal stress fracture of ceramics

Rogers, Wayne Phillip, Ph.D.

University of Washington, 1987

Copyright ©1987 by Rogers, Wayne Phillip. All rights reserved.

INFORMATION TO USERS

The most advanced technology has been used to photograph and reproduce this manuscript from the microfilm master. UMI films the original text directly from the copy submitted. Thus, some dissertation copies are in typewriter face, while others may be from a computer printer.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyrighted material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps. Each oversize page is available as one exposure on a standard 35 mm slide or as a 17" x 23" black and white photographic print for an additional charge.

Photographs included in the original manuscript have been reproduced xerographically in this copy. 35 mm slides or 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.



300 North Zeeb Road, Ann Arbor, MI 48106-1346 USA

PLEASE NOTE:

In all cases this material has been filmed in the best possible way from the available copy.
Problems encountered with this document have been identified here with a check mark ☒.

1. Glossy photographs or pages _____
2. Colored illustrations, paper or print _____
3. Photographs with dark background ☒ _____
4. Illustrations are poor copy _____
5. Pages with black marks, not original copy _____
6. Print shows through as there is text on both sides of page _____
7. Indistinct, broken or small print on several pages ☒ _____
8. Print exceeds margin requirements _____
9. Tightly bound copy with print lost in spine _____
10. Computer printout pages with indistinct print _____
11. Page(s) _____ lacking when material received, and not available from school or author.
12. Page(s) _____ seem to be missing in numbering only as text follows.
13. Two pages numbered _____. Text follows.
14. Curling and wrinkled pages _____
15. Dissertation contains pages with print at a slant, filmed as received _____
16. Other _____

U·M·I

Statistical Model of Thermal Stress Fracture of Ceramics

by

Wayne Phillip Rogers

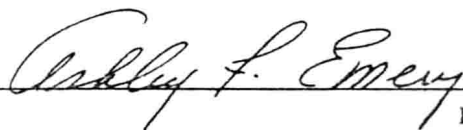
A dissertation submitted in partial fulfillment
of the requirements for the degree of

Doctor of Philosophy

University of Washington

1987

Approved by



Professor Ashley F. Emery

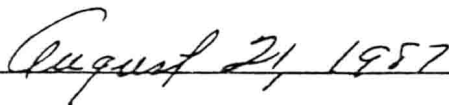
Chairman of the Supervisory Committee

Program Authorized

to Offer Degree



Date



In presenting this dissertation in partial fulfillment of the requirements for the Doctoral Degree at the University of Washington, I agree that the Library shall make its copies freely available for inspection. I further agree that extensive copying of this dissertation is allowable only for scholarly purposes, consistent with "fair use" as prescribed in the U.S. Copyright Law. Requests for copying or reproduction of this dissertation may be referred to University Microfilms, 300 North Zeeb Road, Ann Arbor, Michigan 48106, to whom the author has granted "the right to reproduce and sell (a) copies of the manuscript in microform and/or (b) printed copies of the manuscript made from microform."

Signature Wayne P. Rogers

Date 8/24/87

© Copyright by
Wayne P. Rogers
1987

In presenting this dissertation in partial fulfillment of the requirements for the Doctoral Degree at the University of Washington, I agree that the Library shall make its copies freely available for inspection. I further agree that extensive copying of this dissertation is allowable only for scholarly purposes, consistent with "fair use" as prescribed in the U.S. Copyright Law. Requests for copying or reproduction of this dissertation may be referred to University Microfilms, 300 North Zeeb Road, Ann Arbor, Michigan 48106, to whom the author has granted "the right to reproduce and sell (a) copies of the manuscript in microform and/or (b) printed copies of the manuscript made from microform."

Signature Wayne P. Rogers

Date 8/24/87

University of Washington

Abstract

Statistical Model of Thermal Stress Fracture of Ceramics

by Wayne Phillip Rogers

Chairman of the Supervisory Committee:

Professor Ashley F. Emery

Department of Mechanical Engineering

A general model of fracture of ceramic materials under thermal stress conditions is presented. The model is a hybrid of the Weibull-Batdorf statistical theory of fracture and Stanley's modified approach to strength of brittle materials under transient stress conditions. Both analytical and numerical solution techniques for temperature, stress and probability of failure are presented. A procedure is described for incorporating the statistical theory of fracture into a finite analysis of heat transfer and stress. The probability of failure during a thermal stress cycle is predicted based on thermal and elastic constants, thermal and mechanical boundary conditions, and mechanical strength parameters. A technique is described for obtaining test-independent statistical strength parameters from mechanical tests. Strength was measured at room temperature and at high temperature in three point bending. These results were compared to tests of disk specimens in concentric ring flexure.

Theoretical predictions of failure probability as a function of thermal shock severity and time were compared to experimental results from two thermal shock tests. Four ceramic materials were studied in the water quench test. The model accurately predicted the statistical distributions of critical temperature difference for all four materials. It was found that boiling phenomena lead to a highly temperature dependent convection coefficient which strongly affects the temperature distribution, stress, and probability of failure.

A second thermal shock test, the brass rod contact test, was developed as a means of producing well defined thermal boundary condition, high rates of heat transfer, and a uniform equibiaxial stress state in disk shaped specimens. The specimen temperature was

monitored in order to calibrate the convective heat transfer coefficient directly. The time to failure during the thermal shock test was measured acoustically with a digital data acquisition apparatus. A finite element analysis of the brass rod contact test, combined with the statistical model of fracture, accurately described the experimental results for the probability of failure as a function of initial specimen temperature and time.

