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Representation Theory of Algebras

Proceedings of the Philadelphia Conference

Edited by

Robert Gordon

REPRESENTATION THEORY OF ALGEBRAS

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PREFACE

This collection of papers is the fruit of a National Science Foundation sponsored research conference on representation theory. The conference was held at Temple University's Sugar Loaf facility, May 24-28, 1976. In recognition of his preeminence among American workers in the representation theory of finite dimensional algebras, Maurice Auslander was given the task of delivering approximately half the lectures presented. The reader will find that this is reflected in the text. I wish to thank the contributors for the papers contained therein.

Also I wish to thank Professors G. W. Johnson and M. G. Steisel in their respective capacities as Dean and Chairman of Temple University's College of Liberal Arts and Faculty Senate Lectures and Forums Committee for financial support. The chief donor to the conference was the National Science Foundation, to which institution I am indebted. Finally, I wish to thank my wife, Muriel Gordon, who as secretary to the conference, was instrumental in making things run smoothly.

R. Gordon

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FUNCTORS AND MORPHISMS DETERMINED BY OBJECTS

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Introduction. In the course of our study of the representation theory of artin algebras, I. Reiten and I introduced the notions of right or left almost split morphisms, as well as almost split sequences, of finitely generated modules over artin algebras. In view of the important role these notions have played in elucidating some questions in the representation theory of artin algebras (see [1] for an account of some of the results that have been obtained along these lines), it is natural to wonder if these ideas are special to artin algebras or are of broader interest. This is an especially tempting question since nowhere in the definitions of these notions is the fact that one is dealing with finitely generated modules over artin algebras used. In fact, the formulations given for artin algebras of all these notions make sense in arbitrary abelian categories and some even in arbitrary categories. Moreover, many of the proofs given for artin algebras carry over verbatim to these categorical settings. Thus the problem of generalization boils down basically to the question of when these types of morphisms and exact sequences exist.

This point of view was pursued in [3] where it was shown that right or left almost split morphisms and almost split sequences do indeed exist in more general contexts than artin algebras. That paper dealt with

[1]Written with partial support of NSF Grant MCS 72-04584

various categories of modules over rings Λ which are algebras over complete noetherian local rings R such that Λ is a finitely generated R-module. Amongst other things, it was shown that if R is a complete discrete valuation ring and Λ is an R-order in the classical sense, then the category of Λ-lattices has a theory of right and left almost split morphisms and almost split sequences entirely analogous to that for the category of finitely generated modules over artin algebras.

In this paper we generalize our previous work in a somewhat different direction. Instead of finding various new contexts in which right or left almost split morphisms exist, we introduce a new type of morphisms, those which are either right or left determined by objects. We will presently see that right and left almost split morphisms are special cases of morphisms which are right or left determined by objects. While the notion of a morphism being determined by an object is a general categorical one, as in the case of almost split morphisms, the problem of their existence is not at all clear, and it is with this question that this paper is mainly concerned.

In an effort to make what appears to be a somewhat long and technically involved paper easier to follow, the rest of this introduction is devoted to giving a brief survey of the basic notions and results of the paper. For the most part, the general organization of the survey follows that of the paper, but certain liberties have been taken when this seemed appropriate to aid the exposition of the basic ideas. To simplify the presentation, we assume that all categories are additive, even though much of what is said of a purely categorical nature can be carried out in arbitrary categories.

Let $f: B \longrightarrow C$ be a morphism in the category $\underline{\underline{C}}$. We recall (see [7], [11]) that f is said to be right almost split if a) f is not a splittable epimorphism (f is said to be a splittable epimorphism if there is a morphism $s\colon C \longrightarrow B$ such that $fs = 1_C$) and b) given a morphism $g\colon X \longrightarrow C$ in $\underline{\underline{C}}$ which is not a splittable epimorphism, then there is a morphism $h\colon X \longrightarrow B$ such that $fh = g$. It is easily checked that if $f\colon B \longrightarrow C$ is right almost split, then a) the endomorphism ring $\operatorname{End} C$ of C is a local ring, b) $\operatorname{Im}(\operatorname{Hom}_{\underline{\underline{C}}}(C, B) \xrightarrow{(C,\, f)} \operatorname{End} C)$ is the unique maximal right ideal of $\operatorname{End} C$ and c) if $g\colon Y \longrightarrow C$ is a morphism in $\underline{\underline{C}}$ such that $\operatorname{Im}(\operatorname{Hom}_{\underline{\underline{C}}}(C, Y) \xrightarrow{(C,\, g)} \operatorname{End} C)$ is contained in the unique maximal right ideal of $\operatorname{End} C$, then there is a morphism $h\colon Y \longrightarrow B$ such that $g = fh$. In fact, a little thought suffices to show that a morphism $f\colon B \longrightarrow C$ in $\underline{\underline{C}}$ is right almost split if and only if it satisfies conditions a), b), and c). This formulation suggests the following definition.

