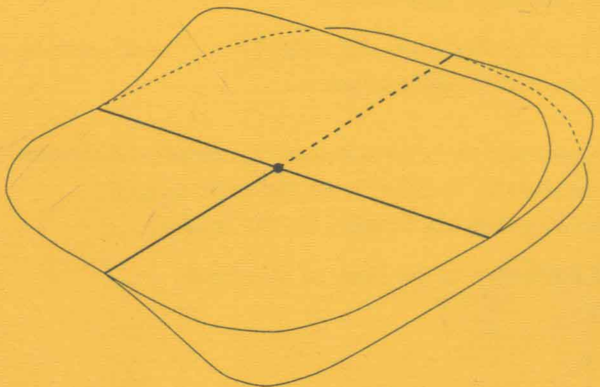
Lecture Notes in Mathematics
1653

Riccardo Benedetti Carlo Petronio

Branched Standard Spines of 3-manifolds



Springer

Riccardo Benedetti Carlo Petronio

Branched Standard Spines of 3-manifolds

Springer

Authors

Riccardo Benedetti
Carlo Petronio
Dipartimento di Matematica
Università di Pisa
Via F. Buonarroti 2
I-56127 Pisa, Italy
e-mail: benedett@dm.unipi.it
petronio@dm.unipi.it

Cataloging-in-Publication Data applied for

Die Deutsche Bibliothek - CIP-Einheitsaufnahme

Benedetti, Riccardo:
Branched standard spines of 3 manifolds / Riccardo Benedetti ; Carlo Petronio. - Berlin ; Heidelberg ; New York ; Barcelona ; Budapest ; Hong Kong ; London ; Milan ; Paris ; Santa Clara ; Singapore ; Tokyo : Springer, 1997
(Lecture notes in mathematics ; 1653)
ISBN 3-540-62627-1
NE: Petronio, Carlo:; GT

Mathematics Subject Classification (1991): Primary: 57N10
Secondary: 57M20, 57R15, 57R25

ISSN 0075-8434
ISBN 3-540-62627-1 Springer-Verlag Berlin Heidelberg New York

Typesetting: Camera-ready T_EX output by the authors
SPIN: 10520303 46/3142-543210 - Printed on acid-free paper

Acknowledgements

This research was carried out while the first named author was a visiting professor at the Institut Fourier of Grenoble and at the Université Louis Pasteur of Strasbourg, and he would like to thank these institutions for their hospitality. The second named author thanks for travel support the following institutions: the Centre Emile Borel of Paris, the University of Oporto and the University of Bochum. Both authors acknowledge partial support by the Italian MURST 40% research project "Geometria reale e complessa"

We warmly thank Vladimir Turaev, Joe Christy, Alexis Marin and Oleg Viro for helpful conversations or for having shown interest in our work.

Lecture Notes in Mathematics

For information about Vols. 1–1469
please contact your bookseller or Springer-Verlag

Vol. 1470: E. Odell, H. Rosenthal (Eds.), Functional Analysis. Proceedings, 1987-89. VII, 199 pages. 1991.

Vol. 1471: A. A. Panchishkin, Non-Archimedean L-Functions of Siegel and Hilbert Modular Forms. VII, 157 pages. 1991.

Vol. 1472: T. T. Nielsen, Bose Algebras: The Complex and Real Wave Representations. V, 132 pages. 1991.

Vol. 1473: Y. Hino, S. Murakami, T. Naito, Functional Differential Equations with Infinite Delay. X, 317 pages. 1991.

Vol. 1474: S. Jackowski, B. Oliver, K. Pawałowski (Eds.), Algebraic Topology, Poznań 1989. Proceedings. VIII, 397 pages. 1991.

Vol. 1475: S. Busenberg, M. Martelli (Eds.), Delay Differential Equations and Dynamical Systems. Proceedings, 1990. VIII, 249 pages. 1991.

Vol. 1476: M. Bekkali, Topics in Set Theory. VII, 120 pages. 1991.

Vol. 1477: R. Jajte, Strong Limit Theorems in Noncommutative L_2-Spaces. X, 113 pages. 1991.

Vol. 1478: M.-P. Malliavin (Ed.), Topics in Invariant Theory. Seminar 1989-1990. VI, 272 pages. 1991.

Vol. 1479: S. Bloch, I. Dolgachev, W. Fulton (Eds.), Algebraic Geometry. Proceedings, 1989. VII, 300 pages. 1991.

Vol. 1480: F. Dumortier, R. Roussarie, J. Sotomayor, H. Żoładek, Bifurcations of Planar Vector Fields: Nilpotent Singularities and Abelian Integrals. VIII, 226 pages. 1991.

