

# Lecture Notes in Control and Information Sciences

Edited by M. Thoma and A. Wyner

158

K. Warwick, M. Kárný,  
A. Halousková (Eds.)

Advanced Methods  
in Adaptive Control  
for Industrial Applications



Springer-Verlag

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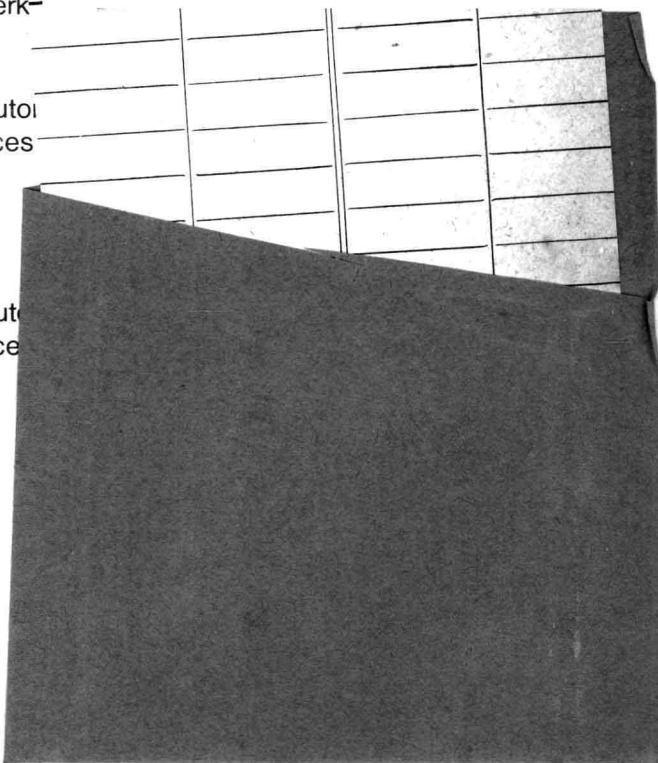
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## PREFACE

This volume contains the selected and collected papers which resulted from a joint Czechoslovak-UK Seminar on "Advanced Methods in Adaptive Control for Industrial Applications", an event held in Praha, 14-16 May 1990. The three day seminar was intended to bring together experts in Czechoslovakia and the UK in order to provide for the presentation and fluent exchange, of ideas and problem solutions in the field of the seminar title. A major aim was to direct the papers towards the actual problems faced by industry, both at the present time and in the near future. However, a number of papers were specifically aimed at reflecting possible trends in the longer term, by looking at particular implementation issues.

The underlying hope with this text, as indeed was the case with the Praha seminar, is that the volume will make an important contribution towards the industrial take-up and usage of advanced computer control systems. The range of papers presented have is quite large, with some being of a more mathematical/theoretical nature, whilst others are very much based on practical application examples. Perhaps the norm is directed towards design and implementation of computer control systems, specifically for general application.

The seminar in Praha was jointly organized and sponsored by the State Committee for Scientific and Technical Development (Praha), the Department of Trade and Industry (London), the Institute of Information Theory and Automation, Czechoslovak Academy of Sciences (Praha) and the University of Reading (Reading). Our thanks go to all of those above, in particular Tony Burwood-Smith and Richard King from the Department of Trade and Industry, for chaperoning the UK delegation, to Ladislav Tichý in the Czechoslovak Embassy in London, to O. Bobko in the State Commission, Praha.

At the end of the Seminar in May, a reception was held for participants, by the British Ambassador to Praha, His Excellency Mr. P. L. O'Keeffe, and our sincere thanks go both to him and to Michael Haddock, Commercial Secretary at the British Embassy for the reception and for their help during the seminar itself.

The editors would also like to thank those concerned with Springer-Verlag, for their swift response in considering this text, particularly Prof. Dr.-Ing. M. Thoma the Series Editor, and Erdmuthe Raufelder, Editorial Assistant. Finally, the editors particularly wish to express their gratitude to those who helped to put the final version of this volume together, namely Jiřina Gajdošová at Praha and Liz Lucas at Reading.

