# Numerical analysis in geomorphology

## *an introduction*

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EDWARD ARNOLD

First published 1971 by
Edward Arnold (Publishers) Ltd
41 Maddox Street
London WIR OAN

ISBN: 0 7131 5589 2



Printed in Great Britain by
Butler & Tanner Ltd, Frome and London

# Acknowledgements

The authors and publisher gratefully acknowledge permission given by the following to reproduce or modify material from copyright works:

The editor and the Institute of British Geographers for eight figures (2.1, 4.1, 5.1, 5.2, 5.3, 5.4, 5.5, 5.6)[1] from 'Multivariate analysis in geography, with particular reference to drainage basin morphometry' by P. M. Mather and J. C. Doornkamp, *Transactions* 51; the editor, the author and the University of Liège for a figure (6.9) from 'The analysis and classification of slope profile forms' by R. A. G. Savigear in *L'Evolution des Versants, Les Congrès et Colloques de l'Université de Liège* 40, 173–93; the editor and the Geologists' Association for a figure (9.2) from 'The development of hillside slopes' by A. Wood, *Proceedings* 53, 128–40; Gebrüder Borntraeger and the editor for two figures (9.4, 9.5) from 'A hypothetical nine unit land surface model' by J. B. Dalrymple, R. J. Blong and A. Conacher, *Zeitschrift für Geomorphologie* 12, 60–76; the editor, the author and the Göttingen Academy of Sciences for a figure (9.12) from 'Descriptive models of slope evolution' by Anthony Young in *Nachrichten der Akademie der Wissenschaften in Göttingen, II: Mathematisch-Physikalische Klasse* 5, 45–66; the McGraw-Hill International Book Company for three tables (appendix tables A, B, C) from *Nonparametric statistics for the behavioral sciences* by S. Siegel and also Oliver and Boyd Ltd, Dr. F. Yates and the Literary Executor of the late Sir Ronald Fisher for two of the three tables above (A and B) which were adapted by Siegel from *Statistical tables for biological, agricultural and medical research* by R. A. Fisher and F. Yates; the editor for two tables (appendix tables D, E) from 'Tables of percentage points of the inverted beta distribution' by M. Merrington and C. M. Thompson, *Biometrika* 33, 73 ff.

[1] Numbers in brackets refer to figures or tables in this book.

# Preface

Land forms and drainage patterns are being studied by more and more elaborate methods all the time. The increasingly accurate methods of measuring land form and geomorphological processes are providing a vast amount of quantitative data. This has to be analysed by numerical methods so that an orderly behaviour may be discerned from amongst the mass of accumulated data. This book sets out to provide an introduction to those numerical methods which we have found to be most useful in the analysis of geomorphological data. In it we have tried to show how even the simplest statistical tests can bring out significant points concerning the landscape. But land forms are sometimes complex, having many measurable properties, and this demands of the geomorphologists an appreciation of the value of multivariate statistical analysis. At its most complex such methods demand the use of a computer. Nevertheless, many of the methods in this book can be applied with no more than a set of mathematical tables or a slide-rule. These are well within the grasp of students in the upper forms of schools, at colleges of higher education and polytechnics as well as those at a university.

Where a computer is required we have tried to bring out the potential of these methods by concentrating on the results that can be obtained. In this way we hope that some may be stimulated to pursue their familiarity with the landscape up to and beyond this level, while those who are already well past the position reached by this book may still find in it some stimulus and guidance.

The book concentrates on four major fields of geomorphological study, namely, drainage basins, slopes, and coastal and glacial land forms; its theme is that of the analysis of form. Some differences in approach have been adopted under each of the main headings. This is partly because the subject matter dictates which techniques and methods are most appropriate, but these differences of approach are also designed to illustrate some of the different ways in which numerical analysis can be used.

We are indebted to many of our colleagues at the University of Nottingham, but in particular we owe our thanks to P. M. Mather and M. J. McCullagh who kindly assisted us by making available computer programs which have been used in some of the analyses presented. We are also grateful to J. A. Dawson and A. Hughes for discussions over earlier drafts of the manuscript. For cartographic assistance we are indebted to M. Cutler who has drawn the maps and diagrams.

