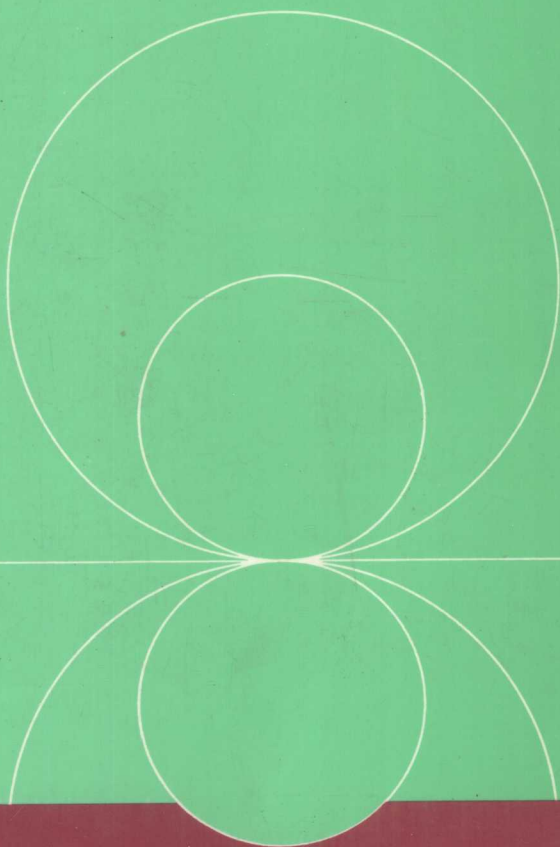


BRIDGE ANALYSIS SIMPLIFIED

**BAIDAR BAKHT
LESLIE G. JAEGER**



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BRIDGE ANALYSIS SIMPLIFIED

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To Anita and Kathleen

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PREFACE

Most highway bridges in North America are designed by the American Association of State Highway and Transportation Officials (AASHTO) specifications. A principal assumption underlying the analysis methods of AASHTO is that bridges of a given type (e.g., slab-on-steel girder bridges) all behave similarly in their live-load distribution properties.

By contrast with the North American tradition, bridge analysis in Europe tends to be highly analytical and, usually, computer-based. Such methods as the grillage analogy method, the orthotropic plate method, and the finite element and finite strip methods are extensively used.

This book presents a number of simple methods of analysis which expand upon the AASHTO approach, making it consistent with the more refined European methods. The simple methods presented in this book are derived from the results of computer-based rigorous analyses.

The book is intended to be useful both to the practicing bridge engineer and to the engineering student. It thus contains both “know-how” and “know-why” material. For the most part the know-why material is to be found in Chapter 1, while the remaining chapters constitute a know-how treatment which is virtually complete in itself.

The book is also intended to bring home to the reader the physical behavior of bridges of different types and to help the designer in establishing a “feel” for the mechanisms of load distributions.

A Note about Units

In order to make this book as widely useful as possible, all measurements in it are given in one of two ways. If the context is that of the AASHTO

specifications, then the measurements are given in the units of the United States Customary System (USCS), followed by their metric equivalents in parentheses. If the context is that of the Ontario Highway Bridge Design Code, the measurements are given in metric units, followed by their USCS equivalents in parentheses. In figures and tables, however, only one of the two systems of units is followed.

Baidar Bakht

Leslie G. Jaeger

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Much of the research upon which this book is based was carried out for the Ministry of Transportation and Communications of Ontario, Canada, in the process of the development of the Ontario Highway Bridge Design Code. The Ministry's help in the preparation of many of the drawings, and the permission of the American Society of Civil Engineers and the *Canadian Journal of Civil Engineering* for the reproduction of others, are gratefully acknowledged. The research also benefited greatly from grants awarded to one of the authors by the Natural Sciences and Engineering Research Council of Canada. This support is also gratefully acknowledged.

