



MODERN CONTROL SYSTEMS

A Manual of Design Methods

John A. Borrie

MODERN CONTROL SYSTEMS: A Manual of Design Methods

JOHN A. BORRIE



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**MODERN CONTROL SYSTEMS:
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PREFACE

This book contains an ordered presentation of practical modern control engineering techniques with explanations, formulas, and examples, but without mathematical proofs. Many detailed suggestions are made for the construction of computer aided design (CAD) algorithms suited to readily available microcomputers supporting BASIC. At the end of each chapter a limited but carefully selected bibliography helps the reader to explore further. Continuous and discrete time systems are given equal emphasis.

Chapter 1 sets the mathematical background with topics such as differential and difference equations, Laplace, z , Fourier transforms, and stochastic system definitions. It includes CAD algorithm designs which are useful in themselves and as subroutines for larger programs. Chapter 2 deals with 'classical' techniques for continuous and discrete time systems. Chapters 3 and 4 describe modern control ideas for linear systems including pole shifting, state estimation, stochastic systems, and Kalman filters. Chapter 5 is devoted to computing methods for function optimization and system identification since these are keys to the future development of this subject. The appendixes contain basic definitions, methods, and algorithms.

This material, which has been warmly welcomed by many short-course students, is suited to professional engineers, postgraduates, and advanced undergraduates. It is not intended as a first introduction to control engineering.

The author is grateful to the considerable number of Cranfield students who have investigated the methods and algorithms outlined in this book. He is also indebted to the staff members who have developed the laboratory experiments. These have provided valuable insight without being unduly complex, and can perhaps be copied by the interested reader.

J. A. B.

**MODERN CONTROL SYSTEMS:
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CONTENTS

Preface xi

1	Basic Definitions and Mathematical Techniques	1
1.1	Introduction	1
1.2	Ordinary Differential Equations	1
	CAD Facility	3
1.3	The Laplace Transform	7
	(i) The Laplace Transform	7
	(a) Linearity	8
	(b) Delayed Function	8
	(c) Initial Value Theorem	8
	(d) Final Value Theorem	8
	(e) Differentiation with respect to t	8
	(f) Integration with respect to t	9
	(ii) The Inverse Laplace Transform	9
1.4	The Solution of Differential Equations using Laplace Transforms	10
	CAD Facility	11
1.5	System Transfer Functions and Dynamic Behavior (Continuous Time)	14
1.6	Transfer Functions of Some Common Circuits	17
1.7	Discrete Time Functions	17
1.8	The z Transform	22
	(i) The z Transform	22
	(a) Linearity	24
	(b) Initial Value Theorem	24
	(c) Final Value Theorem	24
	(d) Shift in Time	24
	(e) Multiplication by e^{an}	24
	(ii) The Inverse z Transform	24
	(a) The General Formula	24

(b)	Table Look-up	28
(c)	Power Series	29
	CAD Facility	30
1.9	The Solution of Difference Equations Using z Transforms	30
1.10	System Transfer Functions and Dynamic Behavior (Discrete Time)	31
1.11	Sampled Data Systems	35
1.12	Relationship Between s and z Plane Poles	38
1.13	Block Diagrams and Composite Systems	40
1.14	Modified z Transforms	40
1.15	The Fourier Transform (Continuous Functions of Time)	42
(a)	Linearity	43
(b)	Shifting in Time	43
(c)	Shifting in Frequency	43
(d)	Differentiation of $f(t)$ with respect to t	43
(e)	Differentiation of $F(j\omega)$ with respect to ω	43
(f)	Real and Imaginary Parts of $F(j\omega)$	43
(g)	Amplitude and Phase Characteristics	44
(h)	Common Fourier Transforms	45
1.16	System Frequency Behavior	46
1.17	The Fourier Transform (Discrete Functions of Time)	48
(a)	Linearity	49
(b)	Shifting in Time	49
(c)	Shifting in Frequency	49
(d)	Amplitude and Phase Characteristics	49
(e)	Common Fourier Transforms	49
1.18	Shannon's Sampling Theorem	50
1.19	Discrete Time System Frequency Behavior Analysis	51
1.20	The Discrete Fourier Transform and its Inverse	52
(i)	Calculation of the Discrete Fourier Transform	53
	CAD Facility	55
(ii)	Calculation of the Inverse Discrete Fourier Transform	57
1.21	Some Properties of Continuous Time Stochastic Processes	58
1.22	Some Properties of Discrete Time Stochastic Processes	65
	References	70
2	Classical Techniques for Continuous and Discrete Time Systems	71
2.1	Introduction	71
2.2	Bode Plots (Continuous Time)	71
2.3	Root Locus Diagrams (Continuous Time)	77
2.4	Nyquist Diagrams (Continuous Time)	83
2.5	Nichols Charts (Continuous Time)	86
2.6	Inverse Nyquist Diagrams (Continuous Time)	89
2.7	Servomechanism Design Using Bode Plots, Root Locus Diagrams, Nyquist Diagrams, and Nichols Charts	91

