Princeton Problems in Physics

with Solutions

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Preface

No one expects a guitarist to learn to play by going to concerts in Central Park or by spending hours reading transcriptions of Jimi Hendrix solos. Guitarists practice. Guitarists play the guitar until their fingertips are calloused. Similarly, physicists solve problems. And hopefully, physicists practice solving problems until doing so seems easy. (Then they find harder problems.)

This book provides a collection of challenging problems for physics students at a range of levels. Some problems, particularly those in the first four chapters, require only an undergraduate physics background. The later chapters cover material that is frequently not encountered until graduate school. Don't be discouraged if some (or most!) of the problems are, in fact, challenging. That's the idea.

However, this is not only a problem book. We also provide complete solutions for each problem. These solutions assume a certain amount of familiarity with the topics, but are not written for experts. For this book to be of maximum benefit, of course, the solutions should be considered a last resort. *Try to solve the problems before looking at the solutions!*

The problems presented here were culled from general examinations written at Princeton University in the last ten years. All physics graduate students at Princeton must pass the generals examination before beginning their theses. The examination is split into two parts: "Prelims," usually taken in the first year of study, and "Generals," typically taken in the second year.

The preliminary examination (Prelims) covers the subjects usually studied at the undergraduate level. It is a six hour exam, taken in two days. Prelims has four sections: mechanics, electricity and magnetism, nonrelativistic quantum mechanics, and thermodynamics and statistical mechanics. The second examination (Generals) covers more advanced topics. The written part of this exam takes place in three three-hour sessions and comprises five sections: condensed matter physics, general relativity and astrophysics, nuclear physics, elementary particle physics, and atomic and "general" physics. Many of the solutions given here are *much* more detailed and complete than would be expected during the general examination. (The passing mark at Princeton is 50%, and students are allowed to choose from several questions in each field.)

We have divided this book into the nine obvious sections suggested by the format of the exams. There are ten problems in each of the sections. We do not make any guarantees that we have provided a random sampling of problems. The problems chosen are those that we found interesting, informative, and well-posed. We have also tried to avoid archetypal problems whose answers have been printed in numerous other books. In several places we have noted useful references. Although each solution has been checked and rechecked, occasional errors may have slipped through. We welcome comments, criticism, and corrections.

A group of five authors can run up an amazing list of debts in the writing of a single book. Our first, and most obvious, is to the many members of the Princeton University Physics Department who have written original, interesting, and instructive problems for the preliminary and general examinations. Their work forms the foundation of this book. We also thank those professors who read chapters of our draft and made numerous useful suggestions: Paul Chaiken, Aksel Hallin, Will Happer, Peter Meyers, Phuan Ong, Jim Peebles, Jeff Peterson, Sam Treiman, and Neil Turok. Early encouragement from Jim Peebles and Jeff Peterson was invaluable, as were the computer facilities of Joe Taylor and Mark Dragovan and the support of Dave Wilkinson. Our most heartfelt thanks, however, goes to all of our fellow graduate students, far too numerous to name, especially the members of our own prelims and generals study groups, who made this book possible.

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Stephen Thorsett

Princeton, August 7, 1990

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Part I

Problems

Chapter 1

Mechanics

Problem 1.1. A Wham-O Super-Ball is a hard spherical ball of radius a. The bounces of a Super-Ball on a surface with friction are essentially elastic and non-slip at the point of contact. How should you throw a Super-Ball if you want it to bounce back and forth as shown in Figure 1.1? (Super-Ball is a registered trademark of Wham-O Corporation, San Gabriel, California.)

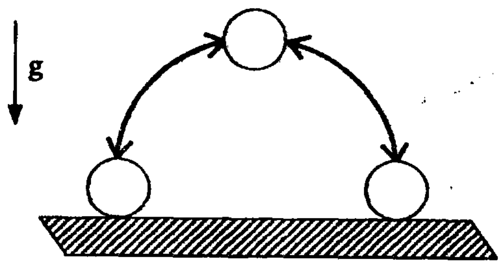

Figure 1.1.

Problem 1.2. Suppose a spacecraft of mass m_0 and cross-sectional area A is coasting with velocity v_0 when it encounters a stationary dust cloud of density ρ. Solve for the subsequent motion of the spacecraft assuming that the dust sticks to its surface and that A is constant over time.

Problem 1.3. The science fiction writer R. A. Heinlein describes a "skyhook" satellite that consists of a long rope placed in orbit at the equator, aligned along a radius from the center of the earth, and moving so that the rope appears suspended in space above a fixed point on the equator (Figure 1.2). The bottom of the rope hangs free just

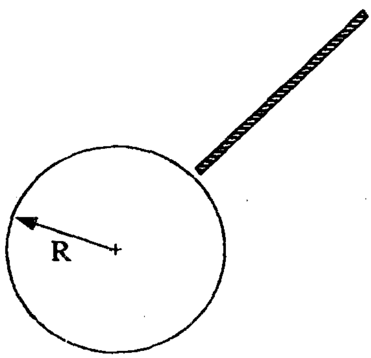

Figure 1.2.

above the surface of the earth (radius R). Assuming that the rope has uniform mass per unit length (and that the rope is strong enough to resist breaking!), find the length of the rope.

Problem 1.4. Three identical objects of mass m are connected by springs of spring constant k, as shown in Figure 1.3. The motion is

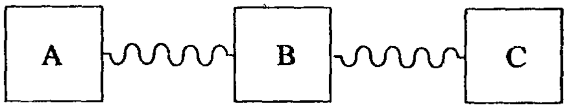

Figure 1.3.

confined to one dimension. At $t = 0$, the masses are at rest at their equilibrium positions. Mass A is then subjected to an external driving force,

$$F(t) = f \cos \omega t, \quad \text{for} \quad t > 0. \tag{1.1}$$

Calculate the motion of mass C.

Problem 1.5. A uniform density ball rolls without slipping and without rolling friction on a turntable rotating in the horizontal plane with angular velocity Ω (Figure 1.4). The ball moves in a circle of radius

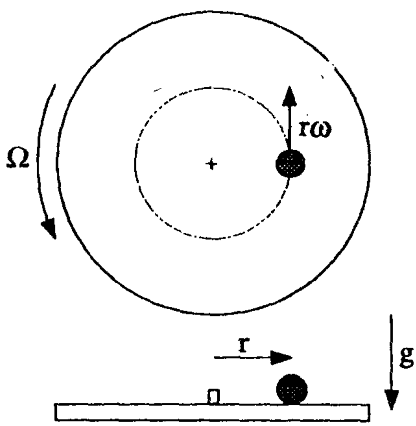

Figure 1.4.

r centered on the pivot of the turntable. Find the angular velocity ω