List of Symbols

a - cylinder radius, disk radius	α - coefficient of thermal expansion
Bi - Biot modulus	β - zero of Bessel function
c - heat capacity	δ - Kroneker delta
d - disk thickness	Δ - temperature difference
E - Young's modulus	ε - strain
G - shear modulus, function of principal stresses	θ - angle
h - convection coefficient	κ - thermal diffusivity
J - Bessel function, Jacobian	ν - Poisson's ratio
k - thermal conductivity	ξ, η - local element coordinates
m - Weibull modulus	ρ - density
n - flaw density function	σ - stress
N - number of specimens	σ_b - mean strength in bending
l - outer span in bending	σ_{mx} - maximum bending stress
L - cylinder length	σ_s - mean strength of unit surface area
P - probability of failure	σ_v - mean strength of unit volume
P_s - probability of survival	σ_0 - Weibull scaling parameter
r - radial position	$\sigma_1, \sigma_2, \sigma_3$ - principal stresses
R - surface resistance	τ - Fourier modulus
s - unit surface area	Ω - solid angle
t - time	
t_f - time to failure	
t_{max} - time to max. surface stress	
S - strength	
T - temperature	
T_0 - initial temperature	
T_∞ - bath temperature	
u_i - displacements	
v - unit volume	
x_i - cartesian coordinates	
X_i - body force	

Acknowledgements

The author wishes to express his sincere appreciation for the guidance and support provided by his advisor, Professor Ashley Emery, during the course of this research. When the path was uncertain Dr. Emery's lightning quick insight frequently showed the way. It has also been a privilege to be associated with the members of the Supervisory Committee from the Department of Materials Science and Engineering, Professors Richard Bradt and William Scott , who contributed in many ways at various stages. This work was made possible by the National Aeronautics and Space Administration under Grant No. NAGW 199.

Table of Contents

	<i>Page</i>
List of Figures	iv
List of Tables	vi
List of Symbols	vii
I. Thermal Stress Fracture	1
Introduction	1
Thermal Shock in Ceramics	2
General Model of Thermal Stress Fracture	10
Objectives of the Present Study	13
II. Theoretical Background	14
Heat Transfer	14
Thermoelasticity	15
Statistical Theory of Fracture	17
Statistical Theory of Thermal Stress Fracture	27
Finite Element Method for Statistical Theory of Fracture	30
III. Experimental Procedure	36
Water Quench Test	36
Materials	36
Thermal Shock	36
Acoustic Emission	39
Brass Rod Thermal Shock	41
Materials	41
Three Point Bending of Alumina Bars	43
Concentric Ring Strength Measurement	44
Thermal Shock	46
Acoustic Emission	48
IV. Results and Discussion	50
Water Quench Test	50
Brass Rod Thermal Shock	59
Mechanical Strength Tests	59
Temperature Measurement	69
Thermal Stress Fracture	73

Effect of Parameters	81
V. Summary and Conclusions	84
Applying the Model to Design with Ceramics	85
Recommendations for Further Research	87
List of References	88
Appendix A: Analytical Solution for Cylinder Cooled by Convection	94
Appendix B: Finite Difference Solution for Cylinder with Variable Convection Coefficient	97
Appendix C: Finite Difference Solution for One Dimensional Conduction in Two Materials with Surface Resistance	99
Appendix D: Analytical Solution for Concentric Ring Biaxial Flexure Test	103
Appendix E: Mechanical Test Data	104
Appendix F: Thermal Shock Time to Failure Data	110
Appendix G: Photographs of Broken Specimens	120
Appendix H: Computer Program for Analytical Solution of Cylinder	127
Appendix I: Computer Program for Finite Difference Solution of cylinder	137
Appendix J: Computer Program for Finite Difference Solution of One Dimensional Conduction	146
Appendix K: Subroutines for Finite Element Statistical Analysis	153
Appendix L: Computer Program for Data Acquisition	160

List of Figures

Number		Page
1.1	Schematic diagram of retained strength of a typical ceramic vs quench temperature difference	6
1.2	Schematic diagram of model of statistical thermal fracture analysis	12
2.1	Schematic of volume elements in modeling probability of failure	19
2.2	Comparison of failure distributions under uniaxial, equibiaxial, and equitriaxial stress	24
2.3	Probability of failure vs. temperature difference for various values of Weibull modulus	28
2.4	Schematic illustration of probability of failure vs time during a thermal stress cycle	30
2.5	Mapping of four node element in x,y space onto ξ, η space	31
2.6	Mapping of four node element onto ξ, η space with surface stress	34
3.1	Weibull plots of retained strength of a) 99% alumina, b) 86% alumina, c) mullite, as measured in bending	38
3.2	Acoustic emission apparatus used to measure time to failure in the water quench test	40
3.3	Three point bending apparatus for high temperature testing	43
3.4	Concentric ring biaxial loading fixture	44
3.5	Diagram of support apparatus for concentric ring test fixture	45
3.6	Brass rod contact thermal shock apparatus	47
3.7	Acoustic emission apparatus used to measure time to failure in the brass rod contact thermal shock test	49
4.1	Temperature and stress distributions in 99% alumina cylinder	51
4.2	Acoustic signal from materials shocked in the water quench test	52
4.3	Probability of failure as a function of time in the water quench test	53
4.4	Probability of failure as a function of quench temperature difference	55
4.5	Functional form for temperature dependence of heat transfer coefficient	58
4.6	Probability of failure as a function of time with variable convection coefficient	58
4.7	Strength of glass tested in concentric ring flexure	61