

The Large-Scale Structure of the Universe

by

P.J.E. Peebles

*Princeton Series
in Physics*

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Peebles, Phillip James Edwin.

The large-scale structure of the universe.

(Princeton series in physics)

Bibliography: p.

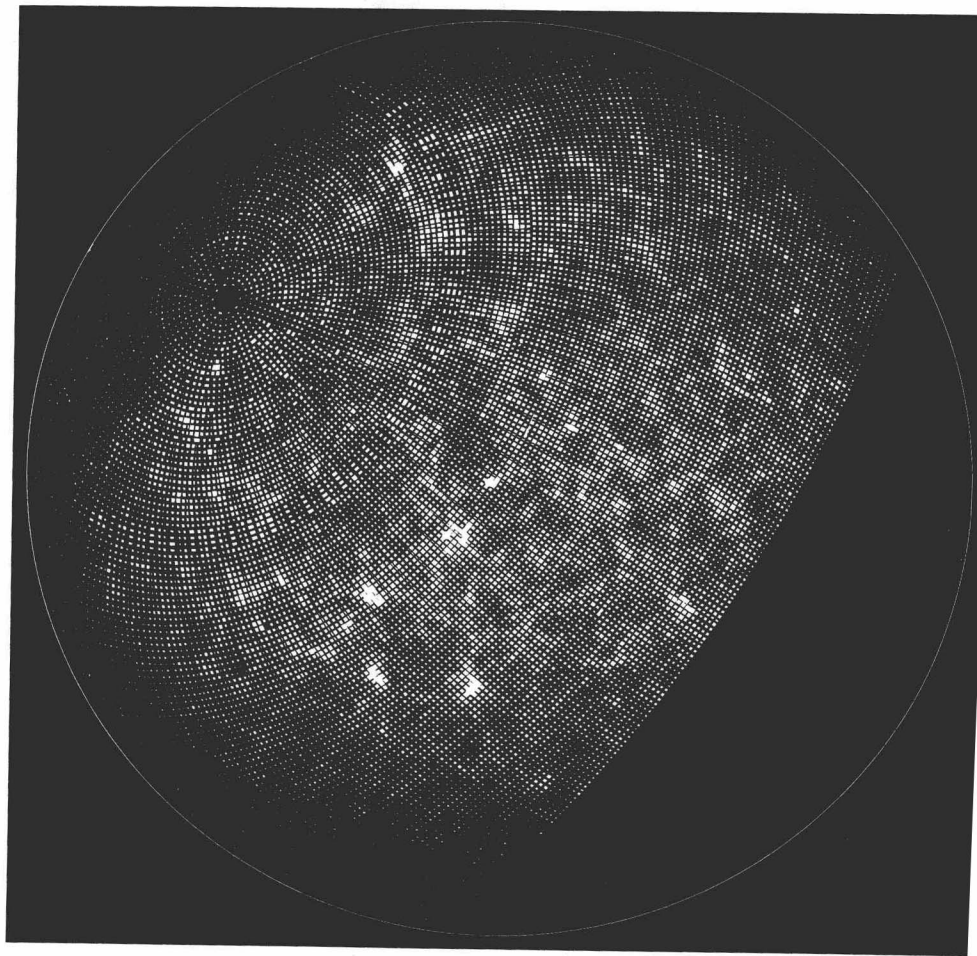
Includes index.

1. Galaxies. 2. Cosmology. I. Title.

QB857.P43 523.1'12 79-84008

ISBN 0-691-08239-1

ISBN 0-691-08240-5 (pbk.)



FRONTISPIECE: The large-scale pattern of the galaxy distribution. Each white square represents a sky cell about one degree by one degree in the Lick sample. The size of the white square is proportional to the number of galaxies brighter than 19th magnitude in the cell. The cells are arranged along lines of fixed right ascension and declination. The north pole of the galaxy is at the center of the map and the equator along the edge. (Map by J. A. Peebles and P.J.E. Peebles.)

