SAUNDERS MAC LANE
SELECTED PAPERS

Saunders Mac Lane
Selected Papers

Edited by I. Kaplansky

Springer-Verlag
New York Heidelberg Berlin

Saunders Mac Lane

Department of Mathematics
University of Chicago
5734 University Avenue
Chicago, Illinois 60637
USA

Editor

I. Kaplansky

Department of Mathematics
University of Chicago
5734 University Avenue
Chicago, Illinois 60637
USA

AMS Subject Classification: 01 A 75

Library of Congress Cataloging in Publication Data

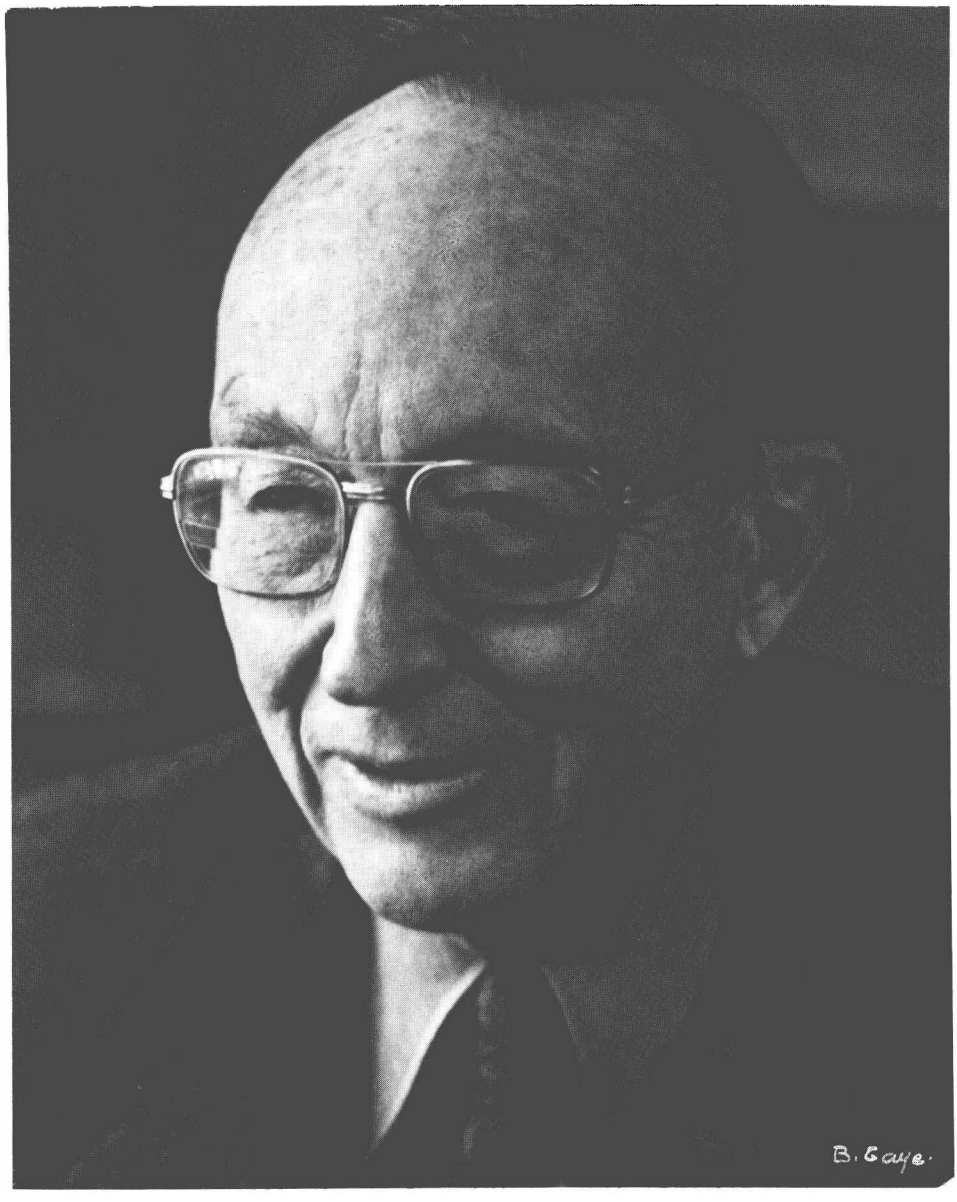
Mac Lane, Saunders,
Selected papers.

