CHASSIS DESIGN

Principles and Analysis

(Part Ⅲ)

Chassis Design
Principles and Analysis

Contents

CHAPTER **8**

Roll, Roll Moments and Skew Rates

"Surely there is nothing very marvelous in a machine weighing a couple of tons, which, even when built so low—as the CG within two feet of the ground—still loses 50% of its potential cornering ability by lateral weight transfer?"

Maurice Olley

8.1 Introduction

In this chapter Olley fills in some details on roll rates that were assumed in earlier chapters. There is an interesting discussion of skew rates, now called warp rates. He also discusses compensated (longitudinally interconnected) suspensions.

The study of static roll rates and roll moments can become fairly complicated. And there is considerable difference between roll caused by an applied moment and roll caused by lateral force applied at the CG which is the usual condition encountered on the road.

8.2 The Roll Axis

The roll axis is discussed in Section 2.5 but we still lack a definition of it. The best definition seems to be that the roll axis defines, in the median plane of the car, a transverse plane in which horizontal lateral forces applied to the rolling mass of the car will move the car sideways **without** causing it to roll; see Figure 8.1 for the basic geometry.

It follows that forces applied above the roll axis (as at the CG) will cause roll. Also that forces, if any, applied below the roll axis will cause "banking" (like a bicycle).

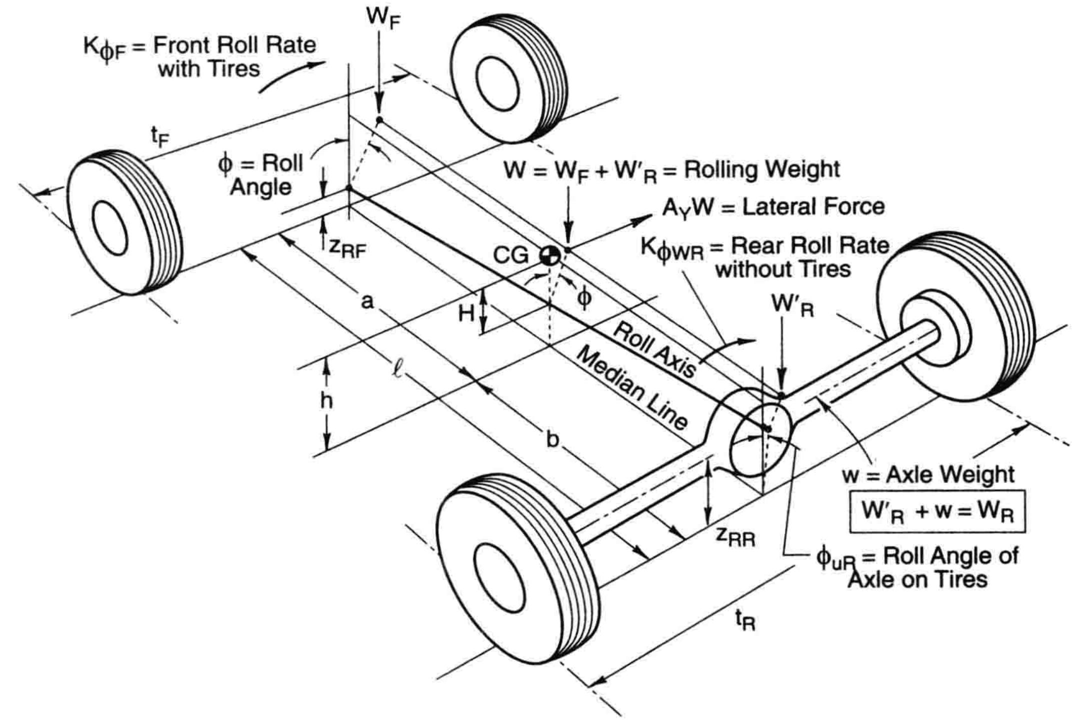

***Figure 8.1** Basic geometry of the roll axis model, including roll of the rear axle on the tires.*

In considering suspension at either end of the car we shall also be discussing "roll centers." These are obviously the points where the roll axis intersects the transverse planes of the front or rear wheel pairs.

There are three main types of suspension to consider:

1. The axle.
2. Independent suspension without tread [*track*] change.
3. Independent suspension with tread change.

Axle

The axle holds the wheels upright because it is a rigid member from wheel to wheel and this forms a traveling "trestle" on which the sprung mass is supported. When the sprung mass is subjected to a roll moment it rotates through an angle about a center in the median plane, the height of which is determined by the suspension mechanism.

In a Hotchkiss rear axle the roll center is normally close to the plane containing the front and rear spring eyes. On other suspensions the roll center height is determined by some form of centering mechanism such as a Panhard rod, or other linkage. (See Section 3.3.)

The conditions governing the distribution of the roll moments to the front and rear tires are rather fully discussed in the following pages. No simple statement confined to one end of the car can fully cover the subject. The point to note here is that in general on an axle with leaf springs, the height of the roll center z_{RR} is controlled by the height of the spring eyes, and that $K_{\phi WR}$ (the rear roll rate without tires) is $\lambda(1/2)K_R t_S^2$ lb.-in./rad. (see Figure 8.2). The constant, λ, which may be 1.30 or so, is due to the fact that the leaf spring rate is raised by the twisting and side bending of the springs in roll.