A morphism $f\colon B \longrightarrow C$ in $\underline{\underline{C}}$ is said to be right determined by an object X in $\underline{\underline{C}}$ (or more simply f is right X-determined) if a morphism $g\colon Y \longrightarrow C$ has the property $\operatorname{Im}(\operatorname{Hom}_{\underline{\underline{C}}}(X, Y) \xrightarrow{(X,\, g)} \operatorname{Hom}_{\underline{\underline{C}}}(X, C))$ is contained in $\operatorname{Im}(\operatorname{Hom}_{\underline{\underline{C}}}(X, B) \xrightarrow{(X,\, f)} \operatorname{Hom}_{\underline{\underline{C}}}(X, C))$ if and only if there is a morphism $h\colon Y \longrightarrow B$ such that $fh = g$. Before discussing the question of the existence of morphisms which are right determined by objects in $\underline{\underline{C}}$, we take up the question of the uniqueness of such morphisms.

Suppose $f\colon B \longrightarrow C$ is a morphism in $\underline{\underline{C}}$ and X an object in $\underline{\underline{C}}$. Then it is easily seen that $H = \operatorname{Im}(\operatorname{Hom}_{\underline{\underline{C}}}(X, B) \xrightarrow{(X,\, f)} \operatorname{Hom}_{\underline{\underline{C}}}(X, C))$ is an $(\operatorname{End} X)^{op}$-submodule of the $(\operatorname{End} X)^{op}$-module $\operatorname{Hom}_{\underline{\underline{C}}}(X, C)$, where $(\operatorname{End} X)^{op}$ denotes the opposite ring of $\operatorname{End} X$ and the abelian group $\operatorname{Hom}_{\underline{\underline{C}}}(X, C)$ is considered an $(\operatorname{End} X)^{op}$-module (all modules are left modules) by means of the usual operation $t \cdot f$ where $t \cdot f$ is the composition ft for all t in $\operatorname{End} X$ and f in $\operatorname{Hom}_{\underline{\underline{C}}}(X, C)$. Now if $f\colon B \longrightarrow C$ is right X-determined then a morphism $g\colon Y \longrightarrow C$ has the property that there is a morphism $h: Y \longrightarrow B$ such that $g = fh$ if and only if the $(\mathrm{End}\ X)^{op}$-submodule $\mathrm{Im}(\mathrm{Hom}_{\underline{\underline{C}}}(X, Y) \xrightarrow{(X, g)} \mathrm{Hom}_{\underline{\underline{C}}}(X, C))$ of $\mathrm{Hom}_{\underline{\underline{C}}}(X, C)$ is contained in the $(\mathrm{End}\ X)^{op}$-submodule $H = \mathrm{Im}(\mathrm{Hom}_{\underline{\underline{C}}}(X, B) \xrightarrow{(X, f)} \mathrm{Hom}_{\underline{\underline{C}}}(X, C))$ of $\mathrm{Hom}_{\underline{\underline{C}}}(X, C)$. Thus two right X-determined morphisms $f: B \longrightarrow C$ and $f': B' \longrightarrow C$ have the property that the two $(\mathrm{End}\ X)^{op}$-submodules $\mathrm{Im}(X, f)$ and $\mathrm{Im}(X, f')$ of $\mathrm{Hom}_{\underline{\underline{C}}}(X, C)$ are the same, if and only if there is a commutative diagram

$$\begin{array}{ccc} B & \xrightarrow{f} & C \\ h \downarrow & & \| \\ B' & \xrightarrow{f'} & C \\ h' \downarrow & & \| \\ B & \xrightarrow{f} & C \end{array}$$