Vol. 1481: D. Ferus, U. Pinkall, U. Simon, B. Wegner (Eds.), Global Differential Geometry and Global Analysis. Proceedings, 1991. VIII, 283 pages. 1991.

Vol. 1482: J. Chabrowski, The Dirichlet Problem with L^2-Boundary Data for Elliptic Linear Equations. VI, 173 pages. 1991.

Vol. 1483: E. Reithmeier, Periodic Solutions of Nonlinear Dynamical Systems. VI, 171 pages. 1991.

Vol. 1484: H. Delfs, Homology of Locally Semialgebraic Spaces. IX, 136 pages. 1991.

Vol. 1485: J. Azéma, P. A. Meyer, M. Yor (Eds.), Séminaire de Probabilités XXV. VIII, 440 pages. 1991.

Vol. 1486: L. Arnold, H. Crauel, J.-P. Eckmann (Eds.), Lyapunov Exponents. Proceedings, 1990. VIII, 365 pages. 1991.

Vol. 1487: E. Freitag, Singular Modular Forms and Theta Relations. VI, 172 pages. 1991.

Vol. 1488: A. Carboni, M. C. Pedicchio, G. Rosolini (Eds.), Category Theory. Proceedings, 1990. VII, 494 pages. 1991.

Vol. 1489: A. Mielke, Hamiltonian and Lagrangian Flows on Center Manifolds. X, 140 pages. 1991.

Vol. 1490: K. Metsch, Linear Spaces with Few Lines. XIII, 196 pages. 1991.

Vol. 1491: E. Lluis-Puebla, J.-L. Loday, H. Gillet, C. Soulé, V. Snaith, Higher Algebraic K-Theory: an overview. IX, 164 pages. 1992.

Vol. 1492: K. R. Wicks, Fractals and Hyperspaces. VIII, 168 pages. 1991.

Vol. 1493: E. Benoît (Ed.), Dynamic Bifurcations. Proceedings, Luminy 1990. VII, 219 pages. 1991.

Vol. 1494: M.-T. Cheng, X.-W. Zhou, D.-G. Deng (Eds.), Harmonic Analysis. Proceedings, 1988. IX, 226 pages. 1991.

Vol. 1495: J. M. Bony, G. Grubb, L. Hörmander, H. Komatsu, J. Sjöstrand, Microlocal Analysis and Applications. Montecatini Terme, 1989. Editors: L. Cattabriga, L. Rodino. VII, 349 pages. 1991.

Vol. 1496: C. Foias, B. Francis, J. W. Helton, H. Kwakernaak, J. B. Pearson, H_∞-Control Theory. Como, 1990. Editors: E. Mosca, L. Pandolfi. VII, 336 pages. 1991.

Vol. 1497: G. T. Herman, A. K. Louis, F. Natterer (Eds.), Mathematical Methods in Tomography. Proceedings 1990. X, 268 pages. 1991.

Vol. 1498: R. Lang, Spectral Theory of Random Schrödinger Operators. X, 125 pages. 1991.

Vol. 1499: K. Taira, Boundary Value Problems and Markov Processes. IX, 132 pages. 1991.

Vol. 1500: J.-P. Serre, Lie Algebras and Lie Groups. VII, 168 pages. 1992.

Vol. 1501: A. De Masi, E. Presutti, Mathematical Methods for Hydrodynamic Limits. IX, 196 pages. 1991.

Vol. 1502: C. Simpson, Asymptotic Behavior of Monodromy. V, 139 pages. 1991.

Vol. 1503: S. Shokranian, The Selberg-Arthur Trace Formula (Lectures by J. Arthur). VII, 97 pages. 1991.

Vol. 1504: J. Cheeger, M. Gromov, C. Okonek, P. Pansu, Geometric Topology: Recent Developments. Editors: P. de Bartolomeis, F. Tricerri. VII, 197 pages. 1991.

Vol. 1505: K. Kajitani, T. Nishitani, The Hyperbolic Cauchy Problem. VII, 168 pages. 1991.

Vol. 1506: A. Buium, Differential Algebraic Groups of Finite Dimension. XV, 145 pages. 1992.

Vol. 1507: K. Hulek, T. Peternell, M. Schneider, F.-O. Schreyer (Eds.), Complex Algebraic Varieties. Proceedings, 1990. VII, 179 pages. 1992.

Vol. 1508: M. Vuorinen (Ed.), Quasiconformal Space Mappings. A Collection of Surveys 1960-1990. IX, 148 pages. 1992.