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*July* 1970

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# Introduction

## Scales of measurement

The application of statistical methods to geomorphological data requires an understanding both of the methods of analysis and the nature of geomorphological data. Both are dependent upon the *scale of measurement* used when the data are collected. Measurement can be made on one of four scales which are equivalent to four levels of generalization. These are the nominal, ordinal, interval and ratio scales. Of these the *nominal scale* is the most generalized in that measurement values are only placed in classes. For example, in one area a hill could be placed in one of the three classes of low, intermediate and high. Each hill belongs to one class only, and every hill must belong to a class. On the *ordinal scale* the measurements can be ranked. For example, pebbles can be arranged in order according to their roundness, and an ordinal scale ranging from an index of 1 up to 10 can be used for this purpose. The *interval scale* is one where it is possible to state the actual difference between two measurements, but an arbitrary zero point has to be adopted for the scale. For example, sea-level is an arbitrary zero point from which to measure altitude. Some other starting point might have been taken instead. The most refined scale of measurement is known as the *ratio scale*, and it applies when a true zero exists. For example, the length of a river or the weight of a pebble can be measured on the ratio scale. In each case the scales used start from an actual (not an arbitrary) zero value.

## Descriptive and inferential statistics

Statistical theory provides many of the techniques useful in the numerical analysis of geomorphological data. *Descriptive statistics* allow the range of values in the data to be summarized by means of simple diagrams or a few calculated values. In geomorphology it is seldom possible to study all examples of a particular feature or process. Instead of studying the total *population*, therefore, it is necessary to study a *sample*. Through descriptive statistics it is possible to estimate the size of an element in the total population from the sample.

During data collection hypotheses often arise in the mind of the investigator. These hypotheses may be tested through *inferential statistics*, by means of *non-parametric* and *parametric* tests. Of these two the non-parametric tests can be used with data measured on the nominal and ordinal scales. Parametric tests are more rigorous in their requirements and data have to be measured on the interval or ratio scales. The conditions which need to be fulfilled before a parametric test can be used are:

.1 observations must be independent and random

2 the data must be drawn from a population for which the descriptive statistics are known.

If the conditions are not met then the data can be reduced to the ordinal or nominal scale so that non-parametric tests can be applied. This means, however, that less rigid tests are being applied to more generalized data, with a corresponding decrease in the precision of the inferences made.

## Geomorphological data

There are varying degrees of complexity associated with geomorphological data. In its simplest form interest may be centred upon a single characteristic of certain land forms. This *variable* can be measured and its proportion analysed through *univariate statistics*. A special kind of univariate statistics involves the analysis of changes which take place in one variable over time. In many instances, however, the geomorphologist needs to know the relation between two, or more, variables. This requires the use of *bivariate* or *multivariate statistics* respectively. For example, bivariate statistics may be used to discover the numerical relationship between the size of a drainage basin and the length of the river which it contains. The analysis can be made more complex, when it will require the use of multivariate statistics, by adding other variables, such as the width or height characteristics of the basin. Skill, however, lies not so much in applying the multivariate technique as in the geomorphological interpretation of the results.

Many of the techniques used in this book are described in textbooks on statistics. Some of these are listed below. In the following chapters statistical techniques are applied, and described, as required by the problem or area under investigation. The use of a particular technique in the text can be found by reference to the index. However, not all of the numerical methods applicable to geomorphology involve statistical tests. Slope form analysis, for example, includes methods of numerical classification. *Model building* allows standards of reference to be set up with which future case studies can be compared, or through which a problem may be simplified for further analysis. Some models are statistical, but others such as simulation models are not. For example, geomorphological events can be simulated by a model through which processes may be isolated and their respective influences on land-form development studied.

## Useful statistical texts

ALDER, H. L. and ROESSLER, E. B. 1962: *Introduction to probability and statistics.* San Francisco and London: Freeman.

BAGGALEY, A. R. 1964: *Intermediate correlation methods.* New York: Wiley.

BLALOCK, H. M. 1960: *Social statistics.* New York: McGraw-Hill.

1961: *Causal inferences in nonexperimental research.* University of North Carolina Press.

BROOKES, B. C. and DICK, W. F. L. 1969: *Introduction to statistical method.* London: Heinemann.

EZEKIEL, M. and FOX, K. A. 1959: *Methods of correlation and regression analysis.* New York: Wiley. (3rd edn.)

KRUMBEIN, W. C. and GRAYBILL, F. A. 1965: *An introduction to statistical models in geology.* New York: McGraw-Hill.

MORONEY, M. J. 1956: *Facts from figures.* London: Penguin. (3rd edn.)

SIEGEL, S. 1956: *Nonparametric statistics for the behavioral sciences.* New York: McGraw-Hill.

# Contents

(Detailed lists of contents precede each chapter.)

