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Some Extensions of de Finetti's Representation Theorem for Interacting Particle Systems

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Introduction

Symmetric probability measures on the infinite product

$$\Omega = F \times F \times \ldots$$

of a set F (equiped with a σ - algebra) are a well-known concept in probability theory: A probability measure on Ω is said to be symmetric if it is invariant under the group of those transformations of Ω which are defined by a permutation of finitely many coordinates. Apparently the first place in which this notion appeared was a contribution of J. Haag to the International Congress of Mathematicians at Toronto in 1924 . B. de Finetti (1931) independently introduced the idea of symmetric probability measures. For the case $F = \{0 , 1\}$ he proved the following theorem which is now well-known: Each symmetric probability measure μ is a mixture of homogeneous product measures, i. e. , μ has a representation

$$(0.1) \qquad \mu = \int m(d\alpha) \quad \alpha \otimes \alpha \otimes \ldots$$

where m is a probability measure on the set of all probability measures on F . Moreover, m is uniquely determined and is obtained as the limit distribution of the empirical distributions. (It is worthwile mentioning that de Finetti was interested in this result for philosophical reasons: It shows that a statement of the form "I believe that the tosses of this coin are independent and identically distributed, with the unknown probability of heads occuring lying somewhere between 1/3 and 2/3" is equivalent to the purely subjective statement "I believe the tosses are symmetrically distributed and that the frequency of heads will fall somewhere between 1/3 and 2/3" .)

De Finetti's theorem was extended to more general F's by A. Khintchine (1932, 1952) , B. de Finetti (1937) , E.B. Dynkin (1953) and finally E. Hewitt and L.J. Savage (1955) (this last paper should be consulted for the earlier references).

Hewitt and Savage pointed out that the representation (0.2) results from a combination of the following two assertions: Each symmetric μ is the mixture of extreme symmetric probability measures, and the extreme symmetric probability measures are just the homogeneous product measures. They proved that the second assertion is true without any condition on F, and found general conditions implying the first. (Their famous 0 - 1 law is equivalent to the statement that the homogeneous product measures are extreme symmetric measures; L.E. Dubins and D.A. Freedman (1979) have shown that the first assertion may fail to hold even when F is a separable metric space.) There are at least two lines of further research which arose from their paper. The first line was concerned with the question of whether the 0 - 1 law could be extended to products of not necessarily identical probability measures, see, for instance, D. Aldous and J. Pitman (1977), G. Simons (1978) and the references therein. A second group of papers dealt with the problem of whether de Finetti's representation theorem has an extension to probability measures with a weaker symmetry condition; for instance a condition which is satisfied by all mixtures of certain Markov chains; see D.A. Freedman (1962), T. Höglund (1974), S.L. Lauritzen (1974), and P. Martin-Löf (1974).

Recently, and independently of this statistical tradition, the need for such representation theorems also arose in Statistical Mechanics. It is the purpose of this text to explain the origins of this need and to give some of the required theorems. So let us describe the problem. Let $F = \{0, 1\}$, choose a countably infinite set S and regard the product $\Omega = F^S$ as the space of all configurations of indistinguishable particles in S, no two of which are allowed to occupy the same site. If (apart from this exclusion rule) the particles do not interact but possibly prefer certain sites then in equilibrium the state of this particle system would be described by a (not necessarily homogeneous) product measure. Clearly, from a physical point of view it is much more natural to consider particle systems with an interaction. Then, as well as specifying a self-potential which describes to what extent the particles prefer to stay at each site, it is also necessary to give the additional energy required in order that a pair of sites should be occupied. (This, of course, is assuming only a pairwise interaction.) In this case the set of equilibrium states (which is possibly not a singleton!) is given by the set of all so-called (grand canonical) Gibbs measures for this interaction. These are defined as those probability measures which have prescribed versions (depending on the interaction) of their conditional probabilities with respect to the configurations outside each finite region.