Praha, November 14, 1990

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# ADAPTATION OF LQG CONTROL DESIGN TO ENGINEERING NEEDS

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## Abstract

The purpose of the paper is twofold: (I) to modify the standard LQG control design procedure so that it might better meet the requirements of practicing engineers, both with respect to quality of the resulting control and with respect to its robustness, (II) to design algorithms for the control synthesis which are robust with respect to numerical errors and are suitable for real-time operation in adaptive systems based on microprocessors with reduced precision of arithmetic operations.

The control synthesis is based on the receding horizon philosophy. A suitable combination of robustness and quality of control is achieved by designing the control for the receding horizon optimally in the LQG sense but with restriction on the admissible control strategy. The process input planned for the receding horizon is restricted to be piece-wise constant for periods longer than the sampling period the control loop operates with. In addition, a modification of the quadratic criterion is recommended in order to reduce the overshoots.

The resulting control is proved to be stable and the increase of robustness is demonstrated on examples.

The algorithms are elaborated for the observable canonical state representation of the incremental ARMA or Delta model of the controlled process.

Simulated examples are given for illustration.

## 1 Introduction

The research reported in this paper has been motivated by the fact that in practice the LQG optimal control often cannot be applied in a straightforward way. One of the reasons is that it is difficult, or impossible, to express all requirements of the user in terms of weights entering the quadratic criterion. The other reason is that the mathematical model used in the design procedure more or less deviates from the true plant. The objective of this paper is to modify the LQG control design procedure so that the both above circumstances may be reflected.

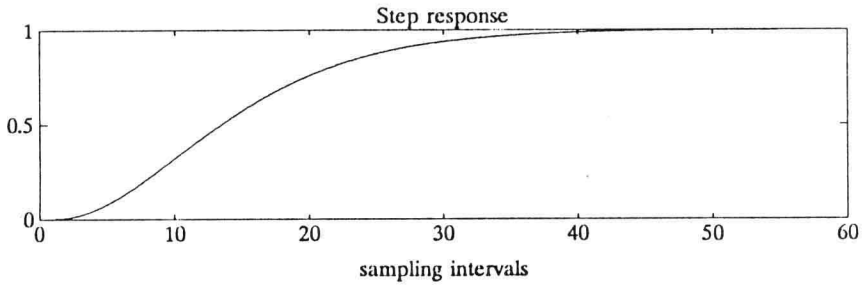


Figure 1: Step response of the system (1). Sampling interval  $T_s = 0.2$  sec .

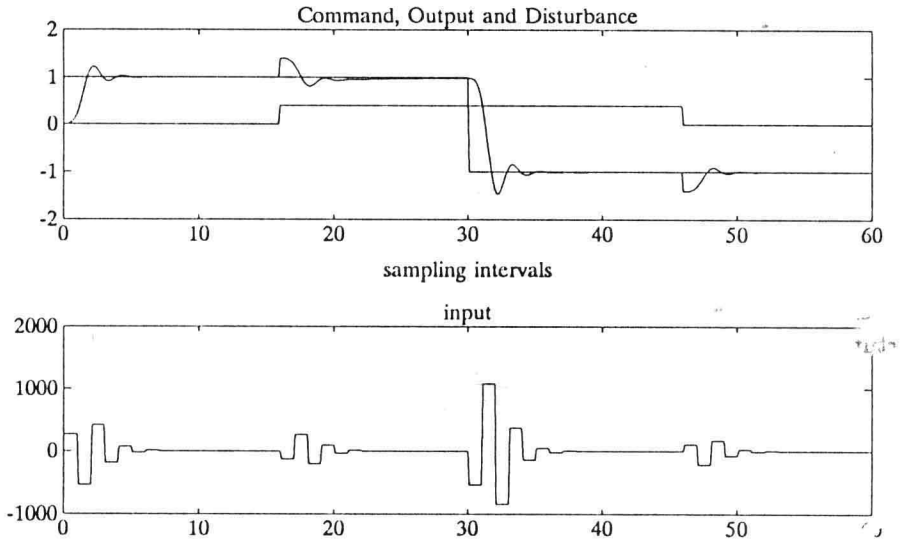


Figure 2: Minimum variance control of the system (1).

