應用機械振動解答

APPLIED MECHAICAL VIBRATIONS

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應用機械振動

2.1 (a) $\ddot{x} + 8x = 0$

$s^2 + 8 = 0 \Rightarrow s = \pm i\sqrt{8} \quad \therefore$ CASE 2

$x(t) = A \cos\sqrt{8}\,t + B \sin\sqrt{8}\,t$

(b) $\ddot{x} + 8\dot{x} + 64 = 0$

$s^2 + 8s + 64 = 0 \qquad a_1^2 - 4a_2 = -192$

$\therefore$ CASE 2, $a_1' = -8/2 = -4$

$a_2' = \sqrt{192}/2 = 6.93$

$x(t) = e^{-4t}(A \sin 6.93t + B \cos 6.93t$

(c) $\ddot{x} + 2\dot{x} + 3x = 0$

$a_1^2 - 4a_2 = 4 - 4(3) = -8 \quad \therefore$ CASE 2

$a_1' = -2/2 = -1 \quad a_2' = \sqrt{8}/2 = \sqrt{2}$

$x(t) = e^{-t}(A \sin\sqrt{2}\,t + B \cos\sqrt{2}\,t)$

(d) $\ddot{x} + 5\dot{x} - 6x = 0$

$s^2 + 5s - 6 = 0$

$(s+6)(s-1) = 0 \Rightarrow s = -6 \quad s = 1$

$x(t) = C_1 e^{-6t} +$

(e) $3\ddot{x} - 7\dot{x} + 8x = 0$

$\ddot{x} - \frac{7}{3}\dot{x} + \frac{8}{3}x = 0$

$a_1^2 - 4a_2 = (-\frac{7}{3})^2 - 4(\frac{8}{3}) = -5.22$

$\therefore$ CASE 2

$$a_1' = -\left(-\tfrac{7}{3}\right)/2 = 1.17$$

$$a_2' = \sqrt{5.22}/2 = 1.14$$

$$x(t) = e^{1.17t}\left(A \sin 1.14t + B \cos 1.14t\right)$$

2.2 (a) $\ddot{x} + 4x = 4e^{2t}$

$$x_p(t) = ae^{2t} \qquad \ddot{x}_p = 4ae^{2t}$$

$$\therefore \quad 4ae^{2t} + 4ae^{2t} = 4e^{2t}$$

$$\Rightarrow \quad a = \tfrac{1}{2}$$

$$\therefore \quad x_p(t) = \tfrac{1}{2}e^{2t}$$

(b) $\ddot{x} + 8\dot{x} + 64x = 128 \cos 6t$

$$x_p(t) = a \sin 6t + b \cos 6t$$

$$\dot{x}_p = 6a \cos 6t - 6b \sin 6t$$

$$\ddot{x}_p = -36a \sin 6t - 36b \cos 6t$$

SUBSTITUTING INTO D.E.:

$$-36a \sin 6t - 36b \cos 6t + 48a \cos 6t - 48b \sin 6t + 64a \sin 6t + 64b \cos 6t = 128 \cos 6t$$

$$\left.\begin{aligned}(-36a - 48b + 64a) &= 0\\ (-36b + 48a + 64b) &= 128\end{aligned}\right\} \quad \begin{aligned}a &= 1.99\\ b &= 1.16\end{aligned}$$

$$x_p(t) = 1.99 \sin 6t + 1.16 \cos 6t$$

2.2 (c) $\ddot{x} + 6x = 2t^2 \sin 4t$

$$x_p(t) = (a + bt + ct^2)\sin 4t + (d + et + ft^2)\cos 4t$$

$$\ddot{x}_p = 2c \sin 4t + 8(b + 2ct)\cos 4t - 16(a + bt + ct^2)\sin 4t + 2f \cos 4t - 8(e + 2ft)\sin 4t - 16(d + et + ft^2)\cos 4t$$