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PREFACE

From the first developments of modern cosmology people have recognized that an important part of cosmology is the large-scale clustering of matter in galaxies and clusters of galaxies. The point was largely eclipsed by the debate over homogeneous world models, but in recent years there has been a considerable revival of interest in the large-scale mass distribution and what it might tell us about the nature and evolution of the universe. The purpose of this book is to review our present understanding of these subjects.

Chapter I is a history of the development of ideas on the large-scale structure of the universe. As is usual in science the story is a mixture of inspired guesses and rational progress with excursions down paths that now seem uninteresting. What makes it somewhat unusual is the slow rate of development that has left ample time for the growth of traditions that are more than commonly misleading, and so it seems worthwhile to examine the evolution of the ideas in some detail. This is a history in the rather loose convention of scientists, that is, it is based on what I could glean from published books and journals. The few conversations I have had with participants have left me only too aware of how limited that is and how much more could be done. On the other hand, the published record is what was readily available to most people who might want to work on the subject and might want to learn what has already been done, though the actual use of the record was just as erratic in the 1930s as it is these days. I have tried to give a complete account of the important developments since about 1927 and have added enough more recent references to serve as a guide to the literature.

Chapter II deals with the behavior of a given mass distribution in the Newtonian approximation. This is only a limiting case of the full relativistic theory, but it is discussed first and in detail because it is a good approximation for most practical applications and is much simpler than the full relativistic theory. There is a considerable variety of methods and results in the analysis of the Newtonian limit. I have collected all those that seem to be useful and interesting.

The statistical pattern of the galaxy distribution is discussed in Chapter III. The descriptive statistics that have proved useful and are analyzed in this chapter are n -point correlation functions (analogs of the autocorre-

lation function and higher moments for a continuous function). The general approach has a long history but it is only in the last several years with the application of fast computers to the large amount of available data that the technique has been extensively developed and applied. This chapter surveys the main theoretical results and observational methods.

The n -point correlation functions have proved useful not only as descriptive statistics but also as dynamic variables in the Newtonian theory of the evolution of clustering. This is discussed in Chapter IV. The functions are generalized to mass correlation functions in position and momentum, and the BBGKY hierarchy of equations for their evolution is derived. This yields a new way to analyze the evolution of mass clustering in an expanding universe. Of course, the main interest in the approach comes from the thought that the observed galaxy correlation functions may yield useful approximations to the mass correlation functions, so the observations may provide boundary values for the dynamical theory of evolution of the mass correlation functions. The test will be whether we can find a consistent theory for the joint distributions in galaxy positions and velocities. The subject still is in a crude state because adequate redshift data do not yet exist. I present some preliminary considerations on how the analysis of the data might proceed.

The full relativistic analysis of the evolution of mass clustering is presented in Chapter V. The important application is to the behavior of the early stages of expansion of the universe when the high mean density would have made even modest density fluctuations strongly relativistic.

The last chapter describes some of the attempts to trace the links between theory and observation showing how the character of the matter distribution we observe developed out of reasonable conditions in the early universe. This is the main point of the subject, but it is not treated at length because I think there are too many options, all apparently viable but none particularly compelling. It seems likely that the game of inventing scenarios will go through several more generations before a secure picture emerges. Perhaps the best we can hope is that the final answer will draw on significant elements of the theory and observations as we now think we understand them.

I have limited the range of the discussion to length scales no smaller than the nominal size of a galaxy or else redshifts no smaller than the epoch at which mass concentrations comparable to present day galaxies appeared, thus excluding the structure and evolution of galaxies. I have excluded a few topics relevant to other areas of cosmology, such as the effect of mass clustering on the standard cosmological tests, and some obviously important subjects where I could find nothing very useful to report, such as the question of intergalactic gas clouds. I have omitted all

discussion of the possibilities offered by nonstandard cosmologies not so much because I am sure the big bang picture is the most likely candidate as that I expect it is neither reasonable nor likely to expect that people will pay much attention to these alternatives until we have a much clearer picture of what the standard model has to offer and what it must deal with.