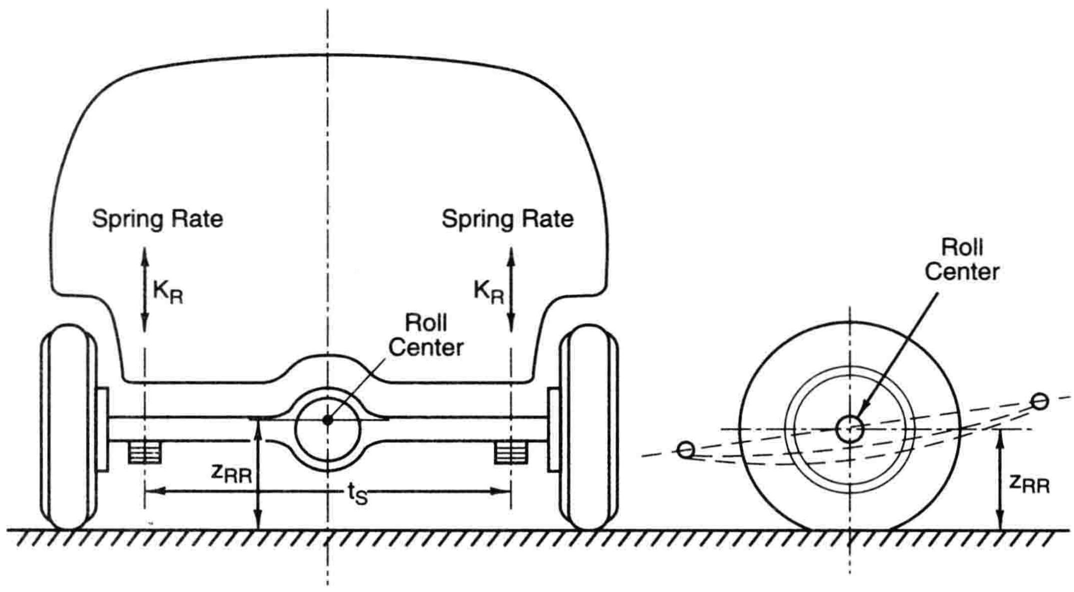

Figure 8.2 *Roll center location, axle with leaf springs.*

The stability in roll of such a suspension is thus dependent on t_S^2, and on the height of the roll center, which in turn depends on the placing of the rear springs.

In a truck rear axle the springs are frequently deep in section and placed **above** the axle, thus raising the roll center and increasing stability. But dual rear wheels may make the spring separation only **half** of the mean wheel track, in which case the rear roll rate is relatively very low.

In other types of suspensions the **height** of the rear springs does not affect the roll axis. In independents this height depends only on the rate of tread-change. And on coil spring rear axles the roll center height is controlled by some form of linkage.

The full treatment for distribution of roll moments, etc., is given below, referring to Figure 8.1.

A_Y = lateral acceleration (in g-units)
$K_{\phi F}$ = front roll rate **with** tires (lb.-ft./rad.)
$K_{\phi WR}$ = rear roll rate **without** tires (lb.-ft./rad.)
$K_{\phi TR}$ = axle roll rate on its tires (lb.-ft./rad.)
ϕ = roll angle of vehicle relative to ground (rad.)
ϕ_{uR} = roll angle of axle relative to ground (rad.)
h = total height of CG of rolling mass above ground (ft.)
H = height of the CG above the roll axis (ft.)
z_{RF} = height of front roll axis (ft.)
z_{RR} = height of CG of axle (assumed same as height of rear roll center) (ft.)
t_F = front wheel track (ft.)
t_R = rear wheel track (ft.)
W_F = front rolling weight (lb.)
W_R' = rear rolling weight (lb.)
w = rear unsprung weight (lb.)
W_R = total rear weight: $W_R = W_R' + w$ (lb.)
ΔW_F = front weight transfer (lb.)
ΔW_R = rear weight transfer (lb.)

Equilibrium of rolling mass:

$$K_{\phi F}\phi + K_{\phi WR}(\phi - \phi_{uR}) = (A_Y + \phi)\left[W_F(h - z_{RF}) + W_R'(h - z_{RR})\right] \tag{8.1}$$

The term $W_F(h - z_{RF}) + W_R'(h - z_{RR})$ is a constant. Call it M_c, the rolling moment constant.

Then
$$K_{\phi F}\phi + K_{\phi WR}(\phi - \phi_{uR}) = M_c(A_Y + \phi) \tag{8.2}$$

From which
$$\phi = \frac{A_Y M_c + K_{\phi WR}\phi_{uR}}{K_{\phi F} + K_{\phi WR} - M_c} \tag{8.3}$$

Equilibrium of axle:

$$K_{\phi TR}\phi_{uR} = K_{\phi WR}(\phi - \phi_{uR}) + W_R z_{RR}(A_Y + \phi_{uR}) \tag{8.4}$$

From which
$$\phi_{uR} = \frac{M_c(A_Y + \phi) + W_R z_{RR} A_Y - K_{\phi F}\phi}{K_{\phi TR} - W_R z_{RR}} \tag{8.5}$$

Then,

Front weight transfer
$$\Delta W_F = \frac{K_{\phi F}\phi + A_Y W_F z_{RF}}{t_F} \tag{8.6}$$

Rear weight transfer $$\Delta W_R = \frac{K_{\phi TR}\phi_{uR}}{t_R} \tag{8.7}$$

The roll moment distribution and its effect upon steering are fully discussed in Chapter 2, Equations 2.40 to 2.49.