This observation easily shows that a right X-determined morphism $f: B \longrightarrow C$ need not necessarily be determined even up to isomorphism by the $(\mathrm{End}\ X)^{op}$-submodule $\mathrm{Im}(X, f)$ of $\mathrm{Hom}_{\underline{\underline{C}}}(X, C)$, whereby an isomorphism from $f: B \longrightarrow C$ to $f': B' \longrightarrow C$ we mean an isomorphism $u: B \longrightarrow B'$ such that $f = f'u$. However, as we now point out, there is a simple condition that can be put on a morphism $f: B \longrightarrow C$ which guarantees that if it is right X-determined, then it is uniquely determined, up to isomorphism, by the $(\mathrm{End}\ X)^{op}$-submodule $\mathrm{Im}(X, f)$ of $\mathrm{Hom}_{\underline{\underline{C}}}(X, C)$.

We say that a morphism $f: B \longrightarrow C$ is right minimal provided an endomorphism $u: B \longrightarrow B$ is an isomorphism whenever $fu = f$. Our previous remarks show that two right X-determined morphisms which are also right minimal $f: B \longrightarrow C$ and $f': B' \longrightarrow C$ are isomorphic if and only if $\mathrm{Im}(X, f) = \mathrm{Im}(X, f')$. With these preliminary general remarks in mind we now turn our attention to the question of the existence of right determined morphisms. All of the existence theorems we know are based on the following theorem which is proven in Section 3 of Chapter I.

Let $\underline{\underline{C}}$ be the category Mod Λ of all Λ-modules for some arbitrary ring Λ. Suppose X is a finitely presented Λ-module and C is an arbitrary Λ-module. Let H be an $(\text{End } X)^{op}$-submodule of $\text{Hom}_\Lambda(X, C)$ containing the $(\text{End } X)^{op}$-submodule $P(X, C)$ consisting of all morphisms $h: X \longrightarrow C$ which are a composition $X \longrightarrow P \longrightarrow C$ with P a projective Λ-module. Then there is an epimorphism $f: B \longrightarrow C$ of Λ-modules which is right minimal and right X-determined such that $\text{Im}(X, f) = H$. Furthermore, this right minimal and right X-determined morphism $f: B \longrightarrow C$ has the following property. Suppose $f': B' \longrightarrow C$ is any right X-determined morphism such that $\text{Im}(X, f') = H$, then $f': B' \longrightarrow C$ is isomorphic to $g: B \coprod B'' \longrightarrow C$ where $B \coprod B''$ is the direct sum of B and some other module B'' and $g|B = f$ while $g|B. = 0$.

An obvious question to ask concerning the right minimal and right X-determined epimorphism $f: B \longrightarrow C$ is what is Ker f? While a complete answer to this question can be found in Section 3, we briefly indicate the form of the answer.

Let $P_1 \xrightarrow{\nu} P_o \xrightarrow{\varepsilon} X \longrightarrow 0$ be a projective presentation of X, i.e. $P_1 \xrightarrow{\nu} P_o \xrightarrow{\varepsilon} X \longrightarrow 0$ is an exact sequence with the P_i finitely generated projective Λ-modules. Then $\text{Hom}_\Lambda(P_i, \Lambda)$ is a finitely generated projective Λ^{op}-module and so

$\text{Tr } X = \text{Coker } (\text{Hom}_\Lambda(P_o, \Lambda) \longrightarrow \text{Hom}_\Lambda(P_1, \Lambda))$ is a finitely presented Λ^{op}-module. Let $\Gamma = \text{End}_{\Lambda^{op}}(\text{Tr } X)$. Then there is an injective Γ-module I such that the abelian group $\text{Hom}_\Gamma(\text{Tr } X, I)$ together with the Λ-module structure induced from the Λ^{op}-module structure of Λ^{op} on $\text{Tr } X$ is isomorphic to the kernel of the right minimal and right

X-determined epimorphism $f: B \longrightarrow C$. The particular injective Γ-module I used in this construction is the injective envelope of $\mathrm{Hom}_\Lambda(X, C)/H$, which is not only an $(\mathrm{End}\ X)^{op}$-module, but also a Γ-module in a natural way, as explained in Section 3.

