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About the Editor

Carlos Canudas de Wit was born in Villahermosa, Tabasco, Mexico, in 1958. He received his B.Sc. degree in electronics and communications from the Technologic of Monterey, Mexico in 1980. From 1981 to 1982 he worked as a research engineer at the Department of Electrical Engineering at the CINVESTAV- del IPN, in Mexico City. In 1984 he received his M.Sc. in the Department of Automatic Control, Grenoble, France. He was visiting researcher in 1985 at The Lund Institute of Technology, Sweden. In 1987 he received his Ph.D. in automatic control from the Polytechnic of Grenoble (Department of Automatic Control), France. Since then he has been working at the same department as a researcher associated with the CNRS, where he teaches and conducts research in the area of adaptive control and robotic control. Dr. Canudas de Wit has written in 1988 a book on Adaptive Control for Partially Known Systems: Theory and Applications (Elsevier Publisher).

control design using adaptation is given in the section of Adaptation and Learning.

Robot control under kinematic singularities. One major difficulty in formulating the control problem in the Cartesian space is due to kinematic singularities. The correct understanding of the treatment of such singularities is fundamental not only for performing feedback control in the work space but also to realize tasks in the framework of force and position control. In here, the impact of kinematic singularities in the feedback control design and its relation with the concepts of nonlinear controllability are analysed.

The three day workshop was an excellent opportunity to exchange ideas and discus common topics. The nice environment of Grenoble with its beautiful mountains combined with the exoticism of the French cuisine and wines propitiated this exchange. I would like to thank the members of the program committee, M. Di Benedetto, F. Nicolo, L. Nielsen, C. Samson, G. Campion, P. Tomei and L. Dugard for making this event possible. I am also indebted to M. Spong, A. De Luca, B. Espiau, S. Nicosia, Y. Nakamura and S. Arimoto for accepting to participate in this conference and for giving interesting tutorial talks. The local organization was an important support. I would like to thank A. Aubin and S. Seleme for helping in the workshop preparation and M.R. Choisy and M.T. Decotes-Genon for ensuring an efficient organization and a warm reception. The laboratory of Automatic Control of Grenoble (LAG), which belongs to the Polytechnic Institute of Grenoble (INPG) and is associated to the National French Research Foundation (CRNS), played the roll of host in the workshop organization. Thanks are also due to I.D. Landau, director of the LAG and to M. Garnier vice-director of the INPG for supporting us in this effort. Finally, I would like to thank the society ALEPH-Technologies for kindly accepting to show their robotics activities and developments to the workshop attendees and to the MNRT-France for its financial support.

Grenoble, France, November 1990

Carlos Canudas de Wit
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Robustness of Adaptive Control of Robots: Theory and Experiment *

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Abstract

It is well known in adaptive control theory that the performance of adaptive controllers can be highly sensitive to the modeling assumptions used to prove convergence. In this paper we discuss the robustness of adaptive control of rigid robots and methods for improving robustness in the face of unmodeled dynamics and external disturbances. Both theoretical and experimental results are presented. Robustness is achieved by modifying the rigid control algorithm in two important ways. First, the rigid robot control law is incorporated into a composite slow/fast control law by adding to it a "fast" control to damp the joint oscillations. Second, so-called σ-modification is used to ensure boundedness of the estimated parameters.

1 Introduction

One of the goals of robotics research is to develop so-called intelligent robots which are capable of adapting their behavior to uncertainties in their environment. Uncertainties arise from many

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sources; unknown loads, grinding forces, part misalignment in assembly, time delays in teleoperation, unknown terrain in mobile robots, etc. Other important sources of uncertainty include uncertainties in the dynamic description of the robot itself, for example, when a "rigid" control algorithm is applied to a flexible robot. For this reason the application of adaptive control techniques in robotics has been an area of intense interest.

At the present time there are a number of "provably correct" adaptive algorithms for motion and force control of rigid robots. By "provably correct" we mean an adaptive control algorithm that uses the full Lagrangian dynamic model (considering the robot as a chain of coupled rigid bodies) and which can be proven to be globally convergent, i.e., the position and velocity tracking errors converge asymptotically to zero with all internal signals (input torque, estimation error, etc.) remaining bounded. A recent tutorial[14] contains details of seven such globally convergent adaptive algorithms. In the meantime, several additional results have been published, including results on exponential stability[25], persistency of excitation[26], improved Lyapunov arguments[27], etc. The result is that rigid robot dynamics are now well understood with respect to the design of adaptive control algorithms.