We are indebted to a number of people for their help in the preparation of this book, but especially to Mrs. Pam Mulcahy, who typed the manuscript, and to Mr. Barry Mitchell, who prepared many of the drawings.

Baidar Bakht

Leslie G. Jaeger

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CHAPTER

1

PRINCIPLES OF LOAD DISTRIBUTION

1.1 Introduction

Within a time span of approximately 30 years, from roughly 1950 to 1980, the science of bridge analysis has undergone major change. Following the advent of the digital computer, and the consequent development of analytical techniques based upon its use, the bridge designer has available today a number of powerful analytical tools in the so-called refined methods of analysis, including the following:

1. The grillage analogy method
2. The orthotropic plate method
3. The articulated plate method
4. The finite element method, including its finite strip formulation

These refined methods are by now well established for the analysis of load distribution in bridges of various types. References 7 to 9, 11, 14, and 25 are representative of a large technical literature.

A principal objective of this book is to provide a simplified approach to the use of these tools so that their fruits may be made available to the designer without the need for performing complicated analysis in the

design office. Thus, the term *simplified methods*, as used repeatedly in later chapters, means the representation of the results of complicated analysis in simple form; it does not mean the use of some overly simplifying assumption about bridge behavior, such as treating the bridge as a simple beam. The simplified methods have been deliberately put into a form so that the sequences of steps which the designer will follow have a marked resemblance to the familiar AASHTO (American Association of State Highway and Transportation Officials) methods which have been in use in North America for many years.

It is important to realize that most of the refined methods have limitations as to the kinds of bridge superstructures which they are capable of representing. For example, the usual grillage analogy and the orthotropic plate methods are suitable for the analysis of bridges in which load distribution takes place mainly through flexure and torsion in the longitudinal and transverse directions, with deflections due to shear being negligibly small. Bridge types which fall within this category include the "shallow superstructure" group, i.e., the solid slab, voided slab, and slab-on-girder types shown in Fig. 1.1. Significantly absent from this group is the cellular or multicell type shown in Fig. 1.2*a*. A bridge having a cross section of this type experiences significant deformation due to shear, which is accompanied by bending of the top and bottom flanges about their own centerlines in the manner shown in Fig. 1.2*b*. For this reason, if the grillage or the orthotropic plate representation is to be employed for the analysis of a multicellular type of bridge, the grillage or orthotropic plate must be different from the normal, and must include provision for significant deflection due to shear. The so-called shear-weak orthotropic plate, which is introduced later in this chapter, meets this need.

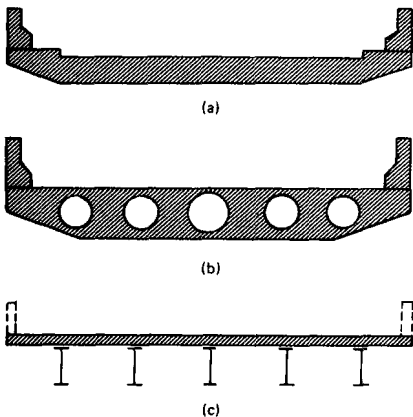


Figure 1.1 Cross sections of shallow superstructures: (a) a slab bridge; (b) a voided slab bridge; (c) a slab-on-girder bridge.

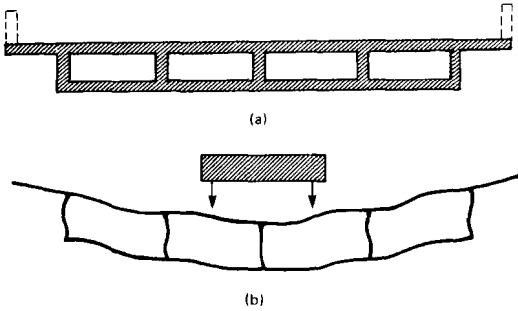


Figure 1.2 A cellular bridge: (a) cross section; (b) deflected cross section under concentrated loads.

