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**COMBINATORIAL  
PROGRAMMING:  
METHODS  
and  
APPLICATIONS**

edited by B.ROY



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# Combinatorial Programming: Methods and Applications

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## PREFACE

"Combinatorial Programming" are two words whose juxtaposition still strike us as unusual, nevertheless their association in recent years adequately reflects the preoccupations underlying differing work fields, and their importance will increase both from methodology and application view points. To those who like definitions and consider the function of this book to furnish one for combinatorial programming, I will simply say that it is precisely this which is exclusively treated here and which in the eyes of the authors is the heart of this branch of applied mathematics. Such was the initial intention of those who in the spring of 1973 gathered together in Paris to state the work of the Advanced Study Institute from which this book arises. As young as combinatorial programming is, it was easy to see that a two week school was insufficient to cover the subject in an exhaustive manner. Finally the decision had to be taken to reduce to book form, and to organise within this particular means of expression, the essential syntheses and communications. Unfortunately the discussions, the round tables, and the majority of the case studies could not be included in this book which is more of a hand-book on the subject.

The choice and orientation of the surveys has been guided by two criteria : the importance of already accomplished work, and the originality of the survey to be undertaken. Accordingly since there is a rich supply of literature on integer programming surveys and shortest path algorithms these two topics have been excluded. On the other hand a new look at the set of branch and bound procedures, analysing them in a systematic way by using general concepts such as "separation principle", "evaluation function",... providing the means of conceiving new ones with respect to the specific problem to be resolved, seems to be self-imposing.

The communications complete the surveys on particular points or sketch the outlines where a survey may seem premature or even unjustifiable according to the two preceding self-imposed criteria.

If for the reasons cited above this book favours the methodology side of combinatorial programming, applications have not been totally neglected. Distribution and routing, investment and location, sequencing and scheduling are problems for which the reader will find throughout this book numerous methods of solution in the form of models, structural properties or even algorithms. Several of the papers included even go into numerical aspects (size of the problems resolved, comparison of algorithms, length and precision of the calculations, ...).

As editor of this book, if I had been confronted with a characteristic combinatorial programming problem this book itself would not really have helped me to solve it. It dealt with arrangement of problem material in such a way so as to form a coherent continuation, graduated in its difficulty, and easily understood by the non-specialist, even if in so doing it is rendered superficial through the eyes of the specialist (for whom in fact it little matters). The four part structure is closely related to the

morphology of combinatorial programming problems presented in the first paper. Within each section the order adopted tends, without being too fragmented, to provide coverage of problems from a relatively vast domain to more specialised subjects. I in no way pretend that the solution finally adopted presents a marked character of optimality, nor even sub-optimality, but only a compromise which realises a satisfactory equilibrium between the antagonistic criteria which are nothing but weaker forms of incompatible constraints. Still one must admit that often the principal actors in a decision making process are confronted with problems formulated in this way (ill-defined in the eyes of certain people), but I leave them the responsibility of their own value of judgement. Thus we are led to predict that, with regard to new structures which are to be formalised and made operational relative to new enumeration generation and elimination procedures, or even to characterise and extract those solutions which appear balanced in the eyes of the expert in that they in no way sacrifice a particular aspect for another, combinatorial programming will undergo during the next few years important developments not only in algorithms and simple heuristics but in concepts and structures.

For this very reason I believe that I am able to say that this book is addressed to those, be they pure mathematicians or scientists, who are interested in the recent developments in this sector of applied mathematics which subtends what we call "operational research" or "decision aid" according to whether emphasis is centred on methods or applications.

I cannot complete this preface without expressing my deep obligation to all those who helped in making the school possible and in consequence this book. Thanks are due first to N.A.T.O. for their financial support and secondly to SEMA for having undertaken the secretarial duties. Although it may seem trivial to add,



without the active participation of each one of the 98 who attended, lecturers or not, this book would not have been possible, each one of these I would like to thank also for the confidence that they confided in me. Finally all those who organised and were responsible for the functioning of the school, the secretaries who typed and reproduced the lecture notes are especially thanked for the often "thankless" work they undertook. Last but not least I am deeply indebted to Michel Gondran for the role he played in liaison with the authors, and for the continuous aid he provided in the realisation of this book. Here he will find the expression of my deep gratefulness.

Bernard Roy

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## PART I

### GENERAL METHODOLOGY