Vol. 1509: J. Aguadé, M. Castellet, F. R. Cohen (Eds.), Algebraic Topology - Homotopy and Group Cohomology. Proceedings, 1990. X, 330 pages. 1992.

Vol. 1510: P. P. Kulish (Ed.), Quantum Groups. Proceedings, 1990. XII, 398 pages. 1992.

Vol. 1511: B. S. Yadav, D. Singh (Eds.), Functional Analysis and Operator Theory. Proceedings, 1990. VIII, 223 pages. 1992.

Vol. 1512: L. M. Adleman, M.-D. A. Huang, Primality Testing and Abelian Varieties Over Finite Fields. VII, 142 pages. 1992.

Vol. 1513: L. S. Block, W. A. Coppel, Dynamics in One Dimension. VIII, 249 pages. 1992.

Vol. 1514: U. Krengel, K. Richter, V. Warstat (Eds.), Ergodic Theory and Related Topics III, Proceedings, 1990. VIII, 236 pages. 1992.

Vol. 1515: E. Ballico, F. Catanese, C. Ciliberto (Eds.), Classification of Irregular Varieties. Proceedings, 1990. VII, 149 pages. 1992.

Vol. 1516: R. A. Lorentz, Multivariate Birkhoff Interpolation. IX, 192 pages. 1992.

Vol. 1517: K. Keimel, W. Roth, Ordered Cones and Approximation. VI, 134 pages. 1992.

Vol. 1518: H. Stichtenoth, M. A. Tsfasman (Eds.), Coding Theory and Algebraic Geometry. Proceedings, 1991. VIII, 223 pages. 1992.

Vol. 1519: M. W. Short, The Primitive Soluble Permutation Groups of Degree less than 256. IX, 145 pages. 1992.

Vol. 1520: Yu. G. Borisovich, Yu. E. Gliklikh (Eds.), Global Analysis – Studies and Applications V. VII, 284 pages. 1992.

Vol. 1521: S. Busenberg, B. Forte, H. K. Kuiken, Mathematical Modelling of Industrial Process. Bari, 1990. Editors: V. Capasso, A. Fasano. VII, 162 pages. 1992.

Vol. 1522: J.-M. Delort, F. B. I. Transformation. VII, 101 pages. 1992.

Vol. 1523: W. Xue, Rings with Morita Duality. X, 168 pages. 1992.

Vol. 1524: M. Coste, L. Mahé, M.-F. Roy (Eds.), Real Algebraic Geometry. Proceedings, 1991. VIII, 418 pages. 1992.

Vol. 1525: C. Casacuberta, M. Castellet (Eds.), Mathematical Research Today and Tomorrow. VII, 112 pages. 1992.

Vol. 1526: J. Azéma, P. A. Meyer, M. Yor (Eds.), Séminaire de Probabilités XXVI. X, 633 pages. 1992.

Vol. 1527: M. I. Freidlin, J.-F. Le Gall, Ecole d'Eté de Probabilités de Saint-Flour XX – 1990. Editor: P. L. Hennequin. VIII, 244 pages. 1992.

Vol. 1528: G. Isac, Complementarity Problems. VI, 297 pages. 1992.

Vol. 1529: J. van Neerven, The Adjoint of a Semigroup of Linear Operators. X, 195 pages. 1992.

Vol. 1530: J. G. Heywood, K. Masuda, R. Rautmann, S. A. Solonnikov (Eds.), The Navier-Stokes Equations II – Theory and Numerical Methods. IX, 322 pages. 1992.

Vol. 1531: M. Stoer, Design of Survivable Networks. IV, 206 pages. 1992.

Vol. 1532: J. F. Colombeau, Multiplication of Distributions. X, 184 pages. 1992.

Vol. 1533: P. Jipsen, H. Rose, Varieties of Lattices. X, 162 pages. 1992.

Vol. 1534: C. Greither, Cyclic Galois Extensions of Commutative Rings. X, 145 pages. 1992.

Vol. 1535: A. B. Evans, Orthomorphism Graphs of Groups. VIII, 114 pages. 1992.

Vol. 1536: M. K. Kwong, A. Zettl, Norm Inequalities for Derivatives and Differences. VII, 150 pages. 1992.

Vol. 1537: P. Fitzpatrick, M. Martelli, J. Mawhin, R. Nussbaum, Topological Methods for Ordinary Differential Equations. Montecatini Terme, 1991. Editors: M. Furi, P. Zecca. VII, 218 pages. 1993.

Vol. 1538: P.-A. Meyer, Quantum Probability for Probabilists. X, 287 pages. 1993.

Vol. 1539: M. Coornaert, A. Papadopoulos, Symbolic Dynamics and Hyperbolic Groups. VIII, 138 pages. 1993.