The above treatment and Figure 8.1 applies particularly to IFS with a rear axle. It is shown in Chapter 2 that it can also be applied to IFS with a swing axle. The modifications necessary for a vehicle with front and rear axles will be obvious. The following are some descriptions of [*other*] suspension types.

Independent without Tread [Track] Change

This applies not only to true parallel action independents, Figure 8.3a, but to any independent, such as Figure 8.3b, with its roll center on the ground. K is now the vertical rate of spring plus tire [*overall ride rate per wheel*], and the roll rate $K_{\phi F}$ will include the tire.

In the absence of a roll stabilizer [*anti-roll bar*]:

$$K_{\phi F} = (1/2)\,Kt_F^2 \text{ lb.-in./radian (if lengths are in inches).} \tag{8.8}$$

The rolling mass in case (a) is the sprung plus unsprung. In case (b) the two wheels usually do not roll quite as far as the sprung mass. But the difference is generally small, and the unsprung is usually added as part of the rolling mass. [*Terry Satchell points out that this is worth accounting for as a function of camber gain, when the unsprung is heavy.*]

The rolling moment at the front end is then $W_F h(A_Y + \phi)$.

It is a mistake to say that this is necessarily resisted by the roll moment $K_{\phi F}\phi$, since the total roll moment is resisted front and rear in proportion to the front and rear roll rates. **What can be said is that in any form of independent, with or without tread change, unless it is fitted with a "stabilizer" or a "de-stabilizer," the roll rate is always $K_{\phi F} = (1/2)\,Kt_F^2$ (lb.-in./rad.).**

or $$K_{\phi F}\,(\text{lb.-ft./deg.}) = \frac{(1/2)\,Kt_F^2}{12 \times 57.3} = \frac{Kt_F^2}{1375} \tag{8.9}$$

$$\text{Roll rate} = \frac{(\text{ride rate, one wheel}) \times (\text{wheel track})^2}{1375} \tag{8.10}$$

The importance of this is that ride rate and roll rate figures are frequently quoted from tests, which do not satisfy these conditions. In such cases it can be confidently stated that errors have been made in one or both of the readings.