In contrast to the grillage analogy and orthotropic plate methods the representation of a bridge as an articulated plate is appropriate when transverse distribution of load occurs mainly through shear forces, with little or no involvement of transverse bending stiffness. Figure 1.3a and b shows two bridge types which can be represented as articulated plates, whereas Fig. 1.3c shows the idealization involved, which is comprised of a number of longitudinal beams freely hinged together along their mating edges.

The finite element method, properly handled, is capable of representing bridge superstructures of all types. The particular kind of finite

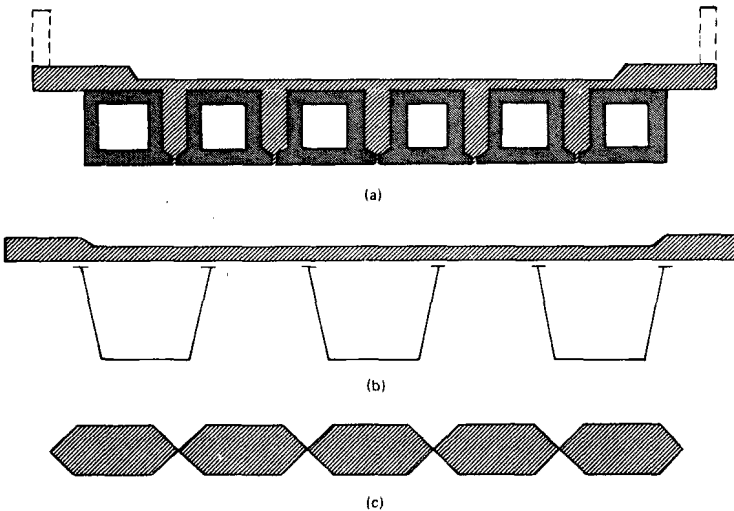


Figure 1.3 Cross sections of structures in which load distribution takes place mainly through transverse shear: (a) a multibeam bridge; (b) a multi-spine bridge; (c) idealized articulated plate.

element in a given case must, however, be chosen with care in order to represent properly the behavior of the bridge type under consideration.

In the remainder of this chapter, the properties of grillages, orthotropic plates, shear-weak orthotropic plates, and articulated plates are developed as a preliminary to the derivation of simplified methods of analysis for various types of bridge superstructures. Readers who prefer the mathematical kind of structural analysis are encouraged to study this first chapter carefully. This should result in a degree of understanding sufficient to enable the reader to use the same generic approach and to develop simplified methods for superstructure types which are not dealt with specifically in this book.

Readers for whom mathematical analysis has little appeal, and whose principal interests lie in the acquisition of design tools, may safely pass over the first chapter in a cursory way. The remaining chapters of the book give immediately applicable design methods which cover most cases.

1.2 The Concept of the Characterizing Parameter

Many simplified methods of bridge analysis are based on the concept of the *characterizing parameter*. This concept may conveniently be explained with the aid of Fig. 1.4.

Two grillages are shown in Fig. 1.4*a* and *b*. Each has the same number of longitudinal girders and the same number of transverse beams. The grillages carry external loadings which have the same pattern. A precise definition of *pattern of load* is given below; meanwhile, it is sufficient to note that the positions of the loads on the two grillages correspond so far as fraction of span and fraction of width are concerned, and that a given load on grillage 1 is a constant multiple of the load at the corresponding position on grillage 2. The two grillages have the same conditions of boundary support.

The following question is posed: What relationships must exist between the structural properties of the two grillages (i.e., such properties as the flexural and torsional stiffnesses of individual girders and beams) in order that they may have the same pattern of deflection when subjected to the same pattern of load? These relationships, once they have been found, will be called the characterizing parameters for deflection. Similarly, consideration of the same patterns of bending moments, twisting moments, etc., will lead to the characterizing parameters for these structural responses.

Figure 1.4*c* and *d* shows two orthotropic plates which are subjected to the same pattern of load. The same question is posed for these two as was posed above for grillages.