SUBSTITUTING INTO D.E. AND COLLECTING TERMS YIELDS:

$$(2c - 10a - 8e)\sin 4t + (-16b - 16f)t \sin 4t - 10c\, t^2 \sin 4t + (8b + 2f - 10d)\cos 4t + (16c - 10e)t \cos 4t - 10f\, t^2 \cos 4t$$

$$= 2t^2 \sin 4t$$

OR

$$2c - 10a - 8e = 0$$
$$-16b - 16f = 0$$
$$-10c = 2$$
$$8b + 2f - 10d = 0$$
$$16c - 10e = 0$$
$$-10f = 0$$

SIMULTANEOUS SOLUTION GIVES:

$$a = {}^{27}/_{125} \qquad b = 0 \qquad c = -{}^{1}/_{5}$$

$$d = 0 \qquad e = -{}^{8}/_{25} \qquad f = 0$$

$$x_p(t) = \frac{27}{125}\sin 4t - \frac{1}{5}t^2 \sin 4t - \frac{8}{25}t\cos 4t$$

(d) $\ddot{x} - 4x = 8t^2$

$$x_p(t) = a + bt + ct^2$$
$$\dot{x}_p = b + 2ct$$
$$\ddot{x}_p = 2c$$

SUBSTITUTING:

$$2c - 4(a + bt + ct^2) = 8t^2$$

$$\left.\begin{aligned} 2c - 4a &= 0 \\ -4b &= 0 \\ -4c &= 8 \end{aligned}\right\} \quad \begin{aligned} a &= -{}^{1}/_{4} \\ b &= 0 \\ c &= -{}^{1}/_{2} \end{aligned}$$

$$x_p(t) = -\frac{1}{4} - \frac{1}{2}t^2$$

(e) $\ddot{x} + 2\dot{x} + x = 2\cos 2t + 3t + 2 + 3e^t$

$$x_p(t) = a + bt + d\sin 2t + f\cos 2t + ge^t$$

$$\dot{x}_p = b + 2d\cos 2t - 2f\sin 2t + ge^t$$

$$\ddot{x}_p = -4d\sin 2t - 4f\cos 2t + ge^t$$

SUBSTITUTING AND COLLECTING TERMS:

$$(a+2b) + bt + (-3d-4f)\sin 2t$$
$$+(4d-3f)\cos 2t + 4g\, e^{t} =$$
$$2\cos 2t + 3t + 2 + 3e^{t}$$

$\therefore$

$$\left.\begin{aligned} a+2b &= 2 \\ b &= 3 \\ -3d-4f &= 0 \\ 4d-3f &= 2 \\ 4g &= 3 \end{aligned}\right\} \quad \begin{aligned} a &= -4 \\ b &= 3 \\ d &= 2/7 \\ f &= -3/14 \\ g &= 3/4 \end{aligned}$$

$$x_p(t) = -4 + 3t + \frac{2}{7}\sin 2t - \frac{3}{14}\cos 2t + \frac{3}{4}e^{t}$$

2.3 (a) $\ddot{x} + 20\dot{x} + 64x = 0 \qquad x(0) = 1/3$

$\dot{x}(0) = 0$

$$a_1^2 - 4a_2 = 400 - 4(64) = 144$$
$$s^2 + 20s + 64 = 0$$
$$(s+4)(s+16) = 0 \Rightarrow s_1 = -4,\ s_2 = -16$$
$$x(t) = C_1 e^{-4t} + C_2 e^{-16t}$$
$$\left.\begin{aligned} x(0) &= C_1 + C_2 = 1/3 \\ \dot{x}(0) &= -4C_1 - 16C_2 = 0 \end{aligned}\right\} \quad \begin{aligned} C_1 &= 4/9 \\ C_2 &= -1/9 \end{aligned}$$
$$\therefore \quad x(t) = \frac{4}{9}e^{-4t} - \frac{1}{9}e^{-16t}$$