In certain situations, however, it is only the interaction potential which is determined by the physical circumstances and not the self-potential. For instance, suppose we have a time evolution of the particle system in which the particles may change their positions but cannot be created or destroyed. Moreover, assume that the particle motion is governed by the interaction in the sense that those particle jumps are favoured which entail the largest gain of total energy. In order to establish that such an evolution is locally in equilibrium it is sufficient to know that in each finite region the configurations have a particular distribution (given in terms of the interaction) *when in addition to the configuration outside the region the particle number in the region is also fixed*. We call a probability measure with this property a *canonical Gibbs measure* because these specific local equilibrium distributions corresponding to given environments and particle numbers are just the so-called canonical Gibbs distributions. If there is no interaction the canonical Gibbs measures are exactly the symmetric measures.

In this text we will ask whether the analogue of de Finetti's result holds, namely whether each canonical Gibbs measure is a mixture of measures for which the distributions of the local particle numbers also have a particular form (being defined in terms of a self-potential), i. e., a mixture of grand canonical Gibbs measures. We will show that the answer is in the negative when the interaction and the self-potential are spatially very inhomogeneous but is positive as soon as these are, in some sense, sufficiently homogeneous. In our framework there will be no difficulty to show that each canonical Gibbs measure is a mixture of extreme canonical Gibbs measures. Thus our main task will be to find natural conditions which ensure that each extreme canonical Gibbs measure is a (grand canonical) Gibbs measure. As a particular result we will obtain some extensions of the Hewitt/Savage 0 - 1 law to inhomogeneous nonproduct measures.

Thus far we have only described particle systems on a discrete set. But clearly the same questions arise if the position space of the particles is continuous, and of course this is the case of most interest in physics. Therefore we will also be concerned with canonical and grand canonical Gibbs point processes, and we will obtain some extensions of the result stated immediately below which is the point process counterpart of de Finetti's theorem, and was first proved by K. Nawrotzki (1962) and D.A. Freedman (1963). Suppose μ is a point process on the real line (i. e., a probability measure on the set of all locally finite point configurations on $\mathbb{R}$) satisfying the following symmetry condition: For each bounded interval Λ and a fixed number of particles in Λ the positions of these particles are independently and uniformly distributed. Then μ is a mixed Poisson process, i. e., μ has a representation

$$(0.2) \qquad \mu = \int m(dz)\ \pi^z ,$$

where m is a (uniquely determined) probability measure on $[0, \infty[$ and π^z denotes the Poisson point process on $\mathbb{R}$ with intensity $z \geq 0$.

The notion of a canonical Gibbs measure has an obvious generalization, namely the concept of a microcanonical Gibbs measure. To obtain its definition we have only to change the italicized phrase above "when ... the particle number in the region is also fixed" into "when ... the values of certain extensive quantities (as, for example, the particle number and the interaction energy) in the region are also specified". However, we think it is reasonable to confine ourselves to canonical Gibbs measures. The reader interested in the microcanonical case is referred to M. Aizenman et al. (1978), C. Preston (1978), and R.L. Thompson (1974).

I am much indepted to C. Preston for help with the English and to U. Sander for typing the manuscript.

Contents

§ 1 Basic concepts

We will consider both discrete and continuous models for an interacting particle system. In both cases we restrict ourselves to the situation which is usually considered in classical statistical mechanics. In the discrete models the particles are allowed to occupy the sites of some countably infinite set S , e.g. the three-dimensional integer lattice. If the particles are indistinguishable, then at each site $x \in S$ we have the alternatives: x is occupied by a particle or not. To stress the symmetry between these two possibilities we imagine the empty sites to be occupied by particles of a species 0 . More generally, we will consider an arbitrary finite set F of particle types. Then the space of all particle configurations is $\Omega = F^S$.

In the continuous case we assume that there is only one type of particle and the particles take their positions in a nice subset S of a Euclidean space. If we exclude the possibility that two particles have the same position the set Ω of all locally finite subsets of S becomes the configuration space. (The generalization to more general spaces S will be clear whenever it is possible. Furthermore, the case of several types of particles can be reduced to the case of a single type by embedding several copies of S in a higher dimensional space.)