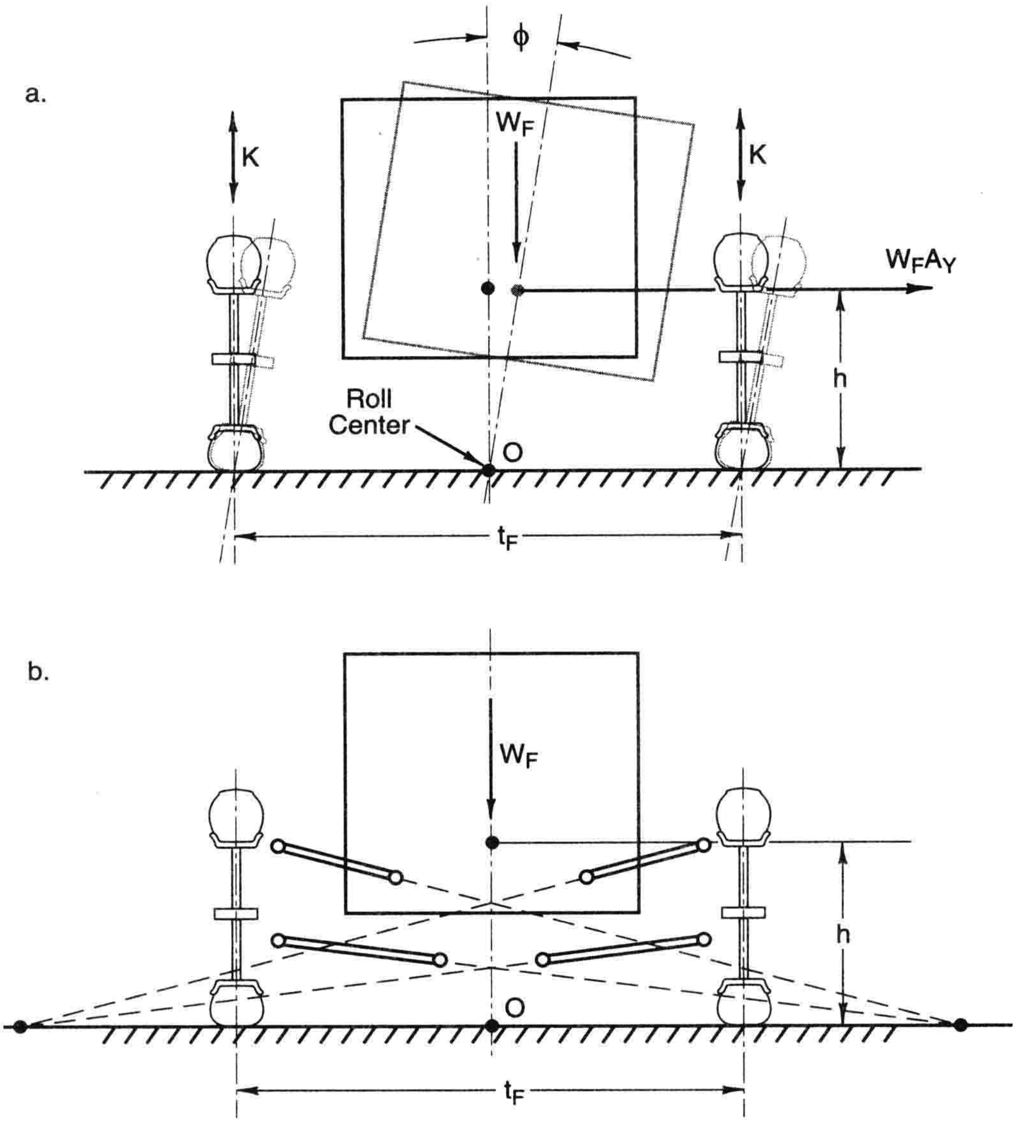

Figure 8.3 *Independent suspensions without tread (track) change.*

The stabilizer [*also called "anti-roll bar"*], principally used in front, adds to the roll rate without increasing the ride rate, Figure 8.4.

The de-stabilizer [*also called "camber compensator"*], principally used on rear swing axles, adds to the ride rate without increasing the roll rate, see Figure 8.5. [*Another common mechanical arrangement is the "Z-bar," which looks like an anti-roll bar but with the lever arms pointing in opposite directions. It also carries load and increases the ride rate, but does not increase the roll rate.*]

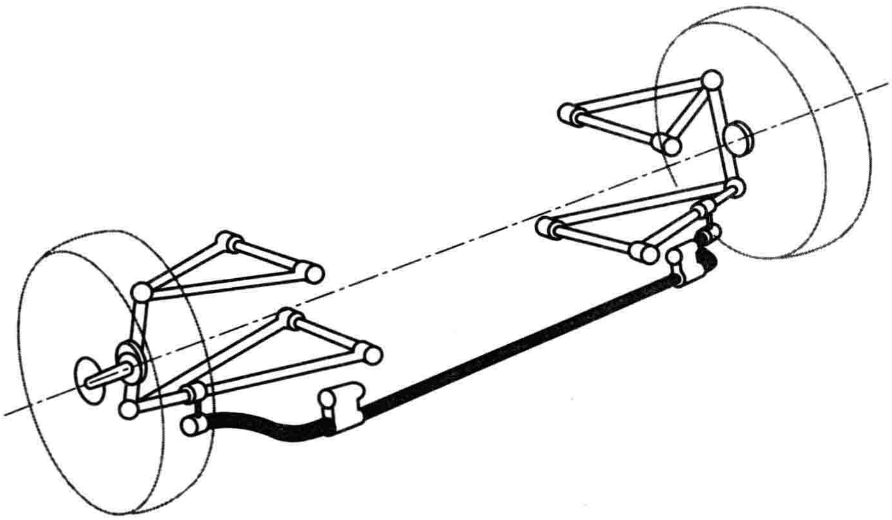

Figure 8.4 *Stabilizer or anti-roll bar.*

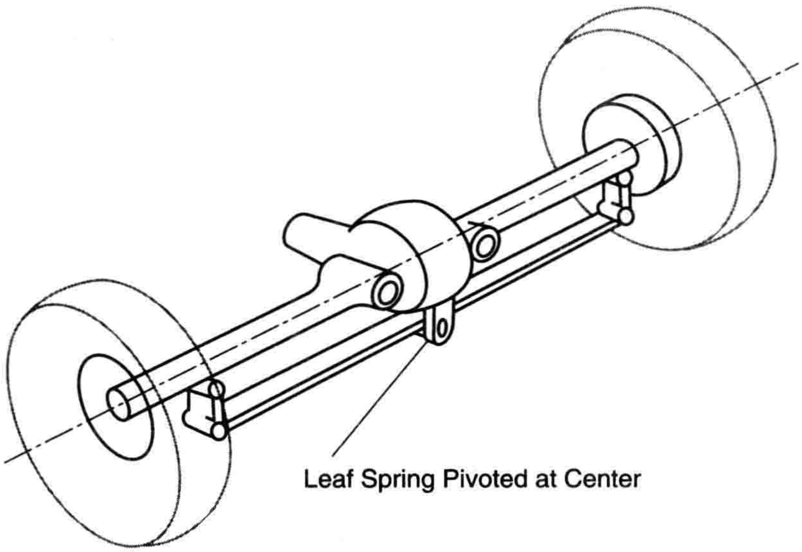

Figure 8.5 *De-stabilizer—pivoted transverse leaf spring (or camber compensator).*

Independent with Tread Change

A variation in the wishbone suspension which is becoming popular [*c. 1960*] is shown in Figure 8.6. In this case, there is tread change as shown by the raised roll center z_{RF}. If $R_1H = R_2(H - h)$, the rate of tread change is constant, i.e., the "scrub line" AB is a straight line, and z_{RF} is constant.