It is known, however, that the stability of adaptive systems can be highly sensitive to disturbances and unmodeled dynamics. These arise in the robotics context from several sources. External disturbances include many types of interaction with the environment. For example, robotic assembly has been described as a sequence of controlled collisions with the environment. These collision forces can be viewed as disturbances to the controller. A repetitive task, for example, subjects the robot to periodic forcing which, even in non-adaptive control, can excite complex nonlinear dynamic behavior, such as period doubling bifurcations and chaos[29].

Unmodeled dynamics include actuator/sensor dynamics, joint flexibility, link flexibility, and environment dynamics. Of these, joint flexibility is dominant in most manipulator designs. Environment dynamics arise in force and impedance controlled tasks such as assembly and grinding and will become increasingly important in future applications.

Several so-called "instability mechanisms" in adaptive control have been identified[10]. In the robotics context these instability mechanisms may manifest themselves when a rigid model is used as a basis for the design of an adaptive control algorithm. Among the mechanisms leading to instability are:

- 1) Reference trajectories which are "too fast." In other words, if the bandwidth of the reference trajectory is in the same frequency range as the unmodeled dynamics, these dynamics can be excited and drive the system unstable.

- 2) Parameter drift. The estimated parameters do not necessarily converge to their true values even in the ideal case without persistency of excitation conditions on the reference signal. In the presence of unmodeled dynamics, or in the presence of external disturbances, the parameters can drift along an equilibrium manifold until an instability results [15].

- 3) High Gain instability. This type of instability, when the controller gains are too high, is actually due to the loss of passivity from the ideal case and can occur even for non–adaptive algorithms [1].

- 4) Fast adaptation instability. This type of instability occurs when the gains in the parameter update law are too large. Due to the complicated nonlinear structure of robotic systems there are few design rules that can be called upon to design these various gains. At present the choice of such gains in adaptive robot control is largely a trial and error process.

In this paper we restrict our discussion to a treatment of the robustness of adaptive control to joint flexibility and to techniques to enhance robustness. Our design methodology, however, can be used as a basis for designing controllers which are robust to other forms of uncertainty such as actuator dynamics, external disturbances, and other effects. Our approach can be explained intuitively as follows: Using the idea of composite control of singularly perturbed systems a fast feedback control law is first designed to damp the oscillations of the fast variables representing the joint flexibility. Once the fast transients have decayed, the slow part of the system should appear nearly like the dynamics of a rigid robot, which can then be controlled using any number of techniques. Our strategy is then summarized as

$$control_{composite} = control_{slow} + control_{fast} \tag{1}$$

where $control_{slow}$ is designed using a rigid robot model and $control_{fast}$ is designed solely to provide sufficient damping of the fast dynamics. In this paper, we base our design of the slow control on the algorithm of Slotine and Li [18] because it is globally convergent in the absence of joint flexibility, and because its implementation requires only position and velocity measurements.

We will see that the presence of unmodeled dynamics greatly complicates the analysis of the tracking properties of the system. Global convergence is no longer guaranteed for all possible reference trajectories. Using the composite Lyapunov theory for singularly perturbed systems we present sufficient conditions for adaptive trajectory tracking. For point-to-point motion we show that there is always a range of joint stiffness for which convergence is achieved and we quantify the region of convergence. For tracking of (smooth and bounded) reference trajectories we give sufficient conditions for closed loop stability and uniform boundedness of the tracking error. A residual set to which the tracking error converges is quantified. We also show that for special classes of trajectories, which include step responses generated from reference models and certain joint interpolated trajectories we can achieve asymptotic tracking. The actuals details and calculations of the proofs are tedious. For this reason we have omitted most of the calculations from the present paper and the interested reader should consult the thesis [4] for complete proofs.