- (i) Bode Plot Design (Continuous Time) 92
- (ii) Root Locus Diagram Design (Continuous Time) 94
- (iii) Nyquist Diagram Design (Continuous Time) 96
- (iv) Nichols Chart Design (Continuous Time) 98
- (v) Inverse Nyquist Diagram Design (Continuous Time) 98
- 2.8 CAD Facility 102
 - Comment 109
- 2.9 Bode Plots (Discrete Systems) 109
- 2.10 Root Locus Diagrams (Discrete Systems) 115
- 2.11 Nyquist Diagrams (Discrete Systems) 119
- 2.12 Nichols Charts (Discrete Systems) 122
- 2.13 Digital Servomechanism Design Using Bode Plots, Root Locus Diagrams, Nyquist Diagrams, and Nichols Charts 123
 - (i) Design for a Short Sampling Period 124
 - (a) Backward Difference Approximation 124
 - (b) Approximation by Bilinear Transformation 125
 - (ii) Bode Plot Design (Discrete Systems) 126
 - (iii) Root Locus Design (Discrete Systems) 130
 - (iv) Nyquist Diagram Design (Discrete Systems) 131
 - (v) Nichols Chart Design (Discrete Systems) 132
- 2.14 CAD Facility 134
- 2.15 Proportional Integral Differential Controllers 134
 - (i) The Reaction Curve Method 136
 - (ii) The Continuous Cycling Method 137
 - Comment 141
- References 141

3 Continuous Time State Space Design Techniques 142

- 3.1 Introduction 142
- 3.2 State Equations 142
- 3.3 Properties of Linear System State Equations 146
 - (i) Solution of the State Equations 146
 - CAD Facility 147
 - (ii) Similar Systems 149
 - (iii) Solution of the State Equations by Laplace Transforms 152
 - CAD Facility 153
- 3.4 Properties of LTI Systems 155
 - (i) Controllability 155
 - (ii) Observability 157
 - (iii) Stability of LTI Systems 158
- 3.5 Realization of State Equations 160
 - (i) Single-input, Single-output Systems 160
 - (ii) Multivariable Systems 163
 - (a) A General Formula 163

	CAD Facility	164
	(b) A Non-general Formula	167
	CAD Facility	168
3.6	Pole Shifting by State Feedback	170
	(i) Single-input Systems	172
	CAD Facility	173
	Comment	176
	(ii) Multi-input Systems – Dyadic Feedback	176
3.7	Optimal Control Strategies	180
	(i) Linear Quadratic Optimal Control	181
	CAD Facility	182
	Comment	188
	(ii) Switching Curve Strategy	189
	Comment	193
3.8	State Estimators for LTI Systems	193
	(i) The Asymptotic State Estimator	194
	CAD Facility	195
	(ii) The Reduced-order Estimator (Luenberger Observer)	195
	CAD Facility	196
	Comment	201
3.9	Modeling and Behavior of Stochastic Systems	201
	(i) Systems Affected by White Noise	201
	(ii) Systems Affected by ‘Colored’ Noise	202
	(iii) The Mean and Covariance of the State and Output	203
3.10	Multivariable System Design Methods	204
	CAD Facility	207
	Comment	214
	References	214

4 Discrete Time State Space System Models 215

4.1	Introduction	215
4.2	State Equations	215
4.3	Properties of LTI System State Equations	216
	(i) Solution of State Equations	216
	CAD Facility	217
	(ii) Similar Systems	220
	(iii) Solution of State Equations by z Transforms	221
	CAD Facility	222
4.4	Properties of LTI Systems	222
	(i) Reachability (or Controllability)	222
	(ii) Observability	222
	(iii) Stability of LTI Systems	222
4.5	Realization of State Equations	223
	(i) Single-input, Single-output Systems	223