In order to completely formulate the models, in both the discrete and the continuous case, it is necessary to specify the interaction between the particles. Then (under a hypothesis of thermodynamic equilibrium) the commonly accepted Ansatz of Gibbs yields the probability distributions for local configurations conditioned with respect to a fixed environment. The objects of our study are probability measures on Ω whose local behaviour is determined by the Gibbs distributions.

We should mention that a unified treatment of both the discrete and the continuous case would be possible using the formalism of point process theory or the abstract setting of Preston (1976, 1978). However, we think that each case is better treated separately, and this facilitates the possible study of only one of the cases.

1.1 The discrete model

Let us start with S , a countably infinite set of particle sites, and F , a finite set of types of particles with cardinality $|F| \geq 2$. The configuration space is $\Omega = F^S$.

For each $V \subset S$ let

$$(1.1) \qquad X_V : \omega = (\omega_x)_{x \in S} \to \omega_V = (\omega_x)_{x \in V}$$

be the projection from Ω on $\Omega_V = F^V$. We use the same symbol for the projection from Ω_W to Ω_V whenever $W \supset V$. If V and W are disjoint subsets of S and $\omega \in \Omega_V$, $\zeta \in \Omega_W$, we denote by $\omega\zeta$ the configuration on $V \cup W$ with $(\omega\zeta)_V = \omega$ and $(\omega\zeta)_W = \zeta$.

For any $a \in F$, $\omega \in \Omega_V$ let

$$(1.2) \qquad N(a, \omega) = |\{x \in V : \omega_x = a\}|$$

be the number of a - particles in the configuration ω . (Which V has to be used will be clear from the context. If $\omega \in \Omega$ and only the particles in V are counted we write $N(a, \omega_V)$.) Finally, let

$$(1.3) \qquad N(\omega) = (N(a, \omega))_{a \in F} .$$

We let

$$(1.4) \qquad \mathcal{S} = \{\Lambda \subset S : 0 < |\Lambda| < \infty\}$$

denote the system of all non-empty finite subsets of S . For singletons in S we usually write x instead of $\{x\}$. Often we use the symbol

$$(1.5) \qquad \lim_{\Lambda \uparrow S}$$

with the meaning that the limit is taken over a fixed sequence $(\Lambda_n)_{n\geq 1}$ in S such that $\Lambda_n \subset \Lambda_{n+1}$ $(n \geq 1)$ and $\bigcup_{n\geq 1} \Lambda_n = S$. This sequence may be chosen arbitrarily unless the contrary is stated.

For any $\Lambda \in S$ let

$$(1.6) \qquad A_\Lambda = \{L \in \{0, 1, 2, \ldots\}^F : \sum_{a\in F} L(a) = |\Lambda|\}$$

be the range of the function $\omega \to N(\omega_\Lambda)$, and for $L \in A_\Lambda$ let

$$(1.7) \qquad \Omega_{\Lambda,L} = \{\omega \in \Omega_\Lambda : N(\omega) = L\}$$

be the set of configurations in Λ with given L.

For any $V \subset S$ we denote by

$$(1.8) \qquad F_V = \sigma(X_x : x \in V)$$

the σ - algebra of the events in V which are generated by the finite-dimensional projections in V. It is well-known that $F = F_S$ is generated by the product topology on Ω (with the discrete topology on F). Since Ω is compact the set of probability measures on (Ω, F) also is compact in the weak topology. Note that a sequence (μ_n) of probability measures on (Ω, F) converges weakly to some μ as soon as all cylinder probabilities converge:

$$\mu_n(X_\Lambda = \zeta) \to \mu(X_\Lambda = \zeta) \qquad (\zeta \in \Omega_\Lambda, \Lambda \in S) \qquad .$$

The following sub - σ - algebras of F will play a central rôle: the tail field

$$(1.9) \qquad F_\infty = \bigcap_{\Lambda \in S} F_{S \setminus \Lambda}$$

and the σ - algebra of symmetric events

$$(1.10) \qquad E_\infty = \bigcap_{\Lambda \in S} E_\Lambda \quad .$$

Here for any $\Lambda \in S$

$$(1.11) \qquad E_\Lambda = \sigma\,(\,N(X_\Lambda),\, F_{S \setminus \Lambda})$$

denotes the σ - algebra of events which are invariant under permutations of the sites in Λ . Note that $E_\Lambda \supset E_{\Lambda'}$ whenever $\Lambda \subset \Lambda'$.