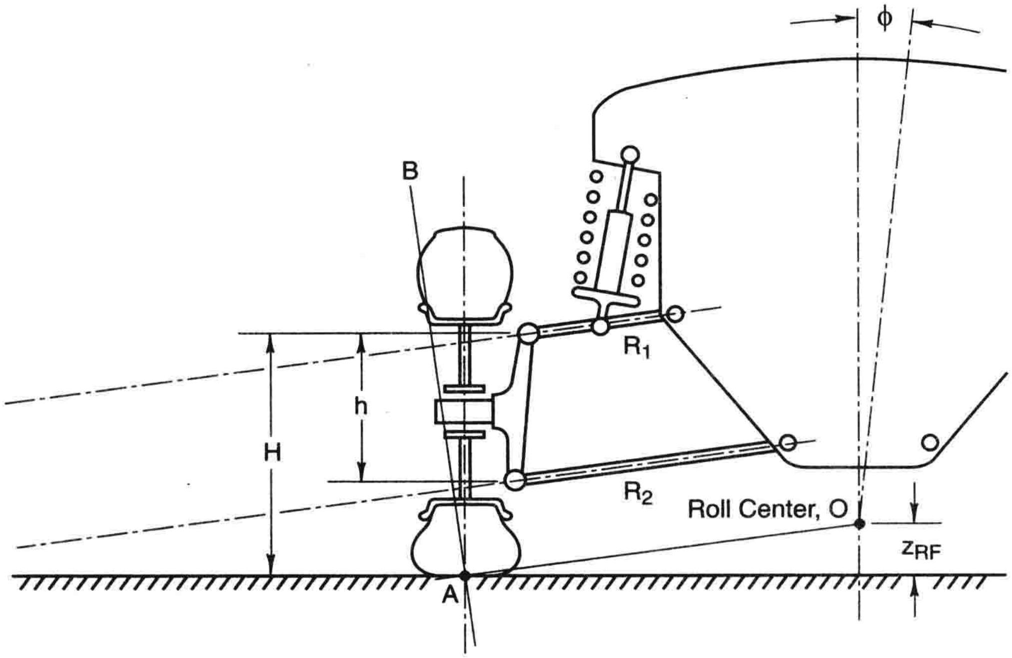

Figure 8.6 *Independent suspension with tread change.*

On a turn, the outer wheel rolls through about the same angle as the car, but the inner wheel rolls **more** (which is undesirable for tire wear and squeal). The difference between the inner and outer wheel is not significant as regards roll moment distribution, and it is usual simply to add the unsprung to the spring for the total rolling mass. When figuring the distribution of roll moments it is [*also*] usual to ignore the slight side shift of the roll center O which occurs when the car rolls. In other words the treatment is like that based on Figure 8.1. [*Depending on the choice of suspension type and actual geometry, the side shift in the roll center with roll may be large and it may make sense to use a more sophisticated type of analysis that includes the full kinematics.*]

Swing Axle

In Chapter 2, the swing axle was dealt with as a simple independent with a raised roll center. That is, the unsprung weight was ignored and the tire deflection was simply added to the spring deflection. This was a reasonable approximation for its purpose. But for a more accurate study it would be better to consider the swing axle as shown in Figure 8.7, in which case it closely resembles a conventional axle, except for the high roll rate due to the wide spring base and the high roll center.

Roll rate **without** tires: $K_{\phi WR} = (1/2)\, K t^2$ (8.11)

Roll rate of unsprung on tires: $K_{\phi TR} = (1/2)\, K_T t^2$ (8.12)