In order to make the ideas as plain as possible let us start with a simple example. Consider, for instance, the third order deterministic system

$$y = \frac{1}{(s+1)^3} u$$

the step response of which is shown in Fig.1. Suppose that this system has to be controlled with a relatively short sampling interval  $T_s = 0.2$  sec. Note that the significant part of the step response is covered, approximately, by 30 sampling intervals. The realization of the "minimum variance" control (with no penalty on the input) is registered in Fig.2. The plotted disturbance acts on the process output. It is clear that such control is unacceptable in practice for both above reasons. Excessive movements of the actuator cannot be practically realized and require an extremely precise knowledge of the dynamics of the system.

There are two standard ways how to make the LQG control more realistic. The first possibility, shown in Fig.3, is to damp the system input by introducing a suitably chosen weight on the input increments into the quadratic criterion. The choice of this weight

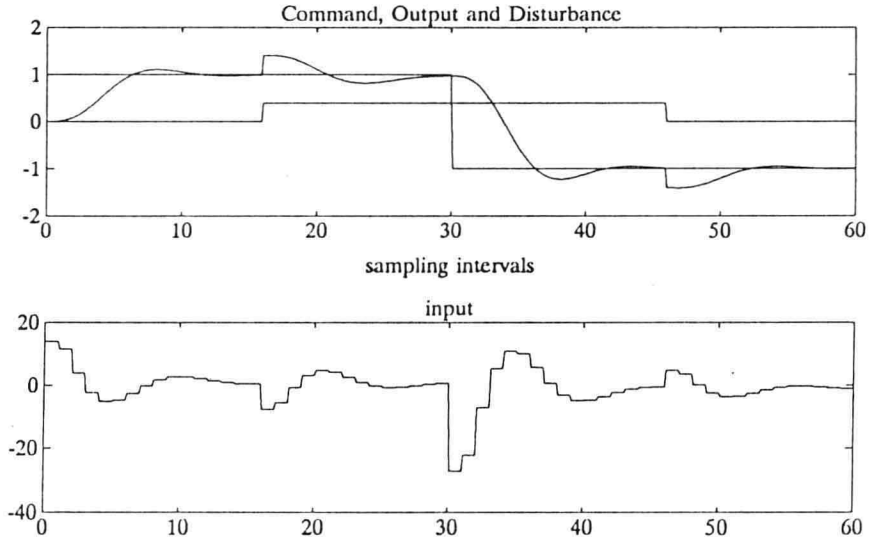


Figure 3: Damping of the control signal by penalization of the input increments

For example  $q_u = 0.00015$ , is left for the user as a “performance oriented knob” but, unfortunately, no simple rule is available for this choice without much experimentation. In self-tuning control applications a procedure has been suggested [1] which introduces penalization of inputs automatically in dependence upon the observed data and the user sets it on the input signal. In this paper another possibility will be followed which has some advantages.

One other way how to reduce the movements of the actuator is to increase the sampling interval. The rule of thumb for the choice of the sampling rate is to cover the significant part of the step response by 5 – 10 sampling intervals. The disadvantage of this method is that the system is left without supervision for relatively long periods. This may cause a slow reaction to disturbances and also undesirable overshoots may appear. This is demonstrated in Fig.4, where the sampling rate is 3 times slower than in Fig.2.