Before giving some of the applications of this main existence theorem, we point out that no comparable theorem exists for the dual notion of morphisms left determined by an object. More precisely, a morphism $f: A \longrightarrow B$ in a category $\underline{\underline{C}}$ is said to be left determined by Y in $\underline{\underline{C}}$ (or is left Y-determined) if a morphism $g: A \longrightarrow X$ has the property $\mathrm{Im}(\mathrm{Hom}_{\underline{\underline{C}}}(X, Y) \xrightarrow{(g, Y)} \mathrm{Hom}_{\underline{\underline{C}}}(A, Y))$ is contained in $\mathrm{Im}(\mathrm{Hom}_{\underline{\underline{C}}}(B, Y) \xrightarrow{(f, Y)} \mathrm{Hom}_{\underline{\underline{C}}}(A, Y))$ if and only if there is a morphism $h: B \longrightarrow X$ such that $hf = g$. All the known existence theorems for morphisms left determined by objects are derived from existence theorems for morphisms right determined by objects by means of some sort of duality. This and other uses of dualities is part of the reason why most of the theory developed in this paper is for algebras rather than rings.

Rather than getting involved in the technicalities needed to give the applications of the main existence theorem to the derivation of other existence theorems in the full generality given in the latter part of Chapter I, I simply give a few sample results in the classical cases of finite dimensional algebras over a field and classical orders over Dedekind rings.

Let Λ be a finite dimensional algebra over a field k. Suppose $0 \longrightarrow A \xrightarrow{g} B \xrightarrow{f} C \longrightarrow 0$ is an exact sequence of finitely generated Λ-modules. Then f is right determined by $\mathrm{Tr}(\mathrm{Hom}_k(A, k))$ and g is left determined by $\mathrm{Hom}_k(\mathrm{Tr}\ C, k)$. More generally, any morphism $f: B \longrightarrow C$ of finitely generated Λ-modules is right determined by $\mathrm{Tr}(\mathrm{Hom}_k(\mathrm{Ker}\ f, k)) \amalg \Lambda$ and is left determined by

$\text{Hom}_k(\text{Tr}(\text{Coker } f), k) \coprod \text{Hom}_k(\Lambda, k)$. Thus we see that as far as finite dimensional algebras are concerned, the property of morphisms being left and right determined by modules is a universal property of morphisms between finitely generated modules.

Suppose now that R is a Dedekind domain (not a field) and Λ an R-order in the classical sense. Let $L(\Lambda)$ be the category of Λ-lattices, i.e. Λ-modules which are finitely generated projective R-modules. Suppose X is a Λ-lattice. Then we define the Λ^{op}-lattice $\text{Tr}_L X$ as follows. Let $P_1 \xrightarrow{b} P_o \xrightarrow{\varepsilon} X \longrightarrow 0$ be a projective presentation of X. Then $\text{Tr}_L X = \text{Im}(\text{Hom}_\Lambda(P_o, \Lambda) \longrightarrow \text{Hom}_\Lambda(P_1, \Lambda))$ by definition. Now suppose $0 \longrightarrow A \xrightarrow{g} B \xrightarrow{f} C \longrightarrow 0$ is an exact sequence of Λ-modules each of which is a Λ-lattice. Viewing g and f as morphisms in $L(\Lambda)$, we have that f is right $\text{Tr}_L(\text{Hom}_R(A, R))$-determined in $L(\Lambda)$ and g is left $\text{Hom}_R(\text{Tr}_L C, R)$-determined in $L(\Lambda)$. More generally, a morphism $f: B \longrightarrow C$ in $L(\Lambda)$ is both left and right determined by objects in $L(\Lambda)$.

Chapter II is devoted to applying some of the results of Chapter I to showing that simple functors on suitable categories of modules are finitely presented. In this connection the notion of a subfunctor of a functor being determined by an object in a category is introduced. Suppose $F: \underline{\underline{C}}^{op} \longrightarrow \text{Ab}$ is a functor (all functors are additive) where $\underline{\underline{C}}^{op}$ is the opposite category of $\underline{\underline{C}}$. A subfunctor G of F is said to be determined by an object X in $\underline{\underline{C}}$ if a subfunctor G' of F is contained in G whenever $G'(X) \subseteq G(X)$. The connection between this notion and that of a morphism $f: B \longrightarrow C$ in $\underline{\underline{C}}$ being right determined by X is given by the following easily verified result. A morphism $f: B \longrightarrow C$ is right X-determined if and only if the induced morphism $(\ ,f): (\ ,B) \longrightarrow (\ ,C)$ of representable functors has the property that the subfunctor $\mathrm{Im}(\ ,f)$ of $(\ ,C)$ is determined by X.