The objects which we are interested in are probability measures on (Ω, F) whose conditional probabilities with respect to either $F_{S \setminus \Lambda}$ or E_Λ have a particular version. This version has the form postulated by Gibbs and is determined by a potential Φ describing the interaction of the particles.

(1.12) Definition: A function

$$\Phi \; : \; S \times \Omega \; \rightarrow \; \mathbb{R}$$

is called a *potential* if

(i) $\Phi\,(A, \cdot\,)$ is F_A - measurable for any $A \in S$. (Sometimes therefore we shall think of $\Phi\,(A, \cdot\,)$ as a function on Ω_A .)

(ii) for any $\Lambda \in S$, $\zeta \in \Omega_\Lambda$, $\omega \in \Omega_\Lambda$ the *energy* of ζ in Λ with boundary condition ω

$$E_\Lambda\,(\zeta \mid \omega\,) = \sum_{A \in S : A \cap \Lambda \neq \emptyset} \Phi\,(A, \zeta\, \omega_{S \setminus \Lambda})$$

is well-defined (as the finite limit of the partial sums over all $A \subset V$ if

V runs through the directed set S) and continuous as a function of ω .

This continuous dependence of the energy on the boundary condition says that the potential decays sufficiently rapidly for sets A with "large diameter" . A well-known sufficient condition for (ii) is

$$\sum_{x \in A \in S} \| \Phi(A, \cdot) \| < \infty \quad (x \in S) \quad , \tag{1.13}$$

where $\| \cdot \|$ denotes the sup-norm.

Without loss of generality we could assume that the potential vanishes for certain configurations, see e.g. Sullivan (1973). We shall not use such a normalization unless $F = \{0, 1\}$. In this case we can assume that there is a function $S \to \mathbb{R}$ (again denoted by Φ) such that

$$\Phi(A, \omega) = \Phi(A) \, \omega^A \quad (A \in S, \omega \in \Omega) \quad , \tag{1.14}$$

where $\omega^A = \prod_{x \in A} \omega_x = 1_{\{X_A \equiv 1\}}(\omega)$.

The so-called chemical potential $\Phi(x, a)$ $(x \in S, a \in F)$ specifies to which extent the site x favours the occurence of an a - particle. In order to favour type - a particles on the whole of S against all other types it is sufficient to add a constant (written in the form $-\log z(a)$) to $\Phi(\cdot, a)$. Since this procedure will be important for us we shall include it from the beginning:

(1.15) Definition: A function

$z : F \to [0, \infty[$ is called an *activity* if $\sum_{a \in F} z(a) > 0$.

A denotes the set of all activities.

As we shall see at once in (1.16) , it makes no difference for us if some $z \in A$ is multiplied by a positive factor. Therefore we write $z_1 \sim z_2$ if $z_1 = c \, z_2$ for some $c > 0$. In the case $F = \{0, 1\}$ the equivalence class of any $z \in A$ usually is represented by the number $z' \in [0, \infty]$ given by

$$z' = \begin{cases} z(1) \ / \ z(0) & \text{if} \quad z(0) > 0 \\ \infty & \text{otherwise} \end{cases} .$$

In the following we fix a potential Φ . Together with an activity $z \in A$, Φ defines a system of conditional probabilities as follows:

(1.16) Definition: Let $\Lambda \in S$, $\omega \in \Omega$. The probability distribution on Ω_Λ defined by

$$\gamma_\Lambda^z (\zeta | \omega) = Z_\Lambda (z, \omega)^{-1} \prod_{a \in F} z(a)^{N(a,\zeta)} \exp [-E_\Lambda (\zeta | \omega)] \qquad (\zeta \in \Omega_\Lambda)$$

is called the *(grand canonical) Gibbs distribution* on Λ with boundary condition ω corresponding to the potential Φ and activity z . The normalization factor

$$Z_\Lambda (z, \omega) = \sum_{\zeta \in \Omega_\Lambda} \prod_{a \in F} z(a)^{N(a,\zeta)} \exp [-E_\Lambda (\zeta | \omega)]$$

is called the *partition function.*

It follows directly from the definition that the family of Gibbs distributions is consistent in the following sense:

$$(1.17) \qquad \gamma_\Delta^z (\zeta | \omega) = \gamma_\Lambda^z (\zeta_\Lambda | \zeta \omega_{S \setminus \Delta}) \ \gamma_\Delta^z (\{X_{\Delta \setminus \Lambda} = \zeta_{\Delta \setminus \Lambda}\} | \omega)$$

whenever $\Lambda \subset \Delta$, $\zeta \in \Omega_\Delta$, $\omega \in \Omega$. Conversely, any reasonable consistent system of conditional probabilities can be described in terms of a potential. The Möbius inversion formula shows (see, for example, Sullivan (1973)) :

(1.18) Remark: *Suppose that a system* $(g_\Lambda(\cdot | \omega))_{\Lambda \in S, \omega \in \Omega}$ *of conditional probabi-*

lities is consistent in the sense of (1.17) *and that the function* $g_\Lambda(\zeta|\cdot)$ *are* $F_{S\setminus\Lambda}$ *- measurable, continuous, and strictly positive. Then there is a potential* Φ *such that* $g_\Lambda = \gamma_\Lambda^1$ *for all* $\Lambda \in S$.

If μ is a probability measure on (Ω, F) and $\Lambda \in S$, $\zeta \in \Omega_\Lambda$ then we let

(1.19) $$\mu_\Lambda (\zeta|\cdot) = \mu (X_\Lambda = \zeta | F_{S\setminus\Lambda})$$

denote the conditional probability of the event $\{X_\Lambda = \zeta\}$ with respect to the σ - algebra $F_{S\setminus\Lambda}$ and the measure μ .

(1.20) Definition: A probability measure μ on (Ω, F) is called a *Gibbs measure* with respect to the activity $z \in A$ and the potential Φ if for any $\Lambda \in S$ and $\zeta \in \Omega_\Lambda$

$$\mu_\Lambda (\zeta|\cdot) = \gamma_\Lambda^z (\zeta|\cdot) \qquad \mu \text{ - a.s.}$$

We let $G(z) = G(z,\Phi)$ denote the set of all such Gibbs measures.

This notion is due to Dobrushin (1968) and Lanford and Ruelle (1969). For this reason, Gibbs measures are often called DLR - states. They have given rise to an extensive theory describing the properties of many-particle systems in thermodynamic equilibrium. Certain aspects of this theory can be found in Preston (1973, 1976) and Ruelle (1978), for instance. In particular, one knows:

(1.21) Theorem: $G(z)$ *is always non-empty, convex, and weakly compact. For some* Φ *and* z *we have* $|G(z)| > 1$.

The non-uniqueness phenomenon $|G(z)| > 1$ has the physical interpretation of a phase transition and is therefore of particular interest. However, since this phenomenon is not the theme of this text we refer the reader to Dobrushin (1968 b), e.g.. Here we will identify the set $G(z)$ only in two simple situations:

(1.22) Example: *Suppose that* $\Phi(A, \cdot) = 0$ *whenever* $|A| > 1$. *Then* $G(z)$

is a singleton consisting of the product measure π^z *whose marginal distributions are given by*

$$\pi^z(X_x = a) \quad = \quad z(a) \exp[-\Phi(x, a)] \;/\; \sum_{b\in F} z(b) \exp[-\Phi(x, b)] \quad .$$

Indeed, it is easy to see that $\gamma_\Lambda^z(\cdot|\omega)$ is a product measure with these marginal distributions and does not depend on ω . Also, it is simple to verify:

(1.23) <u>Example:</u> *Let* $a \in F$ *and suppose that* $z(b) = 0$ *unless* $b = a$. *Then for each* Φ , $G(z)$ *consists of the point mass* ε_a *on the constant configuration whose value at each site is* a .