On the basis of the receding horizon philosophy, in the following Section 2 a modified LQG control problem is formulated which makes it possible to retain the original sampling rate and at the same time can damp the control signal without introducing the penalization of its increments. To solve the problem it is necessary to assume a mathematical model of the controlled process. For the purpose of this paper the elemental ARMA or Delta model will be assumed. A suitable canonical state representation of these models is introduced in Section 3. The solution of the modified LQG control problem is given in Section 4. Since the calculated control law has to operate under conditions which differ from those assumed in the design procedure, the stability of the closed control loop has to be investigated. This is the topic of Section 5. In Section 6 a modification of the design procedure is suggested which reduces the overshoots of the controlled output. The increase of the robustness of the control is demonstrated



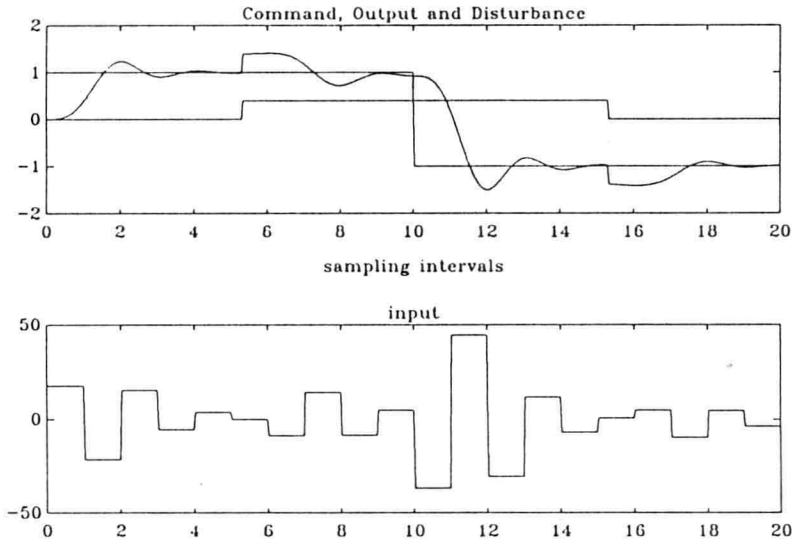


Figure 4: Sampling interval 3 times longer, no penalty on input.

in Section 7. Efficient and numerically reliable algorithm for the computation of the control law is elaborated in Section 8. The Section 9 is concluding and gives the authors recommendation how to apply the results.

## 2 Formulation of the modified LQG control problem

When formulating a control problem it is necessary to define : 1) the system model considered, 2) the admissible control strategies among which the optimal one is to be chosen, and 3) the criterion. The system model will be introduced and discussed in Section 3. Here the attention will be paid to the two remaining parts of the problem formulation.

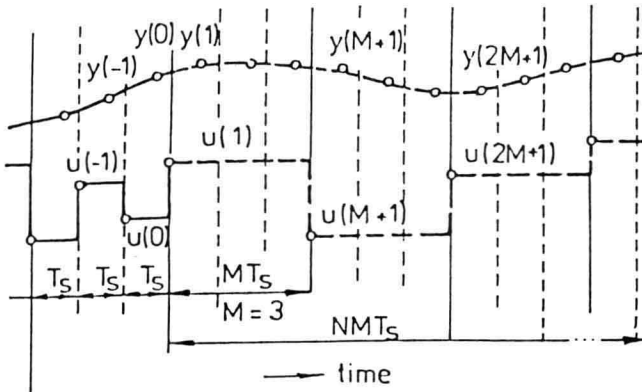


Figure 5: Receding control horizon with piece-wise constant input.

The control strategy will be designed for a receding horizon, finite or infinite, but only those control strategies are taken into the competition which produce the control input which is piece-wise constant for periods longer than the sampling period the control loop operates with. Hence, a fake sampling interval, consisting of  $M$  regular sampling intervals, is introduced as shown in Fig.5. However,  $M$  samples of the system output are measured within this fake sampling interval.

Let the receding horizon, the control strategy is to be design for, be  $N$  fake sampling intervals, ie.  $NM$  regular sampling intervals. To simplify the writing we make use of time indexing so that  $u(1)$  is the first input which is to be determined on the basis of the input-output data  $\mathcal{D}_0 = \{\dots, u(-1), y(-1), u(0), y(0)\}$  observed on the system up to and including the time index 0. Note that  $y(1)$  is not available at the moment when  $u(1)$  is to be determined. The time interval between the samples  $y(0)$  and  $u(1)$  is required to perform the computation. The speed of the computer determines this sample shift.