The choice of emphasis on topics within the boundary conditions, of course, reflects a personal judgment of what is promising. Perhaps the largest omission is the primeval turbulence picture. I have described its origins and some general and well-established results but have not discussed any specific scenarios. That seems reasonable because I doubt the merits of this picture, and there are others who can serve as better and more enthusiastic advocates.

I have provided a short guide to symbols and conventions in the appendix. It probably will prove best to look this over before reading much of the main text. I have given short summaries of concepts of cosmology as they appear in the text, but have left out details available in the standard books. References to my book, *Physical Cosmology*, are indicated by the letters *PC*.

ACKNOWLEDGMENTS

I list with special thanks the people who played the most direct roles in shaping this book: Charles Alcock, Marc Davis, Bob Dicke, Jim Fry, Margaret Geller, Ed Groth, Mike Hauser, Bernard Jones, Jerry Ostriker, Bill Press, Mike Seldner, Joe Silk, Ray Soneira, Juan Uson, Simon White, Dave Wilkinson, and Jer Yu. The process would have been considerably slower and the results less satisfactory without the skill and energy of Marion Fugill.

The first concrete steps toward this book were taken while I enjoyed the hospitality of the Physics Department at the University of California at Berkeley during the 1973–74 academic year. The first draft developed as course notes at Princeton University. I am grateful to John Bahcall for providing hospitality at the Institute for Advanced Study where the final draft was written. The work was supported in part by the National Science Foundation.

CONTENTS

PREFACE	xi
ACKNOWLEDGMENTS	xv
I. HOMOGENEITY AND CLUSTERING	3
1. Homogeneity and clustering	3
2. Is the universe homogeneous?	3
3. Physical principles	11
4. How did galaxies and clusters of galaxies form?	18
5. Summary	35
II. BEHAVIOR OF IRREGULARITIES IN THE DISTRIBUTION OF MATTER: NEWTONIAN APPROXIMATION	37
6. Newtonian approximation	37
7. Particle dynamics in expanding coordinates	41
8. The peculiar acceleration	43
9. Two models: the Vlasov equation and the ideal fluid	45
10. Linear perturbation approximation for δ	49
11. Solutions for $\delta(t)$: $p = \Lambda = 0$	51
12. Solutions for $\delta(t)$: effect of a uniform radiation background	56
13. Solutions for $\delta(t)$: models with $\Lambda \neq 0$	59
14. The peculiar velocity field	63
15. Joining conditions for δ and v	66
16. Critical Jeans length	68
17. Primeval magnetic field as a source for $\delta\rho/\rho$	71
18. Second order perturbation theory for $\delta\rho/\rho$	74
19. Spherical model	77
20. Homogeneous ellipsoid model	86
21. Caustics and pancakes	95
22. Expansion, vorticity, and shear	103
23. Origin of the rotation of galaxies	107
24. Cosmic energy equation	110
25. Spherical accretion model	115
26. Hierarchical clustering model	120

27.	Fourier transform of the equations of motion	124
28.	Coupling of density fluctuations	128
III. n-POINT CORRELATION FUNCTIONS: DESCRIPTIVE STATISTICS		
29.	Statistical measures of the galaxy distribution	138
30.	Fair sample hypothesis	142
31.	Two-point spatial correlation function $\xi(r)$	143
32.	Two-point correlation function: another definition	145
33.	Two-point correlation function: Poisson model	147
34.	Three-point correlation function	148
35.	Four-point correlation function	150
36.	Moments of counts of objects	152
37.	Constraints on ξ and ζ	156
38.	Probability generating function	158
39.	Estimates of P_N	160
40.	Cluster model	163
41.	Power spectrum	166
42.	Power law model for the spectrum	169
43.	Bispectrum	171
44.	Cross correlation function	172
45.	Angular two-point correlation function	174
46.	Angular power spectrum	175
47.	Estimating $w(\theta)$	183
48.	Statistical uncertainty in the estimate of $w(\theta)$	187
49.	Relation between angular and spatial two-point correlation functions	189
50.	Small separation approximation and the scaling relation	191
51.	Decoupling of magnitude and position	194
52.	Relation between ξ and w : some examples	195
53.	Inversion of the equation	200
54.	Angular three-point correlation function	203
55.	Angular four-point correlation function	209
56.	Correction for curvature and expansion	213
57.	Summary of numerical results	221
58.	Power spectrum of the extragalactic light	225
59.	Moments of the number of neighbors	230
60.	Model for P_N	233
61.	Clustering models	236
62.	Continuous clustering hierarchy: Mandelbrot's prescription	243
63.	The mass correlation functions	249