What is the difference in the local behaviour of two Gibbs measures with respect to the same potential but different activities? Obviously, their behaviour will be different when an a - particle at some site is replaced by a particle of a type $b \neq a$. For some models describing the time evolution of a many particle system (the so-called birth - and - death or spin - flip processes, see Liggett (1977), for instance) only this kind of replacements occur during the evolution. Therefore, if a Gibbs measure is invariant under such an evolution then its activity is uniquely determined. A different type of process has been introduced by Spitzer (1970). It models a system of infinitely many moving particles, see section 2.1 for details. In this kind of time evolution, particles of different types interchange their positions, and the behaviour of a Gibbs measure under such interchanges depends only on its potential and not on its activity. Indeed, such interchanges do not alter the total particle numbers in sufficiently large regions, and for each $\Lambda \in S$ and $L \in A_\Lambda$ the conditional probability given $\Omega_{\Lambda,L}$ with respect to a Gibbs distribution $\gamma_\Lambda^z(\cdot|\omega)$ (if it is well-defined) does not depend on z , and is given by $\gamma_{\Lambda,L}(\cdot|\omega)$, which is defined as follows:

(1.24) <u>Definition:</u> Let $\Lambda \in S$, $\omega \in \Omega$, $L \in A_\Lambda$. The probability distribution on Ω_Λ defined by

$$\gamma_{\Lambda,L}(\zeta|\omega) = 1_{\Omega_{\Lambda,L}}(\zeta)\, Z_{\Lambda,L}(\omega)^{-1} \exp[-E_\Lambda(\zeta|\omega)] \qquad (\zeta \in \Omega_\Lambda)$$

is called the *canonical Gibbs distribution* for Φ on Λ with particle numbers $L(a)$, $a \in F$, and boundary condition ω. The normalization factor

$$Z_{\Lambda,L}(\omega) = \sum_{\zeta\in\Omega_{\Lambda,L}} \exp[-E_\Lambda(\zeta|\omega)]$$

is called the *canonical partition function*. If $L \notin A_\Lambda$ we put $Z_{\Lambda,L}(\omega) = 0$.

The following consistency property, which is easily verified, makes precise the statement preceding the definition (1.24):

$$(1.25) \qquad \gamma_\Lambda^z(\zeta|\omega) = \gamma_{\Lambda,L}(\zeta|\omega)\ \gamma_\Lambda^z(\Omega_{\Lambda,L}|\omega)$$

whenever $z \in A$, $\Lambda \in S$, $L \in A_\Lambda$, $\zeta \in \Omega_{\Lambda,L}$, and $\omega \in \Omega$. Furthermore, the family of canonical Gibbs distributions satisfies the consistency condition

$$(1.26) \qquad \gamma_{\Delta,L}(\zeta|\omega) = \gamma_{\Lambda,N(\zeta_\Lambda)}(\zeta_\Lambda|\zeta\omega_{S\setminus\Delta})\ \gamma_{\Delta,L}(\{N(X_\Lambda) = N(\zeta_\Lambda), X_{\Delta\setminus\Lambda} = \zeta_{\Delta\setminus\Lambda}\}|\omega)$$

where $\Lambda \subset \Delta$, $L \in A_\Delta$, $\zeta \in \Omega_{\Delta,L}$, and $\omega \in \Omega$.

Similarly as in (1.20), the system of canonical Gibbs distributions can be used for the definition of canonical Gibbs measures. These will be the central notion of this text. In particular, we will see in § 2 that these measures are invariant under the particle motions mentioned above. Note that for any $\Lambda \in S$ and $\zeta \in \Omega_\Lambda$ the function

$$\omega \to \gamma_{\Lambda,N(\omega_\Lambda)}(\zeta|\omega)$$

is measurable with respect to E_Λ.

(1.27) <u>Definition:</u> A probability measure μ on (Ω, F) is called a *canonical Gibbs measure* corresponding to the potential Φ if for all $\Lambda \in S$ and $\zeta \in \Omega_\Lambda$