Vol. 1540: H. Komatsu (Ed.), Functional Analysis and Related Topics, 1991. Proceedings. XXI, 413 pages. 1993.

Vol. 1541: D. A. Dawson, B. Maisonneuve, J. Spencer, Ecole d´ Eté de Probabilités de Saint-Flour XXI - 1991. Editor: P. L. Hennequin. VIII, 356 pages. 1993.

Vol. 1542: J.Fröhlich, Th.Kerler, Quantum Groups, Quantum Categories and Quantum Field Theory. VII, 431 pages. 1993.

Vol. 1543: A. L. Dontchev, T. Zolezzi, Well-Posed Optimization Problems. XII, 421 pages. 1993.

Vol. 1544: M.Schürmann, White Noise on Bialgebras. VII, 146 pages. 1993.

Vol. 1545: J. Morgan, K. O'Grady, Differential Topology of Complex Surfaces. VIII, 224 pages. 1993.

Vol. 1546: V. V. Kalashnikov, V. M. Zolotarev (Eds.), Stability Problems for Stochastic Models. Proceedings, 1991. VIII, 229 pages. 1993.

Vol. 1547: P. Harmand, D. Werner, W. Werner, M-ideals in Banach Spaces and Banach Algebras. VIII, 387 pages. 1993.

Vol. 1548: T. Urabe, Dynkin Graphs and Quadrilateral Singularities. VI, 233 pages. 1993.

Vol. 1549: G. Vainikko, Multidimensional Weakly Singular Integral Equations. XI, 159 pages. 1993.

Vol. 1550: A. A. Gonchar, E. B. Saff (Eds.), Methods of Approximation Theory in Complex Analysis and Mathematical Physics IV, 222 pages, 1993.

Vol. 1551: L. Arkeryd, P. L. Lions, P.A. Markowich, S.R. S. Varadhan. Nonequilibrium Problems in Many-Particle Systems. Montecatini, 1992. Editors: C. Cercignani, M. Pulvirenti. VII, 158 pages 1993.

Vol. 1552: J. Hilgert, K.-H. Neeb, Lie Semigroups and their Applications. XII, 315 pages. 1993.

Vol. 1553: J.-L- Colliot-Thélène, J. Kato, P. Vojta. Arithmetic Algebraic Geometry. Trento, 1991. Editor: E. Ballico. VII, 223 pages. 1993.

Vol. 1554: A. K. Lenstra, H. W. Lenstra, Jr. (Eds.), The Development of the Number Field Sieve. VIII, 131 pages. 1993.

Vol. 1555: O. Liess, Conical Refraction and Higher Microlocalization. X, 389 pages. 1993.

Vol. 1556: S. B. Kuksin, Nearly Integrable Infinite-Dimensional Hamiltonian Systems. XXVII, 101 pages. 1993.

Vol. 1557: J. Azéma, P. A. Meyer, M. Yor (Eds.), Séminaire de Probabilités XXVII. VI, 327 pages. 1993.

Vol. 1558: T. J. Bridges, J. E. Furter, Singularity Theory and Equivariant Symplectic Maps. VI, 226 pages. 1993.

Vol. 1559: V. G. Sprindžuk, Classical Diophantine Equations. XII, 228 pages. 1993.

Vol. 1560: T. Bartsch, Topological Methods for Variational Problems with Symmetries. X, 152 pages. 1993.

Vol. 1561: I. S. Molchanov, Limit Theorems for Unions of Random Closed Sets. X, 157 pages. 1993.

Vol. 1562: G. Harder, Eisensteinkohomologie und die Konstruktion gemischter Motive. XX, 184 pages. 1993.

Vol. 1563: E. Fabes, M. Fukushima, L. Gross, C. Kenig, M. Röckner, D. W. Stroock, Dirichlet Forms. Varenna, 1992. Editors: G. Dell'Antonio, U. Mosco. VII, 245 pages. 1993.

Vol. 1564: J. Jorgenson, S. Lang, Basic Analysis of Regularized Series and Products. IX, 122 pages. 1993.

Vol. 1565: L. Boutet de Monvel, C. De Concini, C. Procesi, P. Schapira, M. Vergne. D-modules, Representation Theory, and Quantum Groups. Venezia, 1992. Editors: G. Zampieri, A. D'Agnolo. VII, 217 pages. 1993.

Vol. 1566: B. Edixhoven, J.-H. Evertse (Eds.), Diophantine Approximation and Abelian Varieties. XIII, 127 pages. 1993.