# Part 1 Drainage basins

# 1 Drainage basins and stream networks



The analysis of drainage basins, either as single units or as a group of basins which, taken together, comprise a distinct morphological region, has particular relevance to geomorphology. Fluvially eroded landscapes are composed of drainage basins, and these provide convenient units into which an area can be subdivided. The development of a landscape is equal to the sum total of the development of each individual drainage basin of which it is composed. The fact that morphological regions can be recognized suggests not only that within each region the drainage basins have forms similar to each other but also that these basins are evolving in a similar way to each other. Thus, by analysing the development of each drainage basin, greater understanding of the landscape as a whole may be achieved. This is possible if there are definable relationships between the form of a drainage basin and the processes at work within it.

The drainage basin may be thought of as an open system in near steady state (Chorley, 1962; Strahler, 1964; Schumm and Lichty, 1965; Slaymaker, 1966; Morisawa, 1968). In an open system there is, ultimately, a balance between the rates of import and export of material and energy. This is a form of equilibrium. Although the basin and its streams are in this steady state it does not mean that there is no development taking place. On the contrary because sediment is leaving the basin, changes must be taking place within it. The condition of steady state is time-independent for it requires only that the rate of sediment supply from within the basin should be the same as the rate of sediment movement out of the basin. These conditions must be considered in the context of the processes and time periods involved (Schumm and Lichty, 1965). Once steady state has been reached the system becomes self-regulating, for any major changes in the environment (e.g. deforestation) will result in

[1] A summary of the section headings appears at the beginning of each chapter. Section titles are printed bold and sub-titles roman. In addition specific techniques referred to within each section are listed (printed italic and enclosed in square brackets).

compensating changes in the system. Under the condition of steady state it is likely that the morphometry of a drainage basin will display some recognizable regularity from one neighbouring basin to the next. The purpose of part I of this book is to illustrate how this type of orderly arrangement may be discerned through the application of numerical techniques. These techniques are here to be used in order:

1. to study the relationships between the morphometric properties of drainage basins
2. to examine the relationships between basin morphometry and other characteristics of the environment (e.g. rock type, stream discharge)
3. to find an objective grouping of drainage basins having similar morphometric properties, and to see if this grouping coincides with morphological regions originally defined by ground inspection
4. to consider drainage basins as convenient units within which to study slope form (part II).

## 1.1 Defining a drainage basin

*Ordering systems*

The problem of defining a drainage basin does not lie in a definition of the term itself but in locating the boundary of the basin on the ground, or on a map. Fundamental to any numerical analysis of drainage basin characteristics is the concept of stream (or valley) ordering (Fig. 1.1). A system of channel ordering was suggested by Gravelius (1914), but the work of Horton (1932, 1945) marked the beginning of the widespread use of channel ordering systems in geomorphology. Since that time numerical analysis of drainage basins has relied on an ordering system. The usual method of ordering used today is that suggested by Strahler in 1952 (Fig. 1.1B). In this system all streams which have no tributaries are known as first-order streams. When two first-order streams join together they form a second-order stream; when two second-order streams join they form a third-order stream, and so on. If a first-order stream enters a second-order stream then there is no change in the order of the second-order stream. An increase in stream order only occurs when two streams of like order join each other. In this system the head of each second-order stream occurs at the junction of two first-order streams. In the system used prior to 1952 (Fig. 1.1A) it was usual to extend the second-order designation back to the head of the longest of its first-order tributaries, and so on for higher orders. In this study the Strahler system of ordering is employed (Fig. 1.1B). This can be used equally well for the ordering of valley networks as it can for streams.

Alternative systems of ordering streams have been suggested by Scheidegger (1965), Woldenberg (1966) and Shreve (1967). These alternatives were suggested because of weaknesses apparent in the systems of Horton and Strahler. Scheidegger (1965) points out that in the Strahler system where two streams of order $u$ join they form a stream of order $u + 1$. No change in order takes place, however, when a stream of order $u$ is met by one of a lower order. Scheidegger suggests that account should be taken of all tributaries (Fig. 1.1C). This can be achieved by defining