64. Clustering hierarchy: continuity speculation	253
65. Remarks on the observations	255
IV. DYNAMICS AND STATISTICS	257
66. Goals	257
67. Definitions of variables and distribution functions	258
68. BBGKY hierarchy equations	259
69. Fluid limit	262
70. Evolution of the integral of ξ	264
71. Particle conservation equations	266
72. Relative peculiar velocity dispersion	272
73. Similarity solution	275
74. Cosmic energy equation	278
75. Cosmic virial theorem	280
76. Joint distribution in position and velocity	284
77. Behavior of the halo around a cluster of galaxies	291
78. Superclusters	299
79. Problems and prospects	301
V. RELATIVISTIC THEORY OF THE BEHAVIOR OF IRREGULARITIES IN AN EXPANDING WORLD MODEL	304
80. Role of the relativistic theory	304
81. Time-orthogonal coordinates	306
82. The field equations for $h_{\alpha\beta}$	310
83. Gravitational waves	312
84. Newtonian approximation	313
85. Linear perturbation equations for the matter	317
86. Behavior of density perturbations at wavelength $\gg ct$	319
87. Spherical model	324
88. Evolution of acoustic waves	330
89. Nonlinear acoustic waves	333
90. Incompressible flow	341
91. Behavior of collisionless particles	345
92. Linear dissipation of adiabatic perturbations	352
93. Residual fluctuations in the microwave background	363
94. Isothermal perturbations	373
VI. SCENARIOS	379
95. Nature of the universe at high redshift	379
96. Nature of protogalaxies and protoclusters	384

APPENDIX	395
97. Models and notation	395
LIST OF ABBREVIATIONS	401
REFERENCES	402
INDEX	417

**The Large-Scale
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Universe**

I. HOMOGENEITY AND CLUSTERING

1. HOMOGENEITY AND CLUSTERING

Modern discussions of the nature of the large-scale matter distribution can be traced back to three central ideas. In 1917 Einstein argued that a closed homogeneous world model fits very well into general relativity theory and the requirements of Mach's principle. In 1926 Hubble showed that the large-scale distribution of galaxies is close to uniform with no indication of an edge or boundary. In 1927 Lemaître showed that the uniform distribution of galaxies fits very well with the pattern of galaxy redshifts. The homogeneous model, when generalized to allow for evolution, yields a linear redshift-distance relation consistent with what Hubble was finding from his estimates of galaxy distances (as summarized by Hubble in 1929).

The evolving dynamic world model quickly won attention and in the following decades, before the idea became commonplace, it generated some lively discussions. The following sections trace the development of several questions. The first question is whether the universe really is homogeneous (after averaging over a suitable clustering length). Assuming it is, must we be content to say only that this happens to be a reasonable approximation to our neighborhood at the present epoch? Could the homogeneity of the universe have been deduced ahead of time from general principles? Or might it be a useful guide to new principles? The matter distribution in any case is strongly clumped on scales of stars, galaxies, and clusters of galaxies. This clustering is a fossil of some sort, a remnant of processes in the distant past as well as an on-going phenomenon. How does the clustering evolve in an expanding universe? What is its origin? What does it tell us about the nature of the universe?

2. IS THE UNIVERSE HOMOGENEOUS?

In 1917 the phrase "the large-scale distribution of matter" was generally taken to mean the distribution of stars in the Milky Way galaxy. For example, the title of Eddington's (1914) book on the latter subject is *Stellar Movements and the Structure of the Universe*. It was considered well-established from star counts that the stars are concentrated in a flattened roughly spheroidal distribution, the Kapteyn system (after the