"Bibliography of Saunders Mac Lane": p.
1. Algebra—Collected works. I. Kaplansky, Irving, 1917–
QA155.M27 510 79-10105

Printed in the United States of America.

9 8 7 6 5 4 3 2 1

ISBN 0-387-90394-1 Springer-Verlag New York
ISBN 3-540-90394-1 Springer-Verlag Berlin Heidelberg


B. Cayle

Preface

A preface usually ends with appropriate expressions of thanks to the people who have helped. I would like instead to begin in that way. Most important is my gratitude to Samuel Eilenberg, Roger Lyndon, and Max Kelly, who joined me in contributing essays, and to Alfred Putnam who wrote the biography that leads off the volume. Dorothy Mac Lane helped generously with her encouragement (and of course her special source of information). Freda Davidson pitched in gallantly in helping to proofread the selected papers for typographical slips that had somehow eluded the eagle eye of Saunders Mac Lane (there turned out to be mighty few). When tricky problems came up I could always rely on Herman Meyer to solve them. John Gray enriched the year by organizing a highly successful conference in Aspen on May 23–27, 1979. Lastly, Walter Kaufmann-Bühler of the Springer-Verlag brought to bear his immense know-how and met all my wishes (with a smile, I think).

Saunders Mac Lane will be seventy years old on August 4, 1979, and with a little luck, the date of publication will work out to be within epsilon of this target date. Happy birthday, Saunders! (My colleagues in Eckhart Hall who watch his close approximation to perpetual motion will find this hard to believe.)

As Max Kelly remarks in beginning his essay, Saunders has given us bits of his own overview of his scientific career in [93] and [115]. To this I would add [114], where there is a fascinating narrative of the long collaboration with Sammy (which we can now read about on both sides of the ledger), and [116], which offers a broad panorama of the way the cohomology of groups arose in various ways at the hands of various people.

In the February, 1975 issue of the American Mathematical Monthly there is a tribute by Ralph Boas, on the occasion of the selection of Saunders for the Association's Distinguished Service Award.

For both professional and personal reasons, helping to assemble this volume of selected papers has been an enjoyable task. One particularly gratifying aspect is that his Ph.D. thesis is at last appearing in the regular literature, with a candid second look accompanying it as a bonus. Up to now (except for fifty scattered copies) it was invisible except for being listed as number 504 in Church's bibliography of symbolic logic (Journal of Symbolic Logic, vol. 1, p. 214).

There is one final point to be mentioned. The erratum to [109] has not been reprinted, since the paper itself was slightly revised to take account of it.

Irving Kaplansky

A Biographical Note

Alfred Putnam

A biographical note constitutes a form of introduction. Yet to introduce anyone as widely known and universally recognized as Saunders Mac Lane poses a particular problem: Saunders has had his share of formal introductions but he is not a formal person. A solution would be to rehearse informally information available in standard references; that would omit the vitality of the man. An alternative solution would be to recount a selection of more personal and characteristic notes; that would omit the structure of a lifetime. Since my introduction may be serving to acquaint some with Saunders Mac Lane for the first time, I have tried to include both facts from a full life and more personal notes to enable others to gain a sense of this intensely human mathematician and intensely mathematical man. I hope that Saunders himself, from whom I have so often heard—when it especially counted—the admonition "Give me an example" will savor my solution of the problem he presents. I should offer my credentials: for close to forty years Saunders Mac Lane has been my mathematical mentor, colleague, friend.

Of one thing about Saunders Mac Lane I am as certain as when I began to work with him as a graduate student: anyone engaged with him is sure to learn something. What it is could be the significance for mathematics of a complex piece of research. It could be how, over the years, firmly held convictions as to the true nature of mathematics must shape the content and teaching of mathematics. Or it could be the right principles by which the contributions of a mathematician to mathematics ought to be judged. At the right moment, on a walk with him it could even be a simple theory to account for cyclical fluctuations in the level of Lake Michigan. In whatever way learning may be experienced with Saunders Mac Lane, it is certain to have the marks of intense concentration, directness of address, and strictest attention to fundamentals. Saunders Mac Lane is ever the teacher.