(ii)	Multivariable Systems	224
4.6	Sampled Data Systems	224
	CAD Facility	226
4.7	Pole Shifting by State Feedback	228
4.8	Optimal Control by State Feedback	232
	Comment	235
4.9	State Estimators for LTI Systems	235
	(i) The Asymptotic State Estimator	235
	(ii) The Reduced-order Estimator (Luenberger Observer)	236
4.10	Modelling and Behavior of Stochastic Systems	238
	(i) Systems Affected by Colored Noise	238
	(ii) The Mean and Covariance of the State and Output	239
	(iii) Sampled Data Stochastic Systems	240
4.11	The Kalman Filter	241
	(i) Basic Kalman Filter for Linear Time-variant Systems	242
	CAD Facility	243
	(a) A Suboptimal Kalman Filter	247
	(b) An Optimal Filter	248
	(ii) The 'Split' Kalman Filter	250
	(iii) The Extended Kalman Filter	252
	Comment	255
	References	256
5	Useful Computer Techniques	257
5.1	Introduction	257
5.2	General Statement of the Optimization Problem	257
5.3	The Simplex Method	258
	Comment	259
	CAD Facility	259
5.4	Alternating Variable Methods	261
	CAD Facility	264
5.5	Path of Steepest Ascent or Descent	266
	Comment	270
	CAD Facility	270
5.6	Some Useful Two-dimensional Object Functions	271
	(i) Booth's Function	272
	(ii) Zettl's Function	272
	(iii) Rosenbrock's Function	273
5.7	System Identification by Computer Program	273
5.8	Least Squares Estimation of System Parameters	274
	CAD Facility	278
	Comment	279
5.9	Maximum Likelihood Estimation of System Parameters	281
	CAD Facility	283
	Comment	284

5.10 Other Methods of System Parameter Identification	285
References	285

Appendix A Notes on Vectors, Matrices, and Determinants 286

A.1 Vectors	286
A.2 Matrices – Basic Definitions	288
A.3 The Determinant of a Square Matrix	289
A.4 The Minors, Cofactors, Adjoins, and Trace of a Square Matrix	290
A.5 The Rank of a Matrix	291
CAD Facility	294
A.6 The Solution of Simultaneous Equations	294
CAD Facility	295
A.7 The Inverse of a Square Matrix	296
CAD Facility	298
A.8 Some Properties of Matrices and Determinants	298
A.9 The Eigenvalues and Eigenvectors of a Square Matrix; the Cayley–Hamilton Theorem	300
(i) Eigenvalues and Eigenvectors	300
(ii) The Cayley–Hamilton Theorem	301
CAD Facility	301
A.10 Similar Matrices	303
A.11 Definite and Semidefinite Quadratic Forms	304

Appendix B Notes on Probability Functions 306

B.1 Basic Definitions	306
B.2 Mean, Variance, and Standard Deviation	307
B.3 Joint and Conditional Probability Density Functions	309
B.4 The Central Limit Theorem	310
B.5 Pseudo-random Signal Generation	310
(i) Pseudo-random Binary Signals (PRBS)	311
(ii) Evenly Distributed Random Numbers	312
(iii) Random Numbers with Gaussian Distribution	313

Appendix C Notes on Delta Functions 314

C.1 The Dirac Delta Function	314
C.2 The Kronecker Delta Function	315
References	315

<i>Index</i>	317
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1 BASIC DEFINITIONS AND MATHEMATICAL TECHNIQUES

1.1 INTRODUCTION

Some basic definitions and mathematical techniques are set out in this chapter in a compact form, together with suggestions for the design of CAD algorithms. Topics covered include linear differential and difference equations, Laplace, z , and Fourier transforms and basic stochastic system definitions. The reference books listed at the end provide thorough introductions to these topics from a fairly elementary level.

1.2 ORDINARY DIFFERENTIAL EQUATIONS

The behavior of many dynamical systems can be modeled by one or more differential equations of the form:

$$F(y, y^{(1)}, y^{(2)}, \dots, y^{(m)}, t) = 0 \quad (1.1)$$

where t represents time, y represents some aspect of the system, typically its output, and

$$y^{(1)} = \frac{dy}{dt}; \quad y^{(2)} = \frac{d^2y}{dt^2}; \quad \dots \quad ; \quad y^{(m)} = \frac{d^m y}{dt^m}.$$

Equation (1.1) is m th order (the highest order of derivative) and ordinary (only ordinary derivatives involved).