2 Notation and Terminology

In what follows, we use the following standard notation and terminology [3]: $\mathbf{R}_+$ will denote the set of nonnegative real numbers, and $\mathbf{R}^n$ will denote the usual n-dimensional vector space over $\mathbf{R}$ endowed with the Euclidean norm $\|\mathbf{x}\| = \{\sum_{i=1}^n {x_i}^2\}^{\frac{1}{2}}$. $\mathbf{R}^{n\times n}$ denotes the set of all $n \times n$ matrices with real elements. For each matrix $A \in \mathbf{R}^{n\times n}$, we define the induced matrix norm of A corresponding to the Euclidean vector norm $\|A\| = \left\{\lambda_{\max}\left(A^T A\right)\right\}^{\frac{1}{2}}$, where $\lambda_{\max}\left(A^T A\right)$ is the maximum eigenvalue of $A^T A$. We define the standard Lebesgue spaces $\mathbf{L}_\infty$ and $\mathbf{L}_2$ as

$$\mathbf{L}_\infty^n(\mathbf{R}_+) = \{f : \mathbf{R}_+ \to \mathbf{R}^n \ \textit{such that } f \textit{ is Lebesgue measurable and } \|f\|_\infty < \infty\} \tag{2}$$

where $\|f\|_\infty = ess \ \sup_{t\in[0,\infty)} \|f(t)\|$,

$$\mathbf{L}_2^n(\mathbf{R}_+) = \{f : \mathbf{R}_+ \to \mathbf{R}^n \ \textit{such that } f \textit{ is Lebesgue measurable and } \|f\|_2 < \infty\} \tag{3}$$

where $\|f\|_2 = \left\{\int_0^\infty \|f(t)\|^2 dt\right\}^{\frac{1}{2}}$. Denote by $\mathbf{B_x} \subset \mathbf{R}^{2n}$, $\mathbf{B}_{\theta} \subset \mathbf{R}^r$, $\mathbf{B_y} \subset \mathbf{R}^{2n}$ the closed balls centered at $\mathbf{x} = 0$, $\tilde{\theta} = 0$, and $\mathbf{y} = 0$ respectively, and let $\mathbf{B} = \mathbf{B_x} \times \mathbf{B}_{\theta} \times \mathbf{B_y} \subset \mathbf{R}^{2n} \times \mathbf{R}^r \times \mathbf{R}^{2n}$. Also define $\mathcal{B} = \left\{(\|\mathbf{x}\|, \|\tilde{\theta}\|, \|\mathbf{y}\|) : (\mathbf{x}, \tilde{\theta}, \mathbf{y}) \in \mathbf{B}\right\} \subset \mathbf{R}_+^3$.

3 Singular Perturbation Model

The dynamic equations of a flexible joint manipulator are given by [19]

$$\begin{aligned} D(\mathbf{q}_1)\ddot{\mathbf{q}}_1 + C(\mathbf{q}_1,\dot{\mathbf{q}}_1)\dot{\mathbf{q}}_1 + \mathbf{g}(\mathbf{q}_1) + K(\mathbf{q}_1 - \mathbf{q}_2) &= 0 && (4)\\ J\ddot{\mathbf{q}}_2 - K(\mathbf{q}_1 - \mathbf{q}_2) &= \mathbf{u}, && (5) \end{aligned}$$

where the vectors $\mathbf{q}_1 \in \mathbf{R}^n$ and $\mathbf{q}_2 \in \mathbf{R}^n$ represent the link angles and motor angles, respectively, $D(\mathbf{q}_1)$ is the $n \times n$ inertia matrix for the rigid links, J is a diagonal matrix of actuator inertias reflected to the link side of the gears, $C(\mathbf{q}_1,\dot{\mathbf{q}}_1)\dot{\mathbf{q}}_1$ represents the Coriolis and centrifugal terms, $\mathbf{g}(\mathbf{q}_1)$ represents the gravitational terms, and K is a diagonal matrix representing the joint stiffness. For notational simplicity we will assume that all joint stiffness constants are the same in which case K may be taken as a scalar. The composite control law $\mathbf{u}$ that we consider is given by [20] $\mathbf{u} = \mathbf{u}_s(\mathbf{q}_1,\dot{\mathbf{q}}_1,t) + \mathbf{u}_f(\dot{\mathbf{q}}_1,\dot{\mathbf{q}}_2)$, where, $\mathbf{u}_f = K_v(\dot{\mathbf{q}}_1 - \dot{\mathbf{q}}_2)$, K_v is a constant diagonal matrix, and $\mathbf{u}_s$ is designed using the following rigid model, obtained by letting the joint stiffness K tend to infinity, [19]

$$(D(\mathbf{q}_1) + J)\ddot{\mathbf{q}}_1 + C(\mathbf{q}_1,\dot{\mathbf{q}}_1)\dot{\mathbf{q}}_1 + \mathbf{g}(\mathbf{q}_1) = \mathbf{u}_s. \tag{6}$$