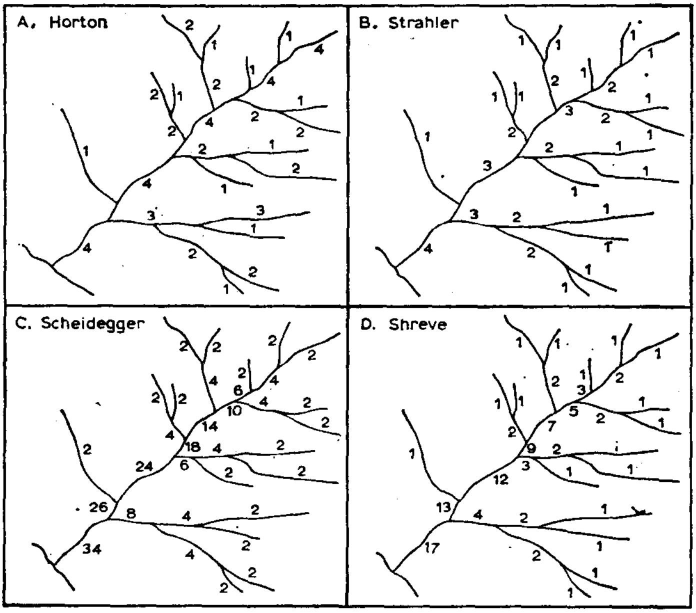


**Fig. 1.1** Channel ordering systems as proposed by **A** Horton; **B** Strahler; **C** Scheidegger; **D** Shreve.

the order ($x$) after the junction of two streams (of order $v$ and $u$ respectively) by:

$$x = \log_2 (2^u + 2^v)$$

This means that in the Scheidegger system all of the extreme outward branches must be of order 2 and not 1 as in the Strahler system.

Shreve (1967) refers not to stream order but to the 'magnitude of a link'. This is defined (Fig. 1.1D):

1 each exterior link has magnitude 1
2 if links of magnitude $\mu_1$ and $\mu_2$ join, then the resultant downstream link has magnitude $\mu_1 + \mu_2$.

By this means account is still taken of all tributary junctions (as demanded by Scheidegger), and the magnitude of a link is also a direct statement of the number of sources ultimately tributary to it. As Shreve points out, magnitude is a purely topological concept. Networks with equal magnitudes have an equal number of links, forks and sources.

The mathematical relationship between stream order ($x$), as defined by Scheidegger, and its magnitude, as defined by Shreve, is:

$$x = \log_2 2\mu$$

Woldenberg (1966) finds that successive orders in the Strahler system are new logarithmic cycles to the base of the bifurcation ratio (this is a measure of the amount of branching in the network, see Table 1.1). He suggests, therefore, that a new absolute order should be derived by raising the bifurcation ratio to successive integer powers equivalent to stream order minus one.

Neither the Scheidegger, Woldenberg, or Shreve systems have been widely used, and they are not used in this book for, by so doing, little or no direct comparison could have been made with results previously obtained in studies using the Strahler system. Nevertheless, there seem to be many advantages in the Shreve system and further research should be directed towards examining its application more fully.

Given that an ordering system, such as the Strahler system, is to form the starting point for the collection of numerical information concerning a drainage basin, there are certain obvious characteristics which can be defined and measured. For example, as a development of the ordering system it is possible to speak not only of first-order streams but also of first-order drainage basins. These are the areas drained by first-order streams. Likewise it is possible to have a second-order drainage basin, but in this case the parallel with the stream ordering system no longer holds good for a second-order basin also includes the area occupied by all of the first-order basins which drain into it, and not just the area around the stream channel which has been designated as a second-order stream. A third-order drainage basin includes not only the slopes which supply water directly to the third-order stream but also the area occupied by the first- and second-order streams which drain into it.

*Morphometric properties*

Once a drainage basin watershed and its stream (or valley) pattern has been defined, measurements can be made of some of its morphometric properties. These variables are defined in Table 1.1, and the symbols used in this account are also listed. The variables fall into several distinct groups. Some of them are measured directly, usually from a map; these include basin area (A), length of first-order streams ($L_1$), total length of all streams ($\Sigma$ L), number of first-order streams ($N_1$), and so on. Other variables are derived from such direct measurements and include measures of basin shape, such as basin circularity ($R_c$), as well as drainage density (D), bifurcation ratio ($R_b$), and basin relief ratio ($R_h$). The variables also differ according to their dimensional characteristics. Thus measures of stream length are one-dimensional (**L**, not to be confused with stream length), while basin area is a two-dimensional measure ($\mathbf{L}^2$). On the other hand drainage density is the reciprocal of a one-dimensional