Before answering these questions, which are addressed in Secs. 1.4 and 1.5 below, it is appropriate to give some definitions.

Corresponding Plane Structures

Two plane structures are said to *correspond* if they have the same conditions of boundary support, and if the planform of one can be made to coincide with that of the other by application of (possibly different) scaling factors in two directions at right angles.

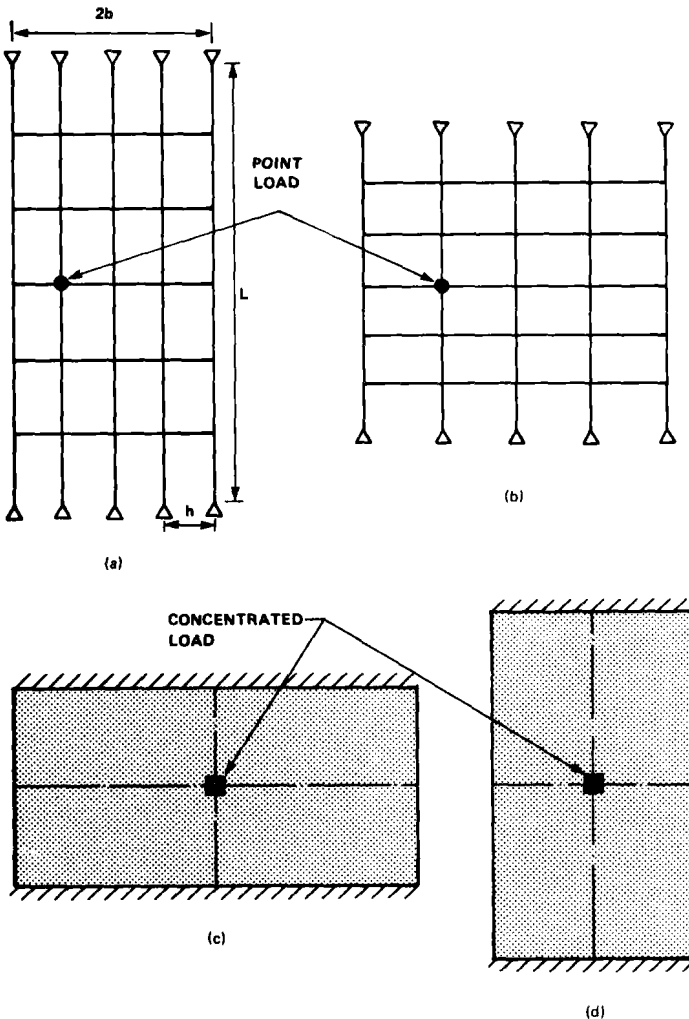


Figure 1.4 Plans for grillages and orthotropic plates: (a) grillage 1; (b) grillage 2; (c) orthotropic plate 1; (d) orthotropic plate 2.

For example, any two rectangles correspond in terms of the definition just given, even though their aspect ratios (length divided by breadth) are different. For rectangular planforms it is then convenient to define nondimensional span and nondimensional width coordinates in the manner shown in Fig. 1.5. In Fig. 1.5a an x coordinate runs from 0 to L and a y coordinate from 0 to $2b$, where L is span and b is half width. Then by defining $x' = x/L$ and $y' = y/2b$, one obtains the nondimensional scheme of Fig. 1.5b, in which x' runs from 0 to 1 and y' runs from 0 to 2. This nondimensional scheme will be used consistently from now on.

Corresponding Points

Points in two plane structures correspond if they coincide when the planform of one is made to coincide with that of the other as defined above. Specifically, for rectangular planforms, the points correspond if they have the same nondimensional coordinates (x' , y').

Pattern of Load

If in a structure number 1 two points, say a_1 and b_1 , are identified, and if in a corresponding structure number 2 the corresponding two points a_2 and b_2 are identified, then the patterns of load are defined to be identical if the ratio of load intensity at a_1 to that at a_2 is the same as the ratio of load intensity at b_1 to that at b_2 for all pairs of corresponding points in the two structures.

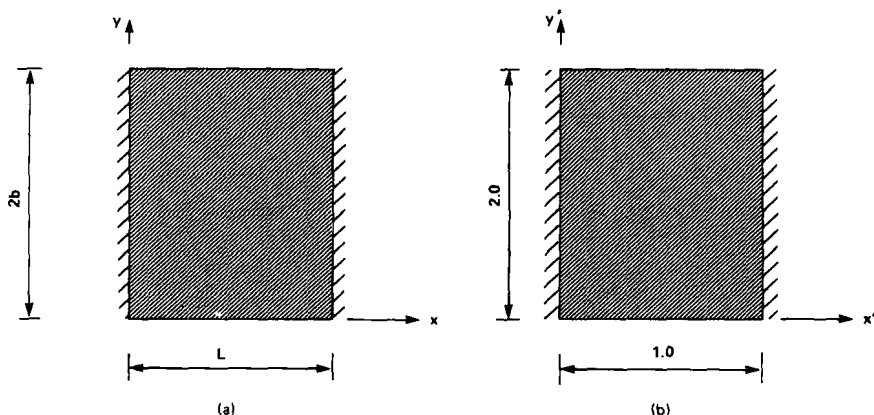


Figure 1.5 Coordinate systems for orthotropic plate planforms: (a) dimensional coordinate system; (b) nondimensional coordinate system.

Characterizing Parameters

If in two corresponding structures the necessary and sufficient condition for identical patterns of distribution of a given structural response, as a consequence of applying the same pattern of external load to the two, is that a certain one or more nondimensional parameters shall have the same value in the two structures, then those one or more parameters are defined to be the characterizing parameters for the structural response concerned.

1.3 The Use of Characterizing Parameters in Analysis

Provided that the number of characterizing parameters is not more than two, or in some cases three, the existence of characterizing parameters is of very great assistance in the development of simplified methods of analysis that are based upon results of often complex analyses. These characterizing parameters are used as axes of suitable design charts or tables. Having calculated the values of the characterizing parameters involved, the engineer can then identify a particular chart or table and read directly the value of the structural response concerned without performing any rigorous analysis. Clearly, a great economy of time and effort is achieved, if, for many structures such as highway bridges, much detailed analysis can be replaced by the calculation of the values of a few parameters, followed by the use of suitable charts.

Further, it is often the case that, for a particular kind of structure, the range of numerical values of its characterizing parameters becomes known to the designer to fall within certain well-recognized limits. The designer using such parameters in the analysis rapidly acquires a "feel" for the expected values of the parameter and thus for the kinds of distribution of the structural responses that are to be expected. With the help of the known ranges of a characterizing parameter, it is easy to ensure that a simplified method of analysis based on the parameter covers the whole range of the structural type concerned.

1.4 Characterizing Parameters for Grillages

A typical grillage, such as that shown in Fig. 1.4*a*, comprises a number of equal longitudinal girders, each having flexural stiffness EI , torsional stiffness GJ , and length L . The longitudinals are spaced a distance h apart

and are interconnected by a number of equally spaced transverse beams each of which has flexural stiffness EI_T and torsional stiffness GJ_T .

If such a grillage is subjected to a given external loading, the problem of finding the responses may be expressed in the form

$$[K]\{a\} = \{W\} \quad (1.1)$$

in which $\{W\}$ is a vector of externally applied forces, $\{a\}$ is a vector whose elements depend upon the deflections and rotations of the nodes and have the dimensions of force, and $[K]$ is a matrix whose elements have no dimensions.

It is readily verified that the elements of $[K]$ depend upon the following nondimensional parameters:

$$\left[\frac{L}{h}\right]^3 \left[\frac{EI_T}{EI}\right] \quad \left[\frac{L}{h}\right] \left[\frac{GJ_T}{EI}\right] \quad \text{and} \quad \left[\frac{h}{L}\right] \left[\frac{GJ}{EI_T}\right]$$

If two different grillages have the same pattern of external load, then the vector $\{W\}$ is the same for both, within a simple scalar multiplier. If, further, the two grillages have the same values of the three nondimensional parameters just identified, then the $[K]$ matrix is the same for both. Hence the vector $\{a\}$ must be the same for both, within a simple scalar multiplier.

Once the vector $\{a\}$ is known, the patterns of distribution of deflections, bending moments, twisting moments, and shear forces follow directly. Hence, it is concluded that the three nondimensional parameters given above are the characterizing parameters for all these structural responses.

For purposes of later comparison with orthotropic plate behavior, it is convenient at this point to express the grillage parameters in terms of an equivalent orthotropic plate, which is obtained by distributing the stiffnesses of the individual girders uniformly across the width and the stiffnesses of the transverse beams uniformly along the length. This means, for example, that the flexural stiffness EI of a girder is written as

$$EI = D_x h \quad (1.2)$$

where D_x is longitudinal flexural stiffness per unit width of the equivalent orthotropic plate. Three suitable nondimensional parameters for the grillage, expressed in equivalent orthotropic plate form, are then

$$\left(\frac{L}{h}\right)^4 \left(\frac{D_y}{D_x}\right) \quad \left(\frac{L}{h}\right)^2 \left(\frac{D_{xy}}{D_x}\right) \quad \text{and} \quad \left(\frac{h}{L}\right)^2 \left(\frac{D_{xy}}{D_y}\right)$$

where plate rigidities are as defined in Sec. 2.2. The quantity h/L can be eliminated from the second and third of these by multiplying or dividing, as the case may be, by the square root of the first one. Further, inverting

the first parameter and then taking the fourth root gives a parameter in which the ratio h/L has the first power. The result of these adaptations is to define the following:

$$\alpha_1 = \frac{D_{xy}}{2(D_x D_y)^{0.5}} \quad (1.3)$$

$$\alpha_2 = \frac{D_{yz}}{2(D_x D_y)^{0.5}} \quad (1.4)$$

$$\theta = \frac{h}{L} \left(\frac{D_x}{D_y} \right)^{0.25} \quad (1.5)$$

If two different grillages have the same values of α_1 , α_2 , and θ , then the two grillages will have the same patterns of distribution of deflections, shearing forces, and bending and twisting moments when subjected to the same pattern of load. These three parameters are the characterizing parameters for grillage behavior.

1.5 Characterizing Parameters for Orthotropic Plates

It is shown in books on the theory of plates, for example, in Ref. 24, that the deflection w of an orthotropic plate is governed by the following equation:

$$D_x \frac{\partial^4 w}{\partial x^4} + (D_{xy} + D_{yx} + D_1 + D_2) \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = q(x, y) \quad (1.6)$$

Since it is *patterns* of deflection that are being sought, Eq. (1.6) is recast in terms of dimensionless quantities $x' = x/L$ and $y' = y/b$, where L is the span and b is the half width, as shown in Fig. 1.5. Then

$$\frac{\partial}{\partial x} \equiv \frac{1}{L} \frac{\partial}{\partial x'} \quad (1.7)$$

and

$$\frac{\partial}{\partial y} \equiv \frac{1}{b} \frac{\partial}{\partial y'} \quad (1.8)$$

so that Eq. (1.6) gives

$$\frac{D_x}{L^4} \frac{\partial^4 w}{\partial x'^4} + \left(\frac{D_{xy} + D_{yx} + D_1 + D_2}{L^2 b^2} \right) \frac{\partial^4 w}{\partial x'^2 \partial y'^2} + \frac{D_y}{b^4} \frac{\partial^4 w}{\partial y'^4} = \phi(x', y') \quad (1.9)$$

where $\phi(x', y')$ is the expression of the externally applied load in terms of x' and y' .