Following the receding horizon philosophy, we are interested mainly in the control law generating  $u(1)$ . This is, actually, the only control law which will be really applied.

It is also necessary to define the availability of the command signal  $w$  (the output reference, the setpoint sequence). In this paper the case of a positional servo will be considered. It is assumed that the command signal is given up to and including the time index 1,  $W_1 = \{\dots, w(0), w(1)\}$ . Note that  $w(1)$  says the controller what the next output  $y(1)$  should be after the input  $u(1)$  is applied. Hence the quadratic criterion, we shall consider, is

$$J = E \left\{ \sum_{j=0}^{N-1} \left[ \sum_{i=1}^M q_r r(jM+i)^2 + q_u \Delta u(jM+1)^2 \right] + x'(NM) P_N x(NM) \right\} \left| \mathcal{D}_0, W_1 \right\} \quad (2)$$

where

$$r(k) = y(k) - w(k) \quad (3)$$

is the control error,  $x(NM)$  is a suitably chosen last state and

$$q_r > 0, \quad q_u \geq 0 \quad P_N \gg 0$$

The criterion has to be minimized under the restriction

$$\Delta u(jM+i) = 0 \quad \text{for } i \neq 1 \quad (4)$$

The last term in the criterion (3) is important for stability reasons as it will be explained later on.

To be able to solve the LQG control problem, modified in the above way, it is necessary to define not only the system model, which will be introduced in the next Section, but also the stochastic model describing the expected evolution of the comand signal within the receding horizon. For the case of positional servo it is suitable to consider the future command signal as the generalized random walk

$$w(k) = w(k-1) + e_w(k) \quad (5)$$

where  $e_w(k)$  is discrete white noise with arbitrarily time varying variance  $Ew^2(k) = \varrho_w(k)$ , which is not correlated with  $y(k)$  and with the previous input-output data  $\mathcal{D}_{k-1}$ .

Note that the model (5) covers also the case of regulation with fixed setpoint when  $\varrho_w(k) = 0$ .

### 3 Process model

The input-output system model is assumed, for the control design purposes, in the incremental ARMA form

$$\Delta y(k) + \sum_{i=1}^n a_i \Delta y(k-i) = \sum_{i=0}^n b_i \Delta u(k-d-i) + e(t) + \sum_{i=1}^n c_i e(k-i) \quad (6)$$

or in the Delta form

$$\Delta^{n+1} y(k) + \sum_{i=1}^n a_i \Delta^{i+1} y(k-i) = \sum_{i=0}^n b_i \Delta^{i+1} u(k-i) + \sum_{i=0}^n c_i \Delta^i e(k-i) \quad (7)$$

where  $\Delta$  denotes the backward difference, eg.

$$\Delta y(k) = y(k) - y(k-1) \quad \Delta^i y(k) = \Delta^{i-1} y(k) - \Delta^{i-1} y(k-1)$$

end  $e(k)$  is a discrete white noise with constant variance  $\varrho$ .

Both the models (6) and (7) can describe the same input-output relation but with different parameters, of course. We shall keep the same notation for the different parameters because, as shown in detail in [8], they can be given the common canonical observable state representation

$$y(k) = y(k-1) + h'_s s(k-1) + b_0 \Delta u(k) + e(t) \quad (8)$$

$$s(k) = F_s s(k-1) + G_s \Delta u(k) + \varsigma(k) \quad (9)$$

If the following parameterless matrices

$$h_s = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad H_s = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \quad (10)$$

and the following parametr vectors are introduced

$$\bar{a} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \quad \bar{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \quad \bar{c} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} \quad (11)$$

then the matrix coefficients of the state equation (9) are

$$F_s = H_s - \bar{a} h'_s + \mu I \quad G_s = \bar{b} - \bar{a} b_0 \quad \varsigma(k) = [\bar{c} - \bar{a}] e(k) \quad (12)$$

where  $\mu = 0$  for ARMA case and  $\mu = 1$  for Delta case.