Vol. 1567: R. L. Dobrushin, S. Kusuoka, Statistical Mechanics and Fractals. VII, 98 pages. 1993.

Vol. 1568: F. Weisz, Martingale Hardy Spaces and their Application in Fourier Analysis. VIII, 217 pages. 1994.

Vol. 1569: V. Totik, Weighted Approximation with Varying Weight. VI, 117 pages. 1994.

Vol. 1570: R. deLaubenfels, Existence Families, Functional Calculi and Evolution Equations. XV, 234 pages. 1994.

Vol. 1571: S. Yu. Pilyugin, The Space of Dynamical Systems with the C^0-Topology. X, 188 pages. 1994.

Vol. 1572: L. Göttsche, Hilbert Schemes of Zero-Dimensional Subschemes of Smooth Varieties. IX, 196 pages. 1994.

Vol. 1573: V. P. Havin, N. K. Nikolski (Eds.), Linear and Complex Analysis – Problem Book 3 – Part I. XXII, 489 pages. 1994.

Vol. 1574: V. P. Havin, N. K. Nikolski (Eds.), Linear and Complex Analysis – Problem Book 3 – Part II. XXII, 507 pages. 1994.

Vol. 1575: M. Mitrea, Clifford Wavelets, Singular Integrals, and Hardy Spaces. XI, 116 pages. 1994.

Vol. 1576: K. Kitahara, Spaces of Approximating Functions with Haar-Like Conditions. X, 110 pages. 1994.

Vol. 1577: N. Obata, White Noise Calculus and Fock Space. X, 183 pages. 1994.

Vol. 1578: J. Bernstein, V. Lunts, Equivariant Sheaves and Functors. V, 139 pages. 1994.

Vol. 1579: N. Kazamaki, Continuous Exponential Martingales and *BMO*. VII, 91 pages. 1994.

Vol. 1580: M. Milman, Extrapolation and Optimal Decompositions with Applications to Analysis. XI, 161 pages. 1994.

Vol. 1581: D. Bakry, R. D. Gill, S. A. Molchanov, Lectures on Probability Theory. Editor: P. Bernard. VIII, 420 pages. 1994.

Vol. 1582: W. Balser, From Divergent Power Series to Analytic Functions. X, 108 pages. 1994.

Vol. 1583: J. Azéma, P. A. Meyer, M. Yor (Eds.), Séminaire de Probabilités XXVIII. VI, 334 pages. 1994.

Vol. 1584: M. Brokate, N. Kenmochi, I. Müller, J. F. Rodriguez, C. Verdi, Phase Transitions and Hysteresis. Montecatini Terme, 1993. Editor: A. Visintin. VII, 291 pages. 1994.

Vol. 1585: G. Frey (Ed.), On Artin's Conjecture for Odd 2-dimensional Representations. VIII, 148 pages. 1994.

Vol. 1586: R. Nillsen, Difference Spaces and Invariant Linear Forms. XII, 186 pages. 1994.

Vol. 1587: N. Xi, Representations of Affine Hecke Algebras. VIII, 137 pages. 1994.

Vol. 1588: C. Scheiderer, Real and Étale Cohomology. XXIV, 273 pages. 1994.

Vol. 1589: J. Bellissard, M. Degli Esposti, G. Forni, S. Graffi, S. Isola, J. N. Mather, Transition to Chaos in Classical and Quantum Mechanics. Montecatini Terme, 1991. Editor: S. Graffi. VII, 192 pages. 1994.

Vol. 1590: P. M. Soardi, Potential Theory on Infinite Networks. VIII, 187 pages. 1994.

Vol. 1591: M. Abate, G. Patrizio, Finsler Metrics – A Global Approach. IX, 180 pages. 1994.

Vol. 1592: K. W. Breitung, Asymptotic Approximations for Probability Integrals. IX, 146 pages. 1994.

Vol. 1593: J. Jorgenson & S. Lang, D. Goldfeld, Explicit Formulas for Regularized Products and Series. VIII, 154 pages. 1994.

Vol. 1594: M. Green, J. Murre, C. Voisin, Algebraic Cycles and Hodge Theory. Torino, 1993. Editors: A. Albano, F. Bardelli. VII, 275 pages. 1994.

Vol. 1595: R.D.M. Accola, Topics in the Theory of Riemann Surfaces. IX, 105 pages. 1994.

Vol. 1596: L. Heindorf, L. B. Shapiro, Nearly Projective Boolean Algebras. X, 202 pages. 1994.

Vol. 1597: B. Herzog, Kodaira-Spencer Maps in Local Algebra. XVII, 176 pages. 1994.