Suppose now that G and G' are two X-determined subfunctors of a functor $F: \underline{\underline{C}}^{op} \longrightarrow Ab$. Then it is obvious that $G = G'$ if and only if $G(X) = G'(X)$. Thus an X-determined subfunctor G of F is uniquely determined by the $(\mathrm{End}\ X)^{op}$-submodule $G(X)$ of $F(X)$. This suggests the question: If H is an $(\mathrm{End}\ X)^{op}$-submodule of $F(X)$ is there an X-determined subfunctor G of F such that $G(X) = H$? It is easy to see that this question has an affirmative answer for each $(\mathrm{End}\ X)^{op}$-submodule H of F. We denote by F_H the unique X-determined subfunctor of F such that $F_H(X) = H$.

In particular, for each C in $\underline{\underline{C}}$ and each right ideal $\underline{a}$ of End C, there is associated the C-determined subfunctor $(\ ,C)_{\underline{a}}$ of $(\ ,C)$. It is not very difficult to see that $\underline{a}$ is a maximal right ideal of End C if and only if $(\ ,C)_{\underline{a}}$ is a maximal subfunctor of $(\ ,C)$. Moreover a subfunctor G of $(\ ,C)$ is maximal if and only if $G(C)$ is a maximal right ideal of End C and $G = (\ ,C)_{G(C)}$. Thus there is a one-to-one correspondence between the maximal right ideals $\underline{m}$ of End C and the maximal subfunctors of $(\ ,C)$, the correspondence being given by $\underline{m} \longmapsto (\ ,C)_{\underline{m}}$ for all maximal right ideals of End C. From this it follows that a simple functor $S: \underline{\underline{C}}^{op} \longrightarrow Ab$ such that $S(C) \neq 0$ is isomorphic to $(\ ,C)/(\ ,C)_{\underline{m}}$ for some maximal right ideal $\underline{m}$ of End C. Moreover, since each simple functor $S \neq 0$, there is some C in $\underline{\underline{C}}$ such that $S(C) \neq 0$. Thus all simple functors from $\underline{\underline{C}}^{op}$ to Ab are isomorphic to $(\ ,C)/(\ ,C)_{\underline{m}}$ for some C in $\underline{\underline{C}}$ and maximal right ideal $\underline{m}$ of End C.

Next we recall that a functor $F: \underline{\underline{C}}^{op} \longrightarrow Ab$ is said to be finitely presented if there is an exact sequence of functors $(\ ,A) \longrightarrow (\ ,B) \longrightarrow F \longrightarrow 0$. Therefore the simple functors are finitely presented if and only if for each C in $\underline{\underline{C}}$ and maximal ideal $\underline{m}$ of End C there is a right C-determined morphism $f: B \longrightarrow C$ such that $\mathrm{Im}((C, B) \xrightarrow{(C, f)} (C, C)) = \underline{m}$.

Chapter II is mainly devoted to showing how the results of Chapter I concerning morphisms which are right determined by objects in certain categories $\underline{\underline{C}}$ of modules over particular rings can be used to prove that the simple functors from $\underline{\underline{C}}^{op}$ to Ab are finitely presented. In particular, if Λ is a finite dimensional algebra over a field k, then the simple functors from $(\mathrm{mod}\ \Lambda)^{op}$ to Ab are finitely presented where mod Λ is the category of all finitely generated Λ-modules. Also if Λ is an order over a Dedekind domain R, then all the simple functors from $(L(\Lambda))^{op}$ to Ab are finitely presented. Similar results for simple functors from a category $\underline{\underline{C}}$ to Ab are also established. These results lead us back to almost split sequences, the starting point of this whole discussion.

While the major portion of this paper is devoted to exploring the notions we just finished discussing, there are sprinkled throughout the paper various applications not directly connected with the development of the general theory. It is hoped that some of these notions will prove useful in studying other such questions as has already happened in the case of finite dimensional algebras over fields as indicated by some of the other papers in this volume (see [1] and [9]).

Finally, I would like to thank Professor Robert Gordon for his skill and diligence in organizing the conference, the NSF and Temple University for their financial backing, and the participants in the conference for their spirit and good humor.