2.3 (b) $\ddot{x} + x = te^t + 3\cos 2t$

$x(0) = 1 \qquad \dot{x}(0) = 0$

HOMOGENEOUS: $\ddot{x}_h + x_h = 0 \Rightarrow s^2 + 1 = 0$

$x_h(t) = A\sin t + B\cos t$

PARTICULAR:

$x_p(t) = (a + bt)e^t + c\cos 2t$

$\ddot{x}_p = be^t + 2bte^t - 4c\cos 2t$

SUBSTITUTING:

$(a+b)e^t + 3bte^t - 3c\cos 2t$
$= te^t + 3\cos 2t$

$$\left.\begin{aligned} a + b &= 0 \\ 3b &= 1 \\ -3c &= 3 \end{aligned}\right\} \quad \begin{aligned} a &= -1/3 \\ b &= 1/3 \\ c &= -1 \end{aligned}$$

$x_p(t) = -\frac{1}{3}e^t + \frac{1}{3}te^t - \cos 2t$

$x(t) = A\sin t + B\cos t - \frac{1}{3}e^t + \frac{1}{3}te^t - \cos 2t$

$x(0) = 1 = B - \frac{1}{3} - 1 \Rightarrow B = 7/3$

$\dot{x}(0) = 0 = A - \frac{1}{3} + \frac{1}{3} + 2 \Rightarrow A = -2$

$x(t) = -2\sin t + 7/3\cos t - 1/3\, e^t + 1/3\, te^t$
$- \cos 2t$

2.3 (a) $\ddot{x} + 3\dot{x} + 4x = 6 \sin 4t$

$x(0) = 3 \qquad x(1) = 1$

HOMOGENEOUS: $a_1^2 - 4a_2 = 9 - 16 = -7$ CASE 2

$a_1' = -3/2 \qquad a_2' = \sqrt{7}/2$

$$x_h(t) = e^{-\frac{3}{2}t}\left(A \sin \frac{\sqrt{7}}{2}t + B \cos \frac{\sqrt{7}}{2}t\right)$$

PARTICULAR:

$$x_p(t) = a \sin 4t + b \cos 4t$$
$$\dot{x}_p = 4a \cos 4t - 4b \sin 4t$$
$$\ddot{x}_p = -16a \sin 4t - 16b \cos 4t$$

$$(-12a - 12b) \sin 4t + (12a - 12b) \cos 4t = 6 \sin 4t$$

$$\left.\begin{aligned} -12a - 12b &= 6 \\ 12a - 12b &= 0 \end{aligned}\right\} \qquad \begin{aligned} a &= -1/4 \\ b &= -1/4 \end{aligned}$$

$$x_p(t) = -\frac{1}{4}(\sin 4t + \cos 4t)$$

$$x(t) = e^{-\frac{3}{2}t}\left(A \sin \frac{\sqrt{7}}{2}t + B \cos \frac{\sqrt{7}}{2}t\right) - \frac{1}{4}(\sin 4t + \cos 4t)$$

$$x(0) = 3 = B - \frac{1}{4} \Rightarrow B = 13/4$$

$$x(1) = 1 = 0.216\,A + 0.178\,B + 0.189 + 0.163$$

$$\Rightarrow A = 2.18$$

$$x(t) = e^{-\frac{3}{2}t}\left(2.18 \sin \frac{\sqrt{7}}{2}t + 3.25 \cos \frac{\sqrt{7}}{2}t\right)$$

$$- \frac{1}{4}\left(\sin 4t + \cos 4t\right)$$

(d) $\ddot{x} - 3\dot{x} + 2x = e^{-t}(1 + \sin 2t)$

$x(0) = 1 \qquad \dot{x}(0) = 2$

HOMOGENEOUS: $\ddot{x}_h + 3\dot{x}_h + 2x = 0$

$$s^2 + 3s + 2 = 0$$

$$(s-2)(s-1) = 0 \qquad s_1 = 1,\ s_2 = 2$$

$$x_h(t) = C_1 e^{t} + C_2 e^{2t}$$

PARTICULAR:

$$x_p(t) = a e^{-t} + b e^{-t} \sin 2t + c\, e^{-t} \cos 2t$$

$$\dot{x}_p = -a e^{-t} + (-b - 2c)\, e^{-t} \sin 2t + (2b - c)\, e^{-t} \cos 2t$$

$$\ddot{x}_p = a e^{-t} + (-3b + 4c)\, e^{-t} \sin 2t + (-4b - 3c)\, e^{-t} \cos 2t$$

SUBSTITUTING AND COLLECTING TERMS:

$$6a\, e^{-t} + (2b + 10c)\, e^{-t} \sin 2t$$
$$+ (-10b + 2c)\, e^{-t} \cos 2t = e^{-t}(1 + \sin 2t)$$

$$\left.\begin{aligned} 6a &= 1 \\ 2b + 10c &= 1 \\ -10b + 2c &= 0 \end{aligned}\right\} \quad \begin{aligned} a &= 1/6 \\ b &= 1/52 \\ c &= 5/52 \end{aligned}$$

$$x(t) = C_1 e^{t} + C_2 e^{2t} + \frac{1}{6} e^{-t} + \frac{1}{52} e^{-t} \sin 2t$$
$$+ \frac{5}{52} e^{-t} \cos 2t$$

$$x(0) = 1 = C_1 + C_2 + \frac{1}{6} + 0 + \frac{5}{52}$$

$$\dot{x}(0) = 2 = C_1 + 2C_2 - \frac{1}{6} + \frac{2}{52} - \frac{5}{52}$$

$$C_1 = 1.25 \qquad C_2 = -0.513$$

$$x(t) = 1.25\, e^{t} - 0.513\, e^{2t} + \frac{1}{6} e^{-t}$$
$$+ \frac{1}{52} e^{-t} \sin 2t + \frac{5}{52} e^{-t} \cos 2t$$

CHAPTER 3

3.1 $W = 5\ \text{LB} \quad k = 20\ \text{LB/IN} \quad x_0 = 3.4'' \quad \dot{x}_0 = 0$

(a) $\ddot{x} + \frac{k}{m} x = 0$

$$\ddot{x} + \frac{20(386.4)}{5} x = 0$$

$$\ddot{x} + 1545.6\, x = 0$$

(b) $\omega = \sqrt{k/m} = \sqrt{1545.6} = 39.3\ \text{RAD/s}$

$f = \omega / 2\pi = 6.26\ \text{Hz}$

(c) $\tau = \frac{1}{f} = 0.16\ \text{s}$

(d) $\dot{x}_{MAX} = X_0 \omega = 3.4(39.3) = 133.6\ \text{IN/s}$

3.2 $m = 6.8\ \text{kg} \quad x_0 = 0 \quad \dot{x}_0 = ?$

$\tau = 0.25\ \text{s} \quad X = 50\ \text{mm}$

$$\omega = \frac{2\pi}{\tau} = 25.13\ \text{RAD/s}$$

$$X = \left[x_0^2 + \left(\frac{\dot{x}_0}{\omega}\right)^2 \right]^{1/2}$$

$$50^2 = 0 + \left(\frac{\dot{x}_0}{\omega}\right)^2$$

$$\dot{x}_0 = \sqrt{(50)^2(20.13)^2} = 1256.5\ \text{mm/s}$$

$$k = m\omega^2 = \frac{6.8(25.13)^2}{1000} = 4.29\ \text{N/mm}$$

3.3 $\ddot{x}_{MAX} = 40 \text{ IN/s}^2 \qquad f = 60 \text{ Hz}$

$$\omega = 2\pi f = 2\pi(60) = 377 \text{ RAD/s}$$

$$X = \frac{\ddot{x}_{MAX}}{\omega^2} = \frac{40}{(377)^2} \cong 0.0003''$$

$$\dot{x}_{MAX} = X\omega = 0.0003(377) \cong 0.113 \text{ IN/s}$$

3.4 $\Delta = 10 \text{ mm}$

STATIC EQUILIBRIUM: $mg = K\Delta$

$$\therefore \quad \omega^2 = \frac{K}{m} = \frac{g}{\Delta} = \frac{9.81 \text{ m/s}^2}{0.01 \text{ m}}$$