Vol. 1598: J. Berndt, F. Tricerri, L. Vanhecke, Generalized Heisenberg Groups and Damek-Ricci Harmonic Spaces. VIII, 125 pages. 1995.

Vol. 1599: K. Johannson, Topology and Combinatorics of 3-Manifolds. XVIII, 446 pages. 1995.

Vol. 1600: W. Narkiewicz, Polynomial Mappings. VII, 130 pages. 1995.

Vol. 1601: A. Pott, Finite Geometry and Character Theory. VII, 181 pages. 1995.

Vol. 1602: J. Winkelmann, The Classification of Three-dimensional Homogeneous Complex Manifolds. XI, 230 pages. 1995.

Vol. 1603: V. Ene, Real Functions – Current Topics. XIII, 310 pages. 1995.

Vol. 1604: A. Huber, Mixed Motives and their Realization in Derived Categories. XV, 207 pages. 1995.

Vol. 1605: L. B. Wahlbin, Superconvergence in Galerkin Finite Element Methods. XI, 166 pages. 1995.

Vol. 1606: P.-D. Liu, M. Qian, Smooth Ergodic Theory of Random Dynamical Systems. XI, 221 pages. 1995.

Vol. 1607: G. Schwarz, Hodge Decomposition – A Method for Solving Boundary Value Problems. VII, 155 pages. 1995.

Vol. 1608: P. Biane, R. Durrett, Lectures on Probability Theory. Editor: P. Bernard. VII, 210 pages. 1995.

Vol. 1609: L. Arnold, C. Jones, K. Mischaikow, G. Raugel, Dynamical Systems. Montecatini Terme, 1994. Editor: R. Johnson. VIII, 329 pages. 1995.

Vol. 1610: A. S. Üstünel, An Introduction to Analysis on Wiener Space. X, 95 pages. 1995.

Vol. 1611: N. Knarr, Translation Planes. VI, 112 pages. 1995.

Vol. 1612: W. Kühnel, Tight Polyhedral Submanifolds and Tight Triangulations. VII, 122 pages. 1995.

Vol. 1613: J. Azéma, M. Emery, P. A. Meyer, M. Yor (Eds.), Séminaire de Probabilités XXIX. VI, 326 pages. 1995.

Vol. 1614: A. Koshelev, Regularity Problem for Quasilinear Elliptic and Parabolic Systems. XXI, 255 pages. 1995.

Vol. 1615: D. B. Massey, Lê Cycles and Hypersurface Singularities. XI, 131 pages. 1995.

Vol. 1616: I. Moerdijk, Classifying Spaces and Classifying Topoi. VII, 94 pages. 1995.

Vol. 1617: V. Yurinsky, Sums and Gaussian Vectors. XI, 305 pages. 1995.

Vol. 1618: G. Pisier, Similarity Problems and Completely Bounded Maps. VII, 156 pages. 1996.

Vol. 1619: E. Landvogt, A Compactification of the Bruhat-Tits Building. VII, 152 pages. 1996.

Vol. 1620: R. Donagi, B. Dubrovin, E. Frenkel, E. Previato, Integrable Systems and Quantum Groups. Montecatini Terme, 1993. Editors:M. Francaviglia, S. Greco. VIII, 488 pages. 1996.

Vol. 1621: H. Bass, M. V. Otero-Espinar, D. N. Rockmore, C. P. L. Tresser, Cyclic Renormalization and Auto-morphism Groups of Rooted Trees. XXI, 136 pages. 1996.

Vol. 1622: E. D. Farjoun, Cellular Spaces, Null Spaces and Homotopy Localization. XIV, 199 pages. 1996.

Vol. 1623: H.P. Yap, Total Colourings of Graphs. VIII, 131 pages. 1996.

Vol. 1624: V. Brînzănescu, Holomorphic Vector Bundles over Compact Complex Surfaces. X, 170 pages. 1996.

Vol.1625: S. Lang, Topics in Cohomology of Groups. VII, 226 pages. 1996.

Vol. 1626: J. Azéma, M. Emery, M. Yor (Eds.), Séminaire de Probabilités XXX. VIII, 382 pages. 1996.

Vol. 1627: C. Graham, Th. G. Kurtz, S. Méléard, Ph. E. Protter, M. Pulvirenti, D. Talay, Probabilistic Models for Nonlinear Partial Differential Equations. Montecatini Terme, 1995. Editors: D. Talay, L. Tubaro. X, 301 pages. 1996.

Vol. 1628: P.-H. Zieschang, An Algebraic Approach to Association Schemes. XII, 189 pages. 1996.

Vol. 1629: J. D. Moore, Lectures on Seiberg-Witten Invariants. VII, 105 pages. 1996.