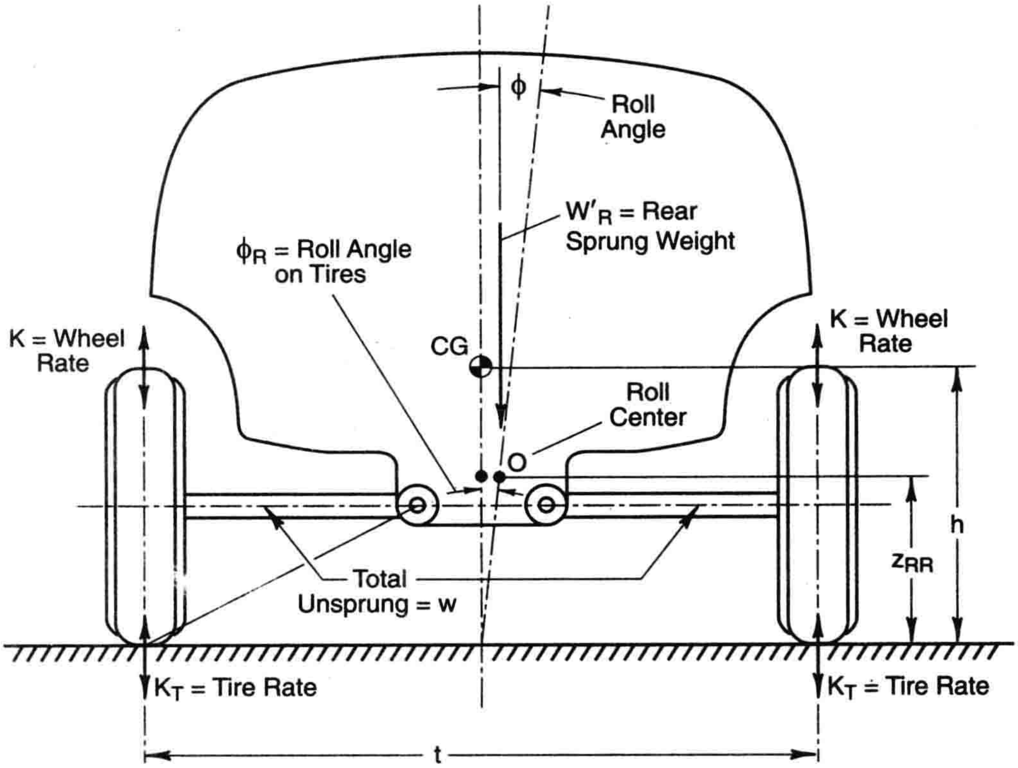

Figure 8.7 *Swing axle (rear view).*

The solution for roll moment distribution is then exactly the same as for the rear axle (earlier in this section). However, this does not intend to say that the swing axle handles or rides exactly like a rigid axle with high roll rate and high roll center.

Attention is directed to Section 7.11 in which it is pointed out that we have no means of testing the actual ride or roll rates on a swing axle, including the effect of camber thrust. Also, the notes on swing axle steering (Section 2.9) are not complete, since they do not cover the important condition of a swing axle running in a lightly loaded position with positive camber on the wheels. This positive camber position of the swing axle, and the resulting oversteer, is probably the greatest fault of the swing axle. However, a compensating advantage is the damping effect of the changing wheel track. (See Section 6.12.)

8.3 Intermediate Designs of Independent Suspension

Between the parallel independent with its roll center on the ground and the swing axle, with its high roll center, there are any number of intermediate designs, of "semi-swing" independents in which the center of swing of each wheel may be at or beyond the center of the opposite wheel, giving a roll center such as O (Figure 8.8) which may be only 6 inches or so above the ground. In calculating roll moment distribution on this type of

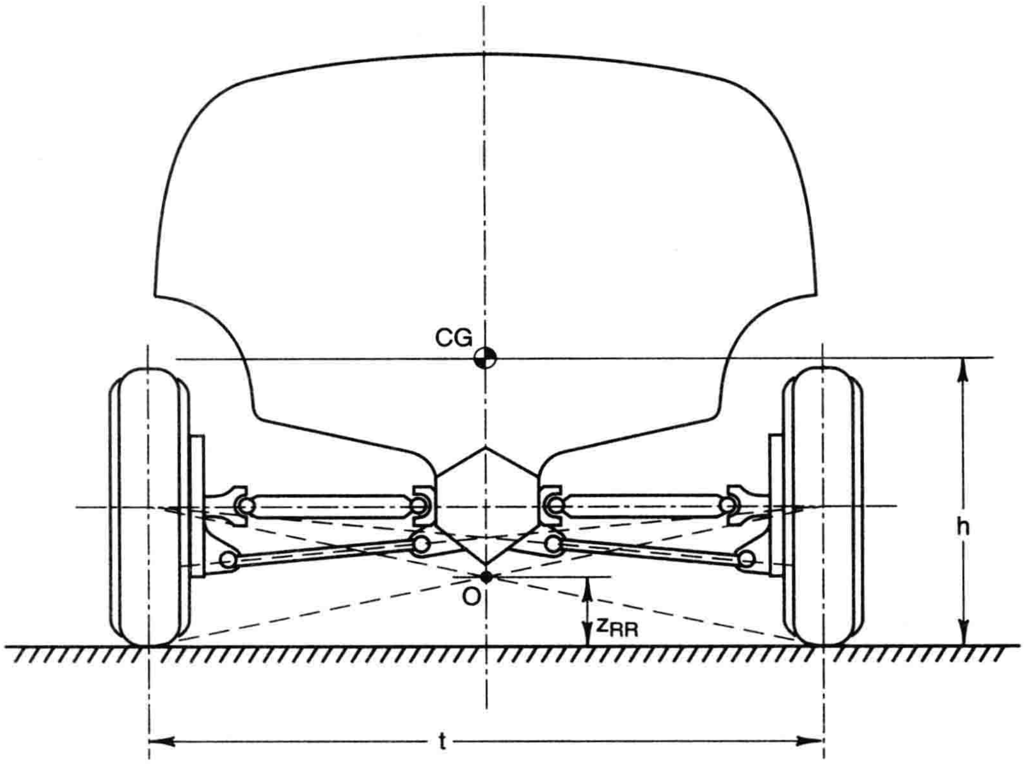

Figure 8.8 *Intermediate design of independent suspension (late Corvair type shown).*

suspension it is probably satisfactory to include 50 percent of the unsprung weight as rolling weight [*sprung weight*], and the other 50 percent as rolling only on the tires. Since these suspensions are independents, it follows that the spring base in roll is equal to the wheel track.