We define the variable $\mathbf{z} := K(\mathbf{q}_2 - \mathbf{q}_1)$, and we assume that K is $O(1/\epsilon^2)$, and K_v is $O(1/\epsilon)$, so that we may write $K = K_1/\epsilon^2$, $K_v = K_2/\epsilon$, where K_1, K_2 are $O(1)$. By substituting the control law $\mathbf{u}$ into (4)-(5), and using the definition of $\mathbf{z}$, we obtain the singularly perturbed system [20]

$$\begin{aligned} D(\mathbf{q}_1)\ddot{\mathbf{q}}_1 + C(\mathbf{q}_1,\dot{\mathbf{q}}_1)\dot{\mathbf{q}}_1 + \mathbf{g}(\mathbf{q}_1) &= \mathbf{z} && (7)\\ \epsilon^2 J\ddot{\mathbf{z}} + \epsilon K_2\dot{\mathbf{z}} + K_1\mathbf{z} &= K_1(\mathbf{u}_s - J\ddot{\mathbf{q}}_1). && (8) \end{aligned}$$

We now choose $\mathbf{u}_s$ as the adaptive control law of [18] designed for the rigid system (6). The whole adaptive system can therefore be written as

i) Plant:

$$\begin{aligned} D(\mathbf{q}_1)\ddot{\mathbf{q}}_1 + C(\mathbf{q}_1,\dot{\mathbf{q}}_1)\dot{\mathbf{q}}_1 + \mathbf{g}(\mathbf{q}_1) &= \mathbf{z} && (9)\\ \epsilon^2 J\ddot{\mathbf{z}} + \epsilon K_2\dot{\mathbf{z}} + K_1\mathbf{z} &= K_1(\mathbf{u}_s - J\ddot{\mathbf{q}}_1). && (10) \end{aligned}$$

ii) Controller (designed for the rigid plant (6)):

$$\mathbf{u}_s = (\hat{D}(\mathbf{q}_1) + \hat{J})\mathbf{a} + \hat{C}(\mathbf{q}_1,\dot{\mathbf{q}}_1)\mathbf{v} + \hat{\mathbf{g}}(\mathbf{q}_1) - K_D\mathbf{r}, \tag{11}$$

where $\hat{D}$, $\hat{J}$, $\hat{C}$ and $\hat{\mathbf{g}}$ represent the terms in (6) with estimated values of the parameters, K_D is a diagonal matrix of positive gains,

$$\tilde{\mathbf{q}}_1 = \mathbf{q}_1 - \mathbf{q}^d, \quad \mathbf{v} = \dot{\mathbf{q}}^d - \Lambda\tilde{\mathbf{q}}_1, \quad \mathbf{r} = \dot{\mathbf{q}}_1 - \mathbf{v} = \dot{\tilde{\mathbf{q}}}_1 + \Lambda\tilde{\mathbf{q}}_1, \quad \mathbf{a} = \dot{\mathbf{v}}. \tag{12}$$

Λ is a constant diagonal matrix, and $\mathbf{q}_d(t)$ is the reference trajectory which is at least three times continuously differentiable.

iii) Parameter Update Law:

$$\dot{\hat{\boldsymbol{\theta}}} = -\Gamma^{-1}Y^T(\mathbf{q}_1, \dot{\mathbf{q}}_1, \mathbf{a}, \mathbf{v})\mathbf{r}, \tag{13}$$

where Γ is a symmetric, positive definite matrix, $\tilde{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}} - \boldsymbol{\theta}$ is the parameter error, and

$$(D(\mathbf{q}_1) + J)\mathbf{a} + C(\mathbf{q}_1, \dot{\mathbf{q}}_1)\mathbf{v} + \mathbf{g}(\mathbf{q}_1) = Y(\mathbf{q}_1, \dot{\mathbf{q}}_1, \mathbf{a}, \mathbf{v})\boldsymbol{\theta}. \tag{14}$$

$Y(\mathbf{q}_1, \dot{\mathbf{q}}_1, \mathbf{a}, \mathbf{v})$ is an $n \times r$ matrix of known functions (regressor), and $\boldsymbol{\theta}$ is an r-dimensional vector of parameters.