The life we are celebrating began in Taftville, Connecticut on August 4, 1909 when Saunders Mac Lane was born, the eldest child of Donald Bradford and Winifred (Saunders) MacLane. His New England descent traced to the Mayflower, his Scots family to the MacLeans of Mull. (Actually, it would be more nearly accurate to record that it was Saunders *MacLane* who was born on that day in 1909, for the Saunders *Mac Lane* we know came into being much later through a creative accident in typing his name.)

Saunders Mac Lane's father and grandfather were both Congregational ministers. His grandfather, who was then a Presbyterian, had espoused evolution in his church in Steubenville, Ohio and found therewith that the more open views of Congregationalism better matched his convictions. Saunders' father was a

conscientious man with a strong sense of obligation. During World War I in Roxbury, Massachusetts, Saunders' father had given his support to pacifism and had felt obliged to relinquish his church there. He moved then to a small and struggling congregation in Wilbraham, Massachusetts. There Saunders attended a two-room school. It must have been with something of his grandfather's and father's spirit that Saunders, in his turn in the 1920's, could be found advocating in the high school paper in Leominster, Massachusetts the abolition of military drill and military spending.

Those who know Saunders Mac Lane at all know him as a man of utmost integrity, steadfast principles, and strong will. On occasion these can have meant pain for himself or others, when feelings and strict application of principles have been at odds, but he is made of stern stuff. In that, as in his gift for teaching, his didactic skill, and his standards of clear and direct expression, we see in Saunders the force of some of that same tradition which informed the lives of his father and grandfather.

When Saunders Mac Lane was fifteen his father died and the family moved to Leominster, where his grandfather had a church. Saunders graduated from high school in 1926 and was sent by an uncle to Yale. In his first year there he proceeded to win the Barge Prize in a freshman mathematics competition. He is reputed also to have achieved the distinction of having worked all the problems in the current redoubtable Yale calculus textbook. His subsequent mathematical development at Yale was powered by his own curiosity and drive, and was channeled by Ore, Wilson and Pierpont into areas represented in the strong but rather stiff department of that day. In another direction, his experience with E. J. Miles, whose love for teaching mathematics to undergraduates extended over a lifetime, must have begun to shape a feeling for teaching into what became later Saunders' own utterly distinctive and colorful style of forceful exposition.

In 1930 Saunders Mac Lane graduated from Yale with the degree of Ph.B. Under what seem now the rather quaint requirements of that day, and even a manifestation of "the two cultures," his was the degree reserved for those who arrived in New Haven with less Latin than required for candidates for the more usual A.B.

The year following graduation Saunders held a fellowship for $1000 at the University of Chicago. In 1929 he and Robert Maynard Hutchins, who had left Yale in that year as Law School dean to become President of the University of Chicago, each had received one of the Yale awards annually provided from Montclair, New Jersey. That departure from the regular custom of recognizing illustrious football performance with an award was a propitious one for the young mathematician in initiating an association with Hutchins and Chicago. When he arrived to take up his fellowship, however, even though Hutchins had personally invited him to Chicago, Saunders' first undertaking was to be admitted as a student, a formality he had completely overlooked.

His fellowship year at Chicago was a significant one. He came into contact with new mathematics and new ways of looking at mathematics. As was true for anyone at Chicago, he felt the influence of the powerful views of E. H. Moore. Then, in the spring of that fellowship year, he met a graduate student in economics who was to have a very important place in his future. Through the spring months Dorothy Jones, the daughter of a faculty family at the University of

Arkansas, and he found and shared together many common interests.

The following fall, with his degree of A. M. from Chicago, plans to continue his mathematics, and an admixture of wanderlust, Saunders Mac Lane was off to Göttingen. There was then at Göttingen an unparalleled group of productive mathematicians, leaders in creating exciting advances in new directions in mathematics. Saunders' studies took shape for him particularly in logic and he undertook his dissertation with Paul Bernays. Because of the ominous movement of events in Germany, however, he hastened to complete his dissertation in the early summer of 1933 and had his examination in mid-July with Hermann Weyl. That July was eventful for him in another way: only a few days after his successful examination he and Dorothy Jones were married in Göttingen. She had come on to Göttingen earlier and had typed his dissertation.

When Saunders and Dorothy returned to the United States it was to a Sterling Fellowship at Yale for 1933–34. His dissertation having been duly published in Germany, he received his D.Phil. from Göttingen in 1934. Then from Yale he moved to Harvard, where for the next two years he held a Benjamin Peirce instructorship.