Generally speaking such compromise designs as these have proven extremely successful in rear suspensions with respect to ride, handling, tire life, lack of wheel hop, etc.

8.4 De Dion Axles

In most cases (Figure 8.9) De Dion axles are mounted similarly to conventional Hotchkiss axles and require no special consideration. Advantages over conventional axles are:

- Elimination of the mass of the drive gears and casing at the axle center. This greatly reduces the tendency of the axle to tramp, and the consequent side shake of the car.
- Rear wheels can be toed-in or given negative camber for improved cornering.

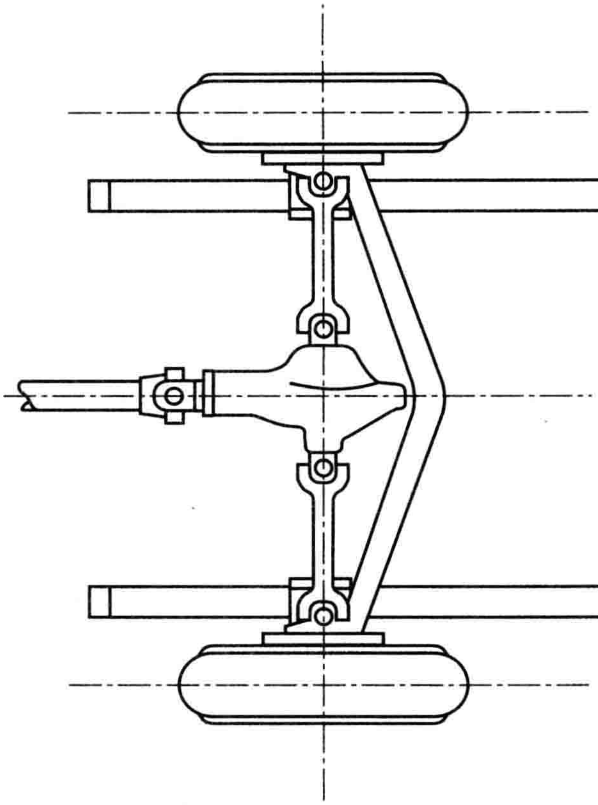

Figure 8.9 *Typical De Dion axle on leaf springs.*

- Unsprung weight is reduced.
- Interference with the rear floor can be reduced.

A particular variation of the De Dion axle used on Mercedes race cars prior to World War II is worthy of note; see Figure 8.10.

The axle is stiff in bending, but made in two halves connected by a rotary sleeve joint. If the car rolls through an angle ϕ, the wheels tilt in the opposite direction through an angle γ, such that $\gamma = -\frac{r}{R}\phi$. [*If the vertical distance moved by the torque arm at the wheel end is x, then* $\phi = \frac{x}{t/2}$, *and the wheel camber change is* $-\frac{r}{R}\frac{x}{t/2} = -\frac{r}{R}\phi$.]

This undoubtedly gives an appreciable advantage in rear roll-steer. [*Olley may have meant "an appreciable advantage in rear roll* ***camber,*** *" which is in the understeer direction with this design.*] (However, Mercedes race cars now [*1950s*] employ a form of low-center swing axle.)