The plant (9)-(10), the controller (11), and the parameter update law (13) are now transformed into a more suitable singularly perturbed form, details of which can be found in [4]

$$\mathcal{S} : \begin{cases} \dot{\mathbf{x}} = A_1\mathbf{x} + \Phi\tilde{\boldsymbol{\theta}} + A_3\mathbf{y} \\ \dot{\tilde{\boldsymbol{\theta}}} = -\Gamma\varphi\mathbf{x} \\ \epsilon\dot{\mathbf{y}} = A_2\mathbf{y} + \epsilon A_2^{-1}B_2\dot{\mathbf{u}}, \end{cases} \tag{15}$$

or equivalently,

$$\mathcal{S} : \begin{cases} \dot{\mathbf{p}} = f(t, \mathbf{p}, \mathbf{y}) = \begin{bmatrix} A_1 & \Phi \\ -\Gamma\varphi & 0_{r\times r} \end{bmatrix}\mathbf{p} + \begin{bmatrix} A_3 \\ 0_{r\times 2n} \end{bmatrix}\mathbf{y} \\ \epsilon\dot{\mathbf{y}} = g(t, \mathbf{p}, \mathbf{y}, \epsilon) = A_2\mathbf{y} + \epsilon A_2^{-1}B_2\dot{\mathbf{u}}, \end{cases} \tag{16}$$

where

- $\mathbf{x} = \begin{bmatrix} \tilde{\mathbf{q}}_1 \\ \mathbf{r} \end{bmatrix} = \mathcal{T}\begin{bmatrix} \tilde{\mathbf{q}}_1 \\ \dot{\tilde{\mathbf{q}}}_1 \end{bmatrix} \in \mathbf{R}^{2n}$, with the nonsingular linear transformation $\mathcal{T}$ (17)

$$\mathcal{T} = \begin{bmatrix} I_{n\times n} & 0_{n\times n} \\ \Lambda & I_{n\times n} \end{bmatrix}, \tag{18}$$

- $$\mathbf{p} = \begin{bmatrix} \mathbf{x} \\ \tilde{\theta} \end{bmatrix} \in \mathbf{R}^{2n+r}, \tag{19}$$

- $$A_1 = A_1(\mathbf{x}, \mathbf{q}_d, \dot{\mathbf{q}}_d) = \begin{bmatrix} -\Lambda & I_{n\times n} \\ -M(\mathbf{q}_1)^{-1}[C(\mathbf{q}_1, \dot{\mathbf{q}}_1) + K_D] & 0_{n\times n} \end{bmatrix} \in \mathbf{R}^{2n\times 2n}, \tag{20}$$

- $$M(\mathbf{q}_1) = D(\mathbf{q}_1) + J, \tag{21}$$

- $$\Phi = \Phi(\mathbf{x}, \mathbf{q}_d, \dot{\mathbf{q}}_d, \ddot{\mathbf{q}}_d) = \begin{bmatrix} 0_{n\times r} \\ M(\mathbf{q}_1)^{-1} Y(\mathbf{q}_1, \dot{\mathbf{q}}_1, \mathbf{v}, \mathbf{a}) \end{bmatrix} \in \mathbf{R}^{2n\times r}, \tag{22}$$

- $$A_3 = A_3(\mathbf{x}, \mathbf{q}_d) = \begin{bmatrix} 0_{n\times n} & 0_{n\times n} \\ M(\mathbf{q}_1)^{-1} & 0_{n\times n} \end{bmatrix} \in \mathbf{R}^{2n\times 2n}, \tag{23}$$

- $$\varphi = \varphi(\mathbf{x}, \mathbf{q}_d, \dot{\mathbf{q}}_d) = \begin{bmatrix} 0_{r\times n} & Y^T(\mathbf{q}_1, \dot{\mathbf{q}}_1, \mathbf{a}, \mathbf{v}) \end{bmatrix} \in \mathbf{R}^{r\times 2n}, \tag{24}$$

- $$A_2 = \begin{bmatrix} 0_{n\times n} & I_{n\times n} \\ -J^{-1}K_1 & -J^{-1}K_2 \end{bmatrix} \in \mathbf{R}^{2n\times 2n}, \tag{25}$$