**Table 1.1 Morphometric and related variables, their dimensions and symbols used**

| Variable | Symbol | Units | Dimensions |
|---|---|---|---|
| *Drainage network* | | | |
| Stream order (used as subscript) | $u$ | enumerative | 0 |
| Number of streams of order $u$ | $N_u$ | ,, | 0 |
| Total number of streams within basin order $u$ | $(\Sigma N)_u$ | ,, | 0 |
| Bifurcation ratio | $R_b = N_u/N_{u+1}$ | ,, | 0 |
| Total length of streams of order $u$ | $L_u$ | miles | L |
| Mean length[1] of streams of order $u$ | $\bar{L}_u = L_u/N_u$ | ,, | L |
| Total stream length within basin of order $u$ | $(\Sigma L)_u = L_1 + L_2 \ldots + L_u$ | ,, | L |
| Stream length ratio | $R_l = \bar{L}_u/\bar{L}_{u-1}$ | | 0 |
| *Basin geometry* | | | |
| Area of basin | $A_u$ | square miles | $L^2$ |
| Length of basin | Lb | miles | L |
| Width of basin | Br | ,, | L |
| Basin perimeter | P | ,, | L |
| Basin circularity | $R_c = A_u$/area of circle having same P | | 0 |
| Basin elongation | $R_e$ = diameter of circle having same $P/L_b$ | | 0 |
| *Measures of intensity of dissection* | | | |
| Drainage density | $D_u = (\Sigma L)_u/A_u$ | miles per miles² | $L^{-1}$ |
| Constant of channel maintenance | $C = 1/D_u$ | miles² per mile | L |
| Stream frequency | $F_u = N_u/A_u$ | number/mile² | $L^{-2}$ |
| Texture ratio | $T_u = N_u/P_u$ | number/mile | $L^{-1}$ |
| *Measures involving heights* | | | |
| Stream channel slope | $\theta_c$ | feet/mile or degrees | 0 |
| Valley-side slope | $\theta_g$ | degrees | 0 |
| Maximum valley-side slope | $\theta_{max}$ | ,, | 0 |
| Height of basin mouth | z | feet | L |
| Height of highest point on watershed | Z | ,, | L |
| Total basin relief | $H = Z - z$ | ,, | L |
| Local relative relief of valley-side | h | ,, | L |
| Relief ratio | $R_h = H/Lb$ | | 0 |
| Ruggedness number | $R_n = D \times H/5280$ | | 0 |
| *Stream flow* | | | |
| Discharge | Q | cubic feet/ second | $L^3$ |

[1] The superscript bar indicates (here and throughout) a mean value.

measure, being equal to total stream length divided by basin area ($L \div L^2 = L^{-1}$). Stream numbers and the ratios such as bifurcation ratio are dimensionless. In the following chapters these variables will, very largely, be referred to by their symbols; it may be necessary, therefore, constantly to return to Table 1.1 for reference.

*Problems in stream ordering*

Problems which are a result of local peculiarities become apparent when stream ordering is undertaken according to the Strahler system. These should be noted, for they can indicate a great deal about the particular character of the geomorphology of the area under examination. For example, it is frequently the case that stream networks are denser in areas of exposed well-jointed bedrock on steep slopes than on neighbouring vegetated slopes. These bedrock slopes therefore introduce many additional first- and even second-order streams into the network. This has a direct consequence on the ordering of the rest of the system.

Figure 1.2A shows another example. Here the ordering of a system is seen to change entirely by the development of one small first-order tributary, and this can lead to an increase of one in each of the higher stream orders. This will only occur, however, in those cases where the first-order stream develops as a tributary to another first-order stream, no change in stream ordering takes place if it develops as a tributary to a stream of higher order. In similar manner, the convergence of two third-order streams just before their junction with a fourth-order stream turns the latter into a fifth-order stream. If, on the other hand, the two third-order streams had managed to take independent paths to the fourth-order stream then no fifth-order would have been involved. This is particularly critical when the two third-order streams meet the main stream (fourth-order in Fig. 1.2D) on a wide alluvial plain. Under such circumstances they might as readily have joined just before reaching the main stream, as shown by the broken line in Figure 1.2D.

Discontinuous drainage lines introduce problems of another kind. Within an area of karst landscape they present special circumstances which require careful assessment (Williams, 1966). Discontinuous flow can also occur on non-limestone areas. Such may be the case where hillside drainage passes below the surface on meeting a scree or footslope predominantly composed of waste material, and the measurement of stream length has to be approximated to the straight-line distance down the footslope or scree. In areas where river capture has recently taken place there may be difficulties over stream ordering. Taken to its extreme ordering becomes impossible where capture is actually taking place today. This is the case in the Kandekye–Koga drainage basins of the Buhweju Mountains in southern Uganda (Fig. 1.3). The Kayania Swamp drains to the south, along the early course of the Kandekye, but it has also been tapped by a tributary of the Koga so that this swamp also drains away