By 1936 the economic depression which had gripped the United States since 1929 had eased, but jobs for beginning mathematicians—no matter how able—were still scarce. It was Saunders' good fortune that among these an instructorship for one year was open at Cornell for 1936–37. In the course of that year came an offer from the University of Chicago, to which he returned for 1937–38 as instructor. It was during that year at Chicago that the Mac Lanes' first daughter, Gretchen, was born and that Saunders was called back to Harvard as assistant professor.

I suppose it must have been when I arrived in Cambridge in the fall of 1938, fresh from Hamilton College and hopeful but untried, that I first met Saunders Mac Lane. During my graduate years I never had a formal course with him, but in courses of his which I attended I met a style of mathematics and mathematical thinking that captured my enthusiasm. I knew my bent was algebra, and when the time came Saunders Mac Lane accepted me as his student. Harvard was an exhilarating place to be for mathematics, and all of us there were fortunate to have had that experience before the anxieties preceding World War II could fully overtake us.

My meetings with Saunders to review my work were sometimes in his study in Eliot House, or on walks along the Charles River, or at the Mac Lanes' home at 7 Avon Street. These meetings were vigorous, stimulating, and productive. I had started out to look at some problems in algebraic geometry, following Saunders' suggestion, but had finally settled down with him to investigate the structure of certain domains complete in a valuation. At our meetings he invariably pressed me for genuine examples of the sometimes unrealistically abstract notions I proposed, and he saw to it that I worked these out as fully as I could. "Give me an example" was a challenge to discipline my speculations.

As Teaching Fellow in one of Harvard's freshman courses I experienced Saunders' style in another way. He visited my Math A class when we were doing some of the analytic geometry of lines. I had prepared diligently, but my exposition of the solution on one problem, though utterly transparently lucid to me, was obviously far less so to the students. At the end of the hour—which finally

arrived—Saunders asked, "Do you have some time?" I agreed I had. "Then come across the hall and I'll show you how you should have taught that class." That next hour was as uncomfortable and as valuable as any in my whole career, and I emerged from it imbued with the makings of a teacher. Even today I find myself returning to some of those lessons. I cannot be sure of everything I learned, but to exhibit natural liveliness was one and to seek to make clear the essential lines of the plot of an argument was another.

In the spring of 1941, while Saunders was on leave at Yale, the Mac Lanes' second daughter, Cynthia, was born. That year also saw the publication of *A Survey of Modern Algebra*, fruit of the collaboration of Garrett Birkhoff and Saunders Mac Lane on his return to Harvard. Birkhoff and Mac Lane, with *A Survey of Modern Algebra*, opened to American undergraduates what had until then been largely reserved for mathematicians in van der Waerden's *Moderne Algebra*, published a decade earlier. The impact of Birkhoff and Mac Lane on the content and teaching of algebra in colleges and universities was immediate and long sustained. What we recognize in undergraduate courses in algebra today took much of its start with the abstract algebra which they made both accessible and attractive.

Pearl Harbor abruptly changed all our lives. I hurried ahead to complete my thesis, then moved to Yale and an undergraduate Navy Program. Saunders early engaged himself in the war effort in the Applied Mathematics Group–Columbia.

Saunders Mac Lane and I became colleagues at the University of Chicago in the fall of 1947. I had preceded him by two years in response to an invitation to join the College Mathematics Staff. This was a separate faculty group, then being organized under the leadership of E. P. Northrop, which was entrusted with the mathematics program in the undergraduate plan of general education in the newly structured College of the University of Chicago, that had been engineered by Robert Maynard Hutchins. Saunders came as professor to the Department of Mathematics from Harvard, where he had become associate professor in 1941 and professor in 1946. The Department of Mathematics, upon the initiative of Hutchins himself, had become by 1947, under the chairmanship of Marshall Stone, a research center unexcelled in the United States. In algebra there was assembled a most remarkable group: Adrian Albert, Irving Kaplansky, Saunders Mac Lane, Otto Schilling, André Weil.

Saunders Mac Lane succeeded Marshall Stone at Chicago in the chairmanship of the Department of Mathematics in 1952. The two three-year terms of his chairmanship were typically vigorous and personal. His advocacy of the research goals of the department was ardent and tireless. It was characteristic of Saunders that he should have made special efforts to know each graduate student and to follow the research of each. For the young mathematicians who held appointments at Chicago he never failed to seek ways to urge them on in their work and to provide them with what he judged to be the most stimulating environment.