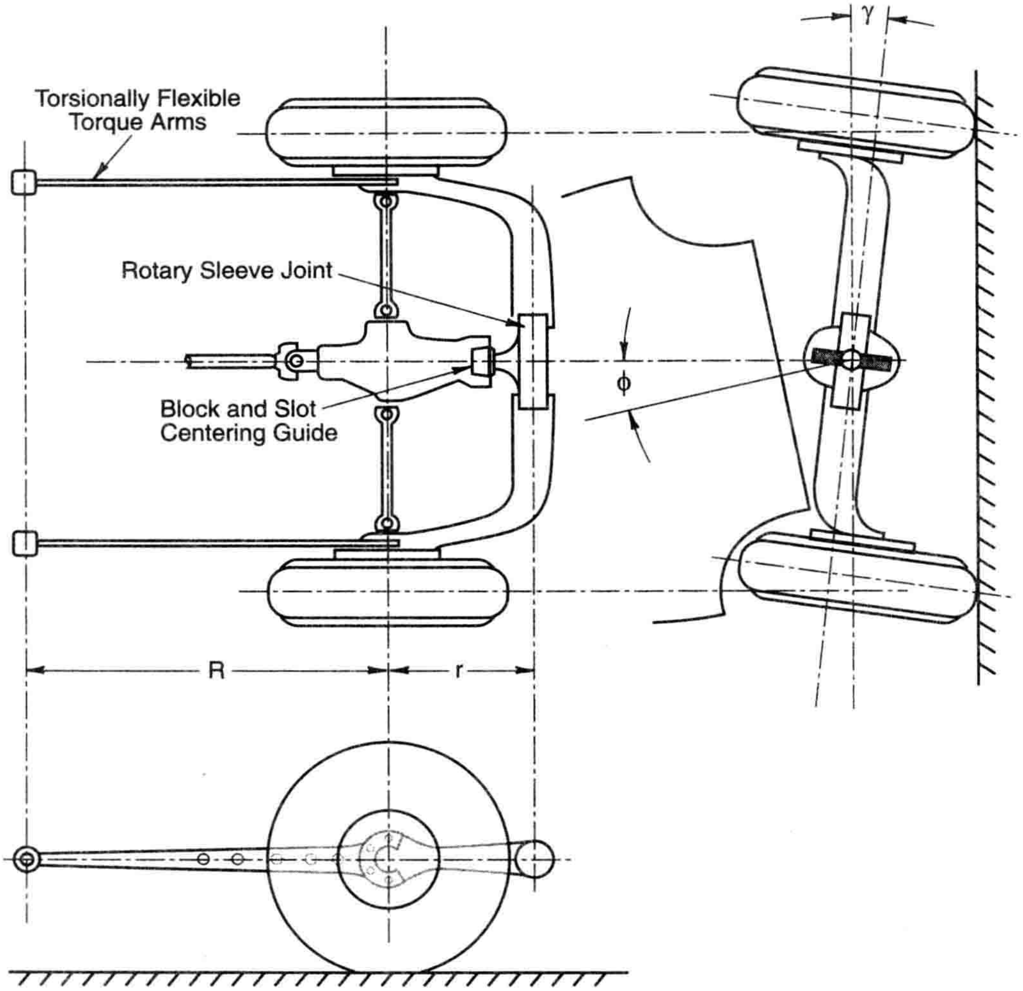

Figure 8.10 Mercedes De Dion axle.

8.5 Skew Rates [*Warp*]

Perhaps skew rates have not previously received enough attention. Under static conditions, or when moving at moderate speed over a warped road surface, the **skew stiffness of a car's suspension controls the torsional distortion forces which fall on the frame and body**.

Imagine the car with its front and rear wheel pairs standing on two seesaws as shown in Figure 8.11, the front plank being pivoted near ground level, while the rear is pivoted somewhere near the rear axle roll center. Then if the two planks are tilted in opposite directions in such a way that the tilting moments on the two planks are equal and opposite, the car will remain upright, and the tilting moment at **either** end divided by the **sum** of the tilt angles of the two planks is the measure of the skew rate of the suspension.

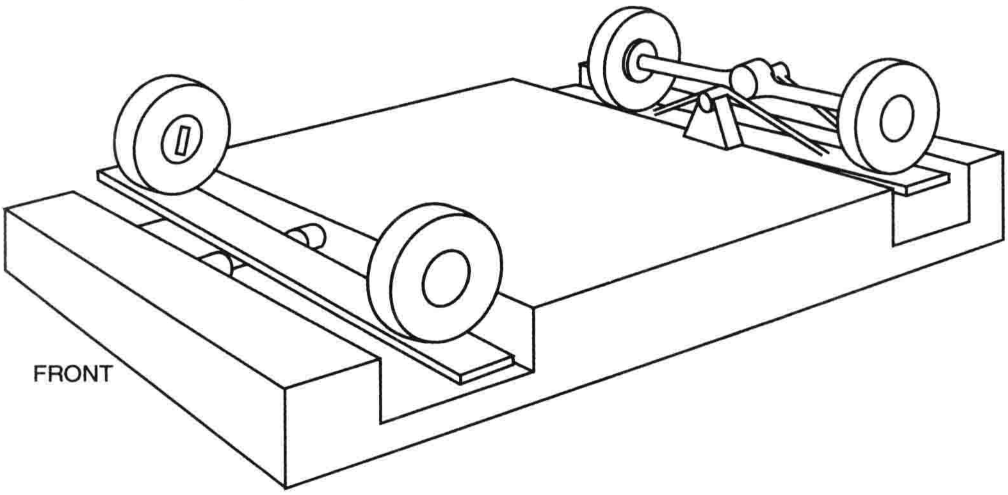

Figure 8.11 *Test rig for determining skew rates.*

For example the roll rate at the front of a conventional passenger car may be 350 lb.-ft./degree, and at the rear 240 lb.-ft./deg. Then the skew rate will be the combination of the two or

$$\frac{350 \times 240}{350 + 240} = 142 \text{ lb.-ft./deg.} \tag{8.13}$$

8.6 Longitudinal Interconnection–Compensated Suspension

A completely different condition exists in a compensated suspension such as the Packard torsion-rod suspension (Figure 8.12) in which, while the total roll rate of the vehicle remains perfectly normal, the skew rate may be exceedingly small. [*Other examples of this are the longitudinal coil springs in the French Citroën 2CV and the Moulton Hydrolastic and Hydragas fluid interconnected suspensions used on English cars from the BMC1100/MG Sedan to the MGF.*[53] *The Lotus Active Suspension*[54] *was programmed "modally" and the skew or warp rate could be set to any desired value over a wide range.*]

The principal feature of this type of suspension is a great reduction in pitch **stability**, and hence a reduction in pitch **frequency**. But the low skew rate is an important incidental advantage.

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[53] Moulton, Dr. A. E. "Moulton Suspension, Past and Future," Sir Henry Royce Memorial Foundation Lecture, IMechE, London, June 2000.

[54] Williams, D. A. and P. G. Wright, US Patent 4,625,993, 1986.