Vol. 1630: D. Neuenschwander, Probabilities on the Heisenberg Group: Limit Theorems and Brownian Motion. VIII, 139 pages. 1996.

Vol. 1631: K. Nishioka, Mahler Functions and Transcendence.VIII, 185 pages.1996.

Vol. 1632: A. Kushkuley, Z. Balanov, Geometric Methods in Degree Theory for Equivariant Maps. VII, 136 pages. 1996.

Vol.1633: H. Aikawa, M. Essén, Potential Theory – Selected Topics. IX, 200 pages.1996.

Vol. 1634: J. Xu, Flat Covers of Modules. IX, 161 pages. 1996.

Vol. 1635: E. Hebey, Sobolev Spaces on Riemannian Manifolds. X, 116 pages. 1996.

Vol. 1636: M. A. Marshall, Spaces of Orderings and Abstract Real Spectra. VI, 190 pages. 1996.

Vol. 1637: B. Hunt, The Geometry of some special Arithmetic Quotients. XIII, 332 pages. 1996.

Vol. 1638: P. Vanhaecke, Integrable Systems in the realm of Algebraic Geometry. VIII, 218 pages. 1996.

Vol. 1639: K. Dekimpe, Almost-Bieberbach Groups: Affine and Polynomial Structures. X, 259 pages. 1996.

Vol. 1640: G. Boillat, C. M. Dafermos, P. D. Lax, T. P. Liu, Recent Mathematical Methods in Nonlinear Wave Propagation. Montecatini Terme, 1994. Editor: T. Ruggeri. VII, 142 pages. 1996.

Vol. 1641: P. Abramenko, Twin Buildings and Applications to S-Arithmetic Groups. IX, 123 pages. 1996.

Vol. 1642: M. Puschnigg, Asymptotic Cyclic Cohomology. XXII, 138 pages. 1996.

Vol. 1643: J. Richter-Gebert, Realization Spaces of Polytopes. XI, 187 pages. 1996.

Vol. 1644: A. Adler, S. Ramanan, Moduli of Abelian Varieties. VI, 196 pages. 1996.

Vol. 1645: H. W. Broer, G. B. Huitema, M. B. Sevryuk, Quasi-Periodic Motions in Families of Dynamical Systems. XI, 195 pages. 1996.

Vol. 1646: J.-P. Demailly, T. Peternell, G. Tian, A. N. Tyurin, Transcendental Methods in Algebraic Geometry. Cetraro, 1994. Editors: F. Catanese, C. Ciliberto. VII, 257 pages. 1996.

Vol. 1647: D. Dias, P. Le Barz, Configuration Spaces over Hilbert Schemes and Applications. VII. 143 pages. 1996.

Vol. 1648: R. Dobrushin, P. Groeneboom, M. Ledoux, Lectures on Probability Theory and Statistics. Editor: P. Bernard. VIII, 300 pages. 1996.

Vol. 1649: S. Kumar, G. Laumon, U. Stuhler, Vector Bundles on Curves – New Directions. Cetraro, 1995. Editor: M. S. Narasimhan. VII, 193 pages. 1997.

Vol. 1650: J. Wildeshaus, Realizations of Polylogarithms. XI, 343 pages. 1997.

Vol. 1651: M. Drmota, R. F. Tichy, Sequences, Discrepancies and Applications. XIII, 503 pages. 1997.

Vol. 1652: S. Todorcevic, Topics in Topology. VIII, 153 pages. 1997.

Vol. 1653: R. Benedetti, C. Petronio, Branched Standard Spines of 3-manifolds. VIII, 132 pages. 1997.

General Remarks

Lecture Notes are printed by photo-offset from the master-copy delivered in camera-ready form by the authors. For this purpose Springer-Verlag provides technical instructions for the preparation of manuscripts.

Careful preparation of manuscripts will help keep production time short and ensure a satisfactory appearance of the finished book. The actual production of a Lecture Notes volume normally takes approximately 8 weeks.

Authors receive 50 free copies of their book. No royalty is paid on Lecture Notes volumes.

Authors are entitled to purchase further copies of their book and other Springer mathematics books for their personal use, at a discount of 33,3 % directly from Springer-Verlag.

Commitment to publish is made by letter of intent rather than by signing a formal contract. Springer-Verlag secures the copyright for each volume.