In the spring of 1952 I had agreed to head the College Mathematics Staff. The divergences between the research interests centered in the department and the perceived commitment of the College Mathematics Staff to mathematics in general education generated tensions which sometimes ruffled the relationship between Saunders and me. He pressed his points forcefully. His disbelief in the appropriateness of the exposition of any formal logic in the mathematics course in the College was consistently maintained. His active belief in the centrality of the concept of function was unwavering. The concomitant exchange of views, which engaged members of the department and staff, opened the way for future constructive changes in the administrative structure of undergraduate mathematics at Chicago, culminating in the consolidation of the College Mathematics Staff into the Department of Mathematics in 1959.

The wider public aspects of Saunders Mac Lane's devotion to the cause of mathematics, and to science generally, have been many. He was President of the Mathematical Association of America from 1950 to 1952, and in 1975 received its Distinguished Service Award in recognition of his sustained and active concern for the advancement of undergraduate mathematics. In 1963 the University of Chicago named him Max Mason Distinguished Service Professor. He served as President of the American Mathematical Society from 1973 to 1975. The following year he headed a delegation of mathematicians for a formal visit to the Peoples Republic of China. In 1973 he became a Vice President of the National Academy of Sciences, to which body he had first been elected in 1949. He was appointed to the National Science Board in 1974.

Beginning as a graduate student with a brief exposure to group extensions, I have watched the development of Saunders Mac Lane's mathematics through homological algebra to category theory. It is a pleasure now to give him a special salute: Saunders Mac Lane belongs in a category by himself.

Table of Contents

*Numbers in brackets refer to Bibliography on pp. 545–553.

ABGEKÜRZTE BEWEISE IM LOGIKKALKUL

Inaugural-Dissertation

zur Erlangung der Doktorwürde
der Mathematisch-Naturwissenschaftlichen Fakultät
der Georg August-Universität zu Göttingen

vorgelegt von

Saunders MacLane
aus Norwalk, Connecticut, U.S.A.

GÖTTINGEN
Gedruckt bei Hubert & Co. G.m.b.H.
1934

Referent : Professor Dr. Hermann Weyl
Tag der mündlichen Prüfung : 19. Juli 1933

Inhaltsverzeichnis.

§ 1. **Der Begriff eines Prozesses:** Symbolik ist vielleicht das Hauptthema der modernen mathematischen Logik. Die logische Symbolik ist jetzt im wesentlichen vollständig, denn mit den Symbolen der modernen Logik kann man jeden mathematischen Satz knapp und bequem ausdrücken. Das Grundproblem der logischen Deduktion bleibt aber durch dieses Studium der Symbolik im wesentlichen ungeändert, denn in jeder Symbolik muß man nichtsymbolische Regeln haben, die verschiedene Gruppen von Zeichen miteinander verknüpfen. Die Aufgabe der Logik ist letzten Endes die Ableitung von Schlüssen aus gegebenen Prämissen, und dazu muß man Methoden haben, die den Aufbau von Schlüssen aus Prämissen erlauben. Eine solche Konstruktionsmethode nennen wir einen Prozeß. Ein *Prozeß* hat also zwei Merkmale: erstens ist er eine Vorschrift, die eine neue Gruppe von Zeichen durch Transformation oder Kombination aus einer oder mehreren gegebenen Gruppen von Zeichen erzeugt; zweitens muß ein Prozeß als Deduktionsmethode anwendbar sein.

Schon die Logik von Aristoteles kann man als eine Logik der Prozesse auffassen: der Syllogismus ist ja ein Prozeß. Man kann z. B. mit der Prämisse (Untersatz) „Sokrates ist ein Mensch" anfangen, und dann, mittels des Bezugstheorems (Obersatz) „Alle Menschen sind sterblich", schließen: „Sokrates ist sterblich". In der Mathematik kommen andere Prozesse vor. Wenn wir z. B. mit der Gleichung

$$(1) \qquad (a + b)\, c = ac + bc$$

als Prämisse anfangen, dann können wir durch Bezugnahme auf das kommutative Gesetz den Schlußsatz (linkes distributives Gesetz)

$$(2) \qquad c(a + b) = ca + cb$$

ableiten. In beiden Fällen haben wir einen Prozeß, der mit einer Prämisse anfängt, und dann durch ein Bezugtheorem einen gewissen Schluß oder ein gewisses Ergebnis erreicht.