$$\omega = 31.3 \text{ RAD/s}$$

$$f = \frac{\omega}{2\pi} = 4.98 \text{ Hz}$$

3.5 $m_1 = ? \qquad K = ? \qquad \tau_1 = 1.5 \text{ s}$

$$m_2 = m_1 + 0.5 \text{ kg} \qquad \tau_2 = 1.15(1.5) = 1.725 \text{ s}$$

$$\omega_1 = \frac{2\pi}{1.5} = 4.19 \text{ RAD/s} \qquad \omega_2 = \frac{2\pi}{1.725} = 3.64 \text{ RAD/s}$$

$$\frac{K}{m_1} = (4.19)^2 \qquad \frac{K}{m_1 + 0.5} = (3.64)^2$$

SIMULTANEOUS SOLUTION GIVES

$$m_1 = 1.54 \text{ kg}$$

$$K = 27 \text{ N/m} \quad \text{OR} \quad 0.027 \text{ N/mm}$$

3.6 $\dot{x}_{MAX} = 15 \text{ IN/S} \qquad \tau = 2 \text{ S}$

$x_0 = 2'' \qquad \dot{x}_0 = ?$

$$\omega = \frac{2\pi}{\tau} = \pi \text{ RAD/S}$$

(a) $$X = \frac{\dot{x}_{MAX}}{\omega} = \frac{15}{\pi} = 4.77''$$

(b) $$\ddot{x}_{MAX} = \dot{x}_{MAX}\,\omega = 15\pi \cong 47.1 \text{ IN/S}^2$$

(c) $$X = \left[x_0^2 + \left(\frac{\dot{x}_0}{\omega}\right)^2\right]^{1/2}$$

$$4.77 = \left[2^2 + \left(\frac{\dot{x}_0}{\pi}\right)^2\right]^{1/2}$$

$$\dot{x}_0 = 13.6 \text{ IN/S}$$

3.7 $\ddot{x} + \frac{k}{m}x = 0$ (INCLINE IS OF NO CONSEQUENCE IF IT IS FRICTIONLESS)

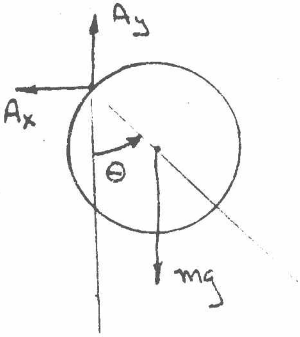
3.8 $\tau = 1.5 \text{ S} \qquad L = ?$

$$\omega = \sqrt{\frac{g}{L}} = \frac{2\pi}{\tau}$$

$$L = \left(\frac{\tau}{2\pi}\right)^2 g = \left(\frac{1.5}{2\pi}\right)^2 (386.4)$$

$$L = 22''$$

3.9



$m = 5.5$ kg $D = 0.75$ m

$\circlearrowleft + \quad \Sigma M_A = I_A \ddot{\theta}$

$$I_A = \frac{1}{2} m R^2 + m R^2 = \frac{3}{2} m R^2$$

$$-(R \sin\theta) mg = \frac{3}{2} m R^2 \ddot{\theta}$$

$$\ddot{\theta} + \frac{2g}{3R} \sin\theta = 0$$

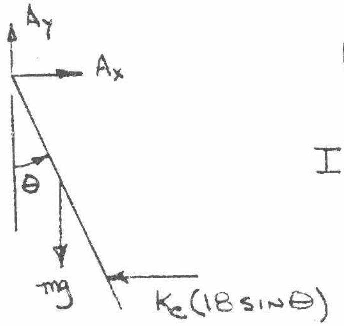
FOR SMALL θ, $\sin\theta \approx \theta$