Addresses:

Professor A. Dold
Mathematisches Institut
Universität Heidelberg
Im Neuenheimer Feld 288
D-69120 Heidelberg
Federal Republic of Germany

Professor F. Takens
Mathematisch Instituut
Rijksuniversiteit Groningen
Postbus 800
NL-9700 AV Groningen
The Netherlands

Springer-Verlag, Mathematics Editorial
Tiergartenstr. 17
D-69121 Heidelberg
Federal Republic of Germany
Tel.: *49 (6221) 487-410

Contents

Chapter 1

Motivations, plan and statements

In this chapter we will describe the general plan of our work and state in full detail our main results. However the reader should refer to the subsequent chapters for a correct interpretation of the *effectiveness* of the maps whose existence is stated in Theorems 1.4.1, 1.4.2, 1.4.3 and 1.4.4. It is in the reconstruction process which underlies the definition of these maps that branched standard spines come into play.

1.1 Combinatorial realizations of topological categories

The main point of this work is the construction of combinatorial realizations of some categories of 3-manifolds with extra structure. Before describing in detail the classes of objects which we handle, let us explain what we exactly mean by *combinatorial realization*. Let $\mathcal{T}$ be a collection of topological objects (for us, 3-dimensional manifolds with a certain extra structure), regarded up to a suitable equivalence relation. The first ingredient of a realization will be an explicitly described set $\mathcal{S}$ of finite combinatorial objects (typically, finite graphs with certain properties and decorations), together with an effective mapping

$$\Psi : \mathcal{S} \to \mathcal{T},$$

called the *reconstruction map*, whose image is the whole of $\mathcal{T}$. The second ingredient of the realization is the *calculus*, namely an explicitly described finite set of local moves on $\mathcal{S}$ with the property that two elements of $\mathcal{S}$ have the same image in $\mathcal{T}$ under Ψ if and only if they are related to each other by a finite combination of the moves of the calculus.

A classical model of a combinatorial realization is the presentation of links up to isotopy in S^3 by means of planar diagrams, where the calculus is generated by Reidemeister moves. In 3-manifold topology some examples of combinatorial representation are known which satisfy at least some of the requirements which we have stated. Let us remark however that the requirements of *finiteness* and *locality* of the calculus are somewhat restrictive. For instance the presentation of closed connected oriented 3-manifolds via longitudinal Dehn surgery on framed links in S^3, with either of the two versions of the Kirby calculus, does not satisfy the requirements. If we use the version of the calculus which includes the band move, then we have a non-local move, whereas, if we use the generalized Kirby move, then we have indeed local moves, but we actually have to take into account infinitely many different ones, parametrized by the number of strands which link the curl removed by the move. In [6], using a slight refinement of the theory of standard spines and an appropriate graphic encoding, we have produced a combinatorial realization of the category of compact connected 3-manifolds with non-empty boundary, and a refined version of the same realization for the oriented case. This realization will be recalled in Chapter 2 because we will use it extensively.

Let us now describe the topological objects of which we provide a combinatorial realization in this work. By a 3-manifold we will always mean a *connected*, *compact* and *oriented* one, with or without boundary. We will denote by $\mathcal{M}$ the class of *closed* 3-manifolds, up to orientation-preserving diffeomorphism. We will call *combing* on a closed 3-manifold a nowhere-vanishing vector field, and *framing* a triple of linearly independent vector fields which pointwise induce the orientation. Combings and framings will always be viewed up to homotopy through objects of the same type. Other objects which we will consider are spin structures. Note that if $f : M \to M'$ is an orientation-preserving diffeomorphism then to any combing or framing or spin structure on M there corresponds under f an object of the same type on M'. We will denote by $\mathcal{M}_{\mathrm{comb}}$ the set of pairs (M, v) where M is a closed manifold and v is a combing on M, viewed up to the natural action on pairs of orientation-preserving diffeomorphisms. We define $\mathcal{M}_{\mathrm{fram}}$ and $\mathcal{M}_{\mathrm{spin}}$ in a similar way. For each of $\mathcal{M}$, $\mathcal{M}_{\mathrm{comb}}$, $\mathcal{M}_{\mathrm{fram}}$ and $\mathcal{M}_{\mathrm{spin}}$ we will give a combinatorial realization as stated above (the realization of $\mathcal{M}$ being different and independent of the one given in [6]).

Before sketching the topological constructions which underlie our presentations, we want to point out a reason for which, to our opinion, it is non-trivial and of some interest to give a combinatorial realization of the classes of "structured manifolds" $\mathcal{M}_{\mathrm{comb}}$, $\mathcal{M}_{\mathrm{fram}}$ and $\mathcal{M}_{\mathrm{spin}}$. Our remark is that, for any given closed manifold M, the set of extra structures of each of the three types in exam can be described as an *affine space* (or a certain "bundle" of affine spaces, in the case of combings), and there