- $$B_2 = \begin{bmatrix} 0_{n\times n} \\ J^{-1}K_1 \end{bmatrix} \in \mathbf{R}^{2n\times n}, \tag{26}$$

- $$\mathbf{u} = \mathbf{u}_s - J\ddot{\mathbf{q}}_1, \tag{27}$$

- $$\mathbf{y} = \begin{bmatrix} \mathbf{z} \\ \epsilon\dot{\mathbf{z}} \end{bmatrix} + A_2^{-1} B_2 \mathbf{u} \quad \in \mathbf{R}^{2n}. \tag{28}$$

4 Analysis of the Singularly Perturbed System $\mathcal{S}$

System $\mathcal{S}$ is a nonautonomous nonlinear singularly perturbed system in the standard form [12]. $\mathbf{p}$ is the slow variable, and $\mathbf{y}$ is the fast variable. The analysis of system $\mathcal{S}$ follows the techniques of composite Lyapunov functions for nonlinear singularly perturbed systems developed in [17]; see also [12].

The boundary layer system, denoted $\mathcal{S}_b$, is defined as

$$\mathcal{S}_b \ : \ \frac{d\mathbf{y}}{d\tau} = g(t, \mathbf{p}, \mathbf{y}(\tau), \epsilon = 0) = A_2\mathbf{y}, \tag{29}$$

where $\tau = t/\epsilon$ is a stretching time scale. Let P be the symmetric positive definite matrix that satisfies the Lyapunov Equation $A_2^TP + PA_2 = -Q$, where Q is a positive definite matrix. We choose, for the boundary layer system, the Lyapunov Function Candidate $W(\mathbf{y}) = \mathbf{y}^TP\mathbf{y}$.

The reduced system, or quasi-steady state, is defined by setting $\epsilon = 0$ in $\mathcal{S}$, that is,

$$\dot{\mathbf{p}} = f(t, \mathbf{p}, \mathbf{y}) = \begin{bmatrix} A_1 & \Phi \\ -\Gamma\varphi & 0_{r\times r} \end{bmatrix} \mathbf{p} + \begin{bmatrix} A_3 \\ 0_{r\times 2n} \end{bmatrix} \mathbf{y} \tag{30}$$

$$0 = g(t, \mathbf{p}, \mathbf{y}, \epsilon = 0) = A_2\mathbf{y}. \tag{31}$$

Since A_2 is invertible, the algebraic equation (31) has the unique root $\mathbf{y} = 0$. The reduced system, denoted $\mathcal{S}_r$, is obtained by replacing $\mathbf{y} = 0$ into (30)

$$\mathcal{S}_r \ : \ \dot{\mathbf{p}} = f(t, \mathbf{p}, \mathbf{y} = 0) = \begin{bmatrix} A_1 & \Phi \\ -\Gamma\varphi & 0_{r\times r} \end{bmatrix} \mathbf{p}, \tag{32}$$

or equivalently,

$$\mathcal{S}_r : \begin{cases} \dot{\mathbf{x}} = A_1\mathbf{x} + \Phi\tilde{\theta} \\ \dot{\tilde{\theta}} = -\Gamma\varphi\mathbf{x}. \end{cases} \tag{33}$$

Fact 1 *The reduced system $\mathcal{S}_r$ is equivalent to the adaptive rigid-joint system* [4].

□

A consequence of Fact 1 is that we can use the same Lyapunov function candidate as that of the adaptive rigid-joint system [14], [22], namely,

$$\begin{aligned} V &= \frac{1}{2}\mathbf{r}^TM(\mathbf{q}_1)\mathbf{r} + \tilde{\mathbf{q}}_1^T\Lambda^TK_D\tilde{\mathbf{q}}_1 + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} \\ &= V(\mathbf{q}_d, \tilde{\mathbf{q}}_1, \mathbf{r}, \tilde{\theta}) = V(t, \mathbf{x}, \tilde{\theta}) = V(t, \mathbf{p}). \end{aligned} \tag{34}$$