$$\ddot{\theta} + \frac{2g}{3R}\theta = 0$$

$$\omega = \sqrt{\frac{2g}{3R}} = \sqrt{\frac{2(9.81)}{3(0.375)}} = 4.18 \text{ RAD/s}$$

$$f = \frac{\omega}{2\pi} = 0.67 \text{ Hz}$$

3.10



$k_e = k_1 + k_2 = 40$ LB/IN

$$I_A = I_G + md^2$$
$$= \frac{1}{12} m L^2 + m\left(\frac{L}{2}\right)^2$$
$$= \frac{1}{3} m L^2$$

$\circlearrowleft + \quad \Sigma M_A = I_A \ddot{\theta}$

$$-mg\left(\frac{L}{2}\sin\theta\right) - k_e(18\sin\theta)(18\cos\theta) = \frac{1}{3} m L^2 \ddot{\theta}$$

FOR SMALL OSCILLATIONS, $\sin\theta \approx \theta$
$\cos\theta \approx 1$

$$\frac{1}{3} m L^2 \ddot{\Theta} + \left[\frac{mgL}{2} + K_e (18)^2 \right] \Theta = 0$$

SUBSTITUTING $m = \frac{10}{g}$, $L = 20''$, $K_e = 40$ LB/IN

AND REARRANGING:

(a) $$\ddot{\Theta} + 3785\,\Theta = 0$$

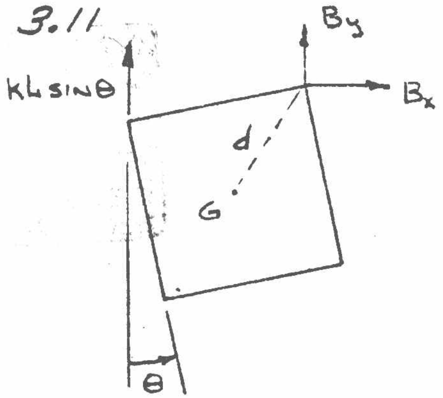
(b) $$\omega = \sqrt{3785} = 61.5 \text{ RAD/s}$$

(c) $$\tau = \frac{2\pi}{\omega} = 0.10 \text{ s}$$

(d) $$\Theta_0 = 5\left(\frac{\pi}{180}\right) \text{ RAD}$$

$$\dot{\Theta}_{MAX} = \Theta_0 \omega = 5\left(\frac{\pi}{180}\right)(61.5) = 5.37 \text{ RAD/s}$$

3.11



DYNAMIC FBD

$$I_B = I_G + md^2$$

$$I_B = \frac{1}{12} m \left(L^2 + L^2\right) + m\left[2\left(\frac{L}{2}\right)^2\right]$$

$$I_B = \frac{2}{3} m L^2$$

$$\circlearrowleft + \; \sum M_B = I_B \ddot{\Theta}$$

$$-kL\sin\Theta(L\cos\Theta) = \frac{2}{3} m L^2 \ddot{\Theta}$$

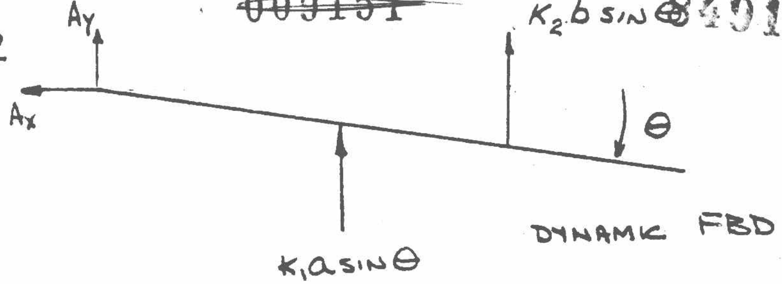
$$\sin\Theta \approx \Theta \qquad \cos\Theta \approx 1$$

$$\frac{2}{3} m L^2 \ddot{\Theta} + kL^2 \Theta = 0$$

$$\ddot{\Theta} + \frac{3k}{2m} \Theta = 0$$

$$\tau = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{2m}{3k}} \text{ s}$$