The general solution, i.e. the function $y(t)$ which satisfies Eq. (1.1), is of the form:

$$y(t) = y(t, c_0, c_1, \dots, c_{m-1})$$

where c_0, c_1, \dots, c_{m-1} are constants which can usually be determined if values of $y, y^{(1)}, y^{(2)}, \dots, y^{(m-1)}$ are known for some specific value of t , say t_0 . Typically, 'initial' values $y(0), y^{(1)}(0), \dots, y^{(m-1)}(0)$ are known.

EXAMPLE

Consider the first-order differential equation:

$$\frac{dy}{dt} + 2yt - e^{-t^2} = 0. \quad (1.2)$$

This has the general solution:

$$y = e^{-t^2}(t + c_0)$$

where c_0 is a constant.

Given the 'initial' value $y(0) = 1$, it is easy to show by substitution that the solution is:

$$y = e^{-t^2}(t + 1)$$

While some useful types of differential equation do have analytic solutions (ref. 1), numerical solutions can very often be found for these and less tractable cases. This topic is of interest here.

If Eq. (1.1) can be rewritten with the highest-order derivatives on the left-hand side (LHS)—and it usually can—it can be recast as two equations, one first and one $(m - 1)$ th order. Repeating this process, Eq. (1.1) can finally be cast as a set of m first-order equations which in vector format (App. A.1) is:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t) \quad (1.3)$$

with initial conditions:

$$\mathbf{x}(0) = \mathbf{x}_0$$

where

$$\mathbf{x} = (x_1 x_2 x_3 \dots x_m)^T.$$

EXAMPLE

Consider the third-order differential equation:

$$\frac{d^3y}{dt^3} + 9\frac{d^2y}{dt^2} + 26\frac{dy}{dt} + 24y - 1 = 0 \quad (1.4)$$

with initial conditions:

$$y(0) = 4, \quad y^{(1)}(0) = 3, \quad y^{(2)}(0) = 2.$$

By setting $x_1 = y$, rewriting Eq. (1.4) with d^3x_1/dt^3 on the LHS and substituting $x_2 = dx_1/dt$, two equations are generated, and by repeating the process with $x_3 = dx_2/dt$, Eq. (1.4) is recast:

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_3 \\ -9x_3 - 26x_2 - 24x_1 + 1 \end{bmatrix}$$

$$y = x_1$$

with initial conditions:

$$\mathbf{x}(0) = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}.$$

This is in the form of Eq. (1.3)

Once a differential equation, or set of equations, has been cast in this form, it can be fed to a fairly simple CAD algorithm to yield a numerical solution.

CAD Facility

Equations in the form of Eq. (1.3) with $\mathbf{x}(0) = \mathbf{x}_0$ can be solved numerically; i.e. the value $\mathbf{x}(nh)$, $n = 0, 1, 2, \dots, h$ a calculation interval, can be found by several well-known methods.

An interactive CAD algorithm is shown in the flow diagram of Fig. 1.1.

The operation of the algorithm is as follows.

BLOCK 1

The differential equation is input in the form of Eq. (1.3) or in a manner allowing easy conversion to this format. It is displayed and corrected if necessary.

BLOCK 2

The calculation interval h , a printout ratio, R , and the total number of iterations required, N , are input.

BLOCK 3

$\mathbf{x}(nh)$, $n = 0, 1, 2, \dots, N$, are calculated. Two types of mathematically stable method are outlined here.

(a) *Runge-Kutta Methods* (refs. 5, 7). These are based on the Taylor expansion:

$$\mathbf{x}((n+1)h) = \mathbf{x}(nh) + h\mathbf{x}'^{(1)}(nh) + \frac{h^2}{2!}\mathbf{x}''^{(2)}(nh) + \frac{h^3}{3!}\mathbf{x}'''^{(3)}(nh) + \dots \quad (1.5)$$

Commonly, four terms in this series are used to derive the fourth-order Runge-Kutta algorithm:

Calculate:

$$\mathbf{k}_1 = h\mathbf{f}(\mathbf{x}(nh), nh)$$

$$\mathbf{k}_2 = h\mathbf{f}(\mathbf{x}(nh) + \frac{1}{2}\mathbf{k}_1, nh + \frac{1}{2}h)$$

$$\mathbf{k}_3 = h\mathbf{f}(\mathbf{x}(nh) + \frac{1}{2}\mathbf{k}_2, nh + \frac{1}{2}h)$$

$$\mathbf{k}_4 = h\mathbf{f}(\mathbf{x}(nh) + \mathbf{k}_3, nh + h).$$

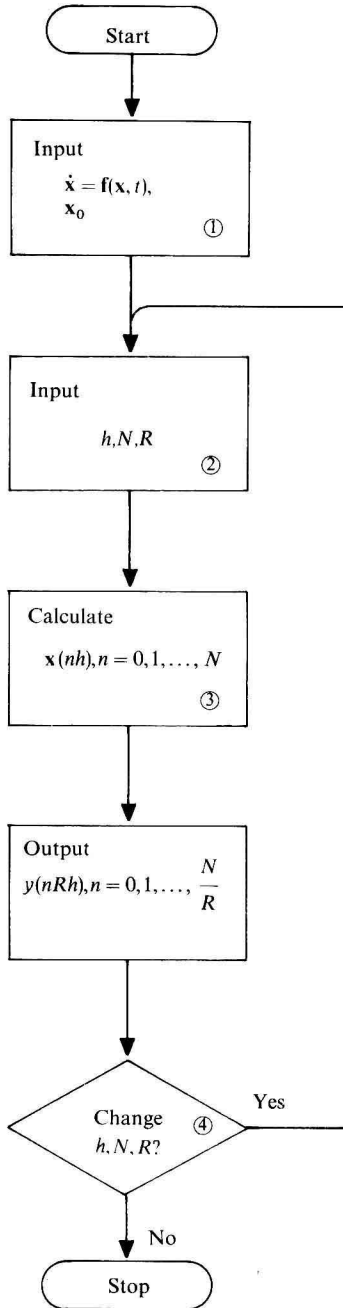


Fig. 1.1 CAD program for solving differential equations.

Then:

$$\mathbf{x}((n+1)h) = \mathbf{x}(nh) + \frac{1}{6}(\mathbf{k}_1 + 2\mathbf{k}_2 + 2\mathbf{k}_3 + \mathbf{k}_4).$$

Higher accuracy, at the cost of added complexity, can be achieved using a fifth-order Runge–Kutta algorithm (ref. 7). For simple cases, only two terms need be considered to yield the second-order algorithm:

Calculate:

$$\mathbf{k}_1 = h\mathbf{f}(\mathbf{x}(nh), nh)$$

$$\mathbf{k}_2 = h\mathbf{f}(\mathbf{x}(nh) + \frac{1}{2}\mathbf{k}_1, nh + \frac{1}{2}h).$$

Then:

$$\mathbf{x}((n+1)h) = \mathbf{x}(nh) + \mathbf{k}_2.$$

In each of these algorithms, since $\mathbf{x}((n+1)h)$ can be found from $\mathbf{x}(nh)$, the starting data $\mathbf{x}(0)$ are sufficient to allow the algorithm to proceed.

The Runge–Kutta method yields estimates of calculation errors only with some difficulty (ref. 7). Predictor–corrector methods are better in this respect, and one of these is outlined next.

(b) *Predictor–Corrector Methods*. Perhaps the simplest such method, due to Adams–Moulton (ref. 5), uses the information $\mathbf{x}(nh)$, $\mathbf{x}((n+1)h)$, to predict $\mathbf{x}((n+2)h)$ by linear extrapolation.

$$\bar{\mathbf{x}}(n+2)h = \mathbf{x}(nh) + \frac{h}{2}\{3\mathbf{f}[\mathbf{x}((n+1)h), (n+1)h] - \mathbf{f}(\mathbf{x}(nh), nh)\}.$$

This is then used in the correction formula:

$$\begin{aligned} \mathbf{x}((n+2)h) = \mathbf{x}((n+1)h) + \frac{h}{2}\{ & \mathbf{f}[\mathbf{x}((n+1)h), (n+1)h] \\ & + \mathbf{f}[\bar{\mathbf{x}}((n+2)h), (n+2)h]\}. \end{aligned}$$

The second equation can be used a number of times to improve $\mathbf{x}((n+2)h)$ until a stable value is obtained.

More sophisticated algorithms of this kind are based on nonlinear extrapolation formulas using three or more initial known values, typically $\mathbf{x}(0)$, $\mathbf{x}(h)$, $\mathbf{x}(2h)$. The initial information required by such algorithms is usually generated by Runge–Kutta methods.

BLOCK 4

The calculation interval may be shortened after a run, and the process repeated. If compatible results are obtained, the solution may be judged satisfactory and the longer interval selected for further runs. Otherwise the process is repeated.