Define

$$F \quad := \quad I_{n\times n} + JD(\mathbf{q}_1)^{-1}, \tag{35}$$

$$\rho(t) \quad := \quad \frac{\partial \mathbf{u}}{\partial \mathbf{q}_d}\dot{\mathbf{q}}_d + \frac{\partial \mathbf{u}}{\partial \dot{\mathbf{q}}_d}\ddot{\mathbf{q}}_d + \frac{\partial \mathbf{u}}{\partial \ddot{\mathbf{q}}_d}\mathbf{q}_d^{(3)}. \tag{36}$$

For $\forall(\mathbf{x}, \tilde{\theta}, \mathbf{y}) \in \mathbf{B}$, define positive constants k_1, k_2, k_3, and positive functions $k_4(t)$, $k_5(t)$ and $k_g(t)$ such that

$$\bullet(\text{a1}) \ : \ \left\| F\frac{\partial \mathbf{u}}{\partial \mathbf{x}}A_3\mathbf{y} + \frac{1}{\epsilon}F\frac{\partial \mathbf{u}}{\partial \mathbf{y}}A_2\mathbf{y}\right\| \leq (k_3 + \frac{1}{\epsilon}k_2)\,\|\mathbf{y}\|, \tag{37}$$

$$\begin{aligned}
\bullet(\text{a2}) \ : \ & \left\| \left\{ F\frac{\partial \mathbf{u}}{\partial \mathbf{x}}A_1 - F\frac{\partial \mathbf{u}}{\partial \tilde{\theta}}\Gamma\varphi \right\}\mathbf{x} + F\frac{\partial \mathbf{u}}{\partial \mathbf{x}}\Phi\tilde{\theta} \right\| \\
& \leq \left\| \left\{ F\frac{\partial \mathbf{u}}{\partial \mathbf{x}}A_1 - F\frac{\partial \mathbf{u}}{\partial \tilde{\theta}}\Gamma\varphi \right\} \right\| \|\mathbf{x}\| \\
& + \left\| F\frac{\partial \mathbf{u}}{\partial \mathbf{x}} \begin{bmatrix} 0 \\ M^{-1}\left(-\tilde{M}(\mathbf{q}_1)\Lambda\dot{\tilde{\mathbf{q}}}_1 - \tilde{C}(\mathbf{q}_1, \dot{\mathbf{q}}_1)\Lambda\tilde{\mathbf{q}}\right) \end{bmatrix} \right\| \\
& + \left\| F\frac{\partial \mathbf{u}}{\partial \mathbf{x}} \begin{bmatrix} 0 \\ M^{-1}\left(\tilde{M}(\mathbf{q}_1)\ddot{\mathbf{q}}_d + \tilde{C}(\mathbf{q}_1, \dot{\mathbf{q}}_1)\dot{\mathbf{q}}_d\right) \end{bmatrix} \right\| \\
& + \left\| F\frac{\partial \mathbf{u}}{\partial \mathbf{x}} \begin{bmatrix} 0 \\ M^{-1}Y_g(\mathbf{q}_1)\tilde{\theta}_g \end{bmatrix} \right\| \\
& \leq k_1\,\|\mathbf{x}\| + k_5(t) + k_g(t).
\end{aligned} \tag{38}$$

$$\bullet(\text{a3}) \ : \ \|F\rho(t)\| \leq k_4(t). \tag{39}$$

Note that the existence of the various constants k_i in the above estimates requires only continuity of the functions involved since the set $\mathbf{B}$ is compact. Note also that $k_5(t)$ is proportional to the norms of the $\dot{\mathbf{q}}_d$ and $\ddot{\mathbf{q}}_d$, and that $k_g(t)$ is proportional to the norm of $\tilde{\mathbf{g}} = Y_g(\mathbf{q}_1)\tilde{\theta}_g$.

Consider the following composite Lyapunov function candidate for the singularly perturbed system $\mathcal{S}$

$$\mathcal{V}_1(t, \mathbf{x}, \tilde{\theta}, \mathbf{y}) = (1-d)\ V(t, \mathbf{x}, \tilde{\theta}) + d\ W(\mathbf{y}) \quad , \quad 0 < d < 1, \tag{40}$$

which represents a weighted sum of $V(t, \mathbf{x}, \tilde{\theta})$, the Lyapunov function of the reduced system $\mathcal{S}_r$, and $W(\mathbf{y})$, the Lyapunov function of the boundary layer system $\mathcal{S}_b$. Taking into account (a1),(a2),