3.12



$$I_A = I_G + md^2 = \frac{1}{12} mL^2 + m\left(\frac{L}{2}\right)^2 = \frac{1}{3} mL^2$$

$$\circlearrowright + \; \sum M_A = I_A \ddot{\theta}$$

$$-K_1 a \sin\theta (a \cos\theta) - K_2 b \sin\theta (b \cos\theta) = \frac{1}{3} mL^2 \ddot{\theta}$$

USING THE SMALL ANGLE APPROXIMATIONS GIVES

$$\frac{1}{3} mL^2 \ddot{\theta} + (K_1 a^2 + K_2 b^2)\theta = 0$$

$$\ddot{\theta} + \frac{3(K_1 a^2 + K_2 b^2)}{mL^2} \theta = 0$$

SUBSTITUTING NUMERICAL VALUES:

$$\ddot{\theta} + 1159.2\,\theta = 0$$

$$\omega = \sqrt{1159.2} = 34 \text{ RAD/s}$$

(a) $\tau = \frac{2\pi}{\omega} = 0.185$ s

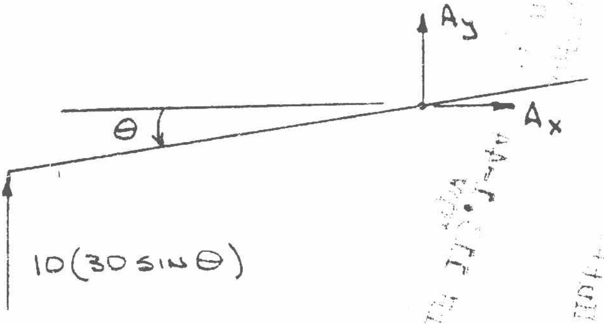
(b) $f = \frac{1}{\tau} = 5.41$ Hz

(c) $\Theta = \theta_0 = 5\left(\frac{\pi}{180}\right) = 0.087$ RAD

(d) $\theta(5) = 0.087 \cos[34(5)]$

$\theta(5) = 0.082$ RAD

3.13



$$I_A = I_G + md^2 = \frac{1}{12}mL^2 + m\left(\frac{L}{4}\right)^2 = \frac{7}{48}mL^2$$

$$\circlearrowleft\!+\ \Sigma M_A = I_A\ddot{\theta}$$

$$-10(30\sin\theta)(30\cos\theta) = \frac{7}{48}\left(\frac{25}{g}\right)(40)^2\ddot{\theta}$$

FOR SMALL OSCILLATIONS:

$$\frac{7}{48}\left(\frac{25}{g}\right)(40)^2\ddot{\theta} + 10(30)^2\theta = 0$$

(a) $$\ddot{\theta} + 596\,\theta = 0$$

$$\omega = \sqrt{596} = 24.4 \text{ RAD/s}$$

(b) $$\tau = \frac{2\pi}{\omega} = 0.258 \text{ s}$$

$$\theta_0 = \sin^{-1}\left(\frac{2}{30}\right) = 0.067 \text{ RAD}$$

$$\dot{\theta}_{MAX} = \theta_0\omega = 0.067(24.4) = 1.63 \text{ RAD/s}$$

(c) $$(V_B)_{MAX} = 30\,\dot{\theta}_{MAX} = 48.9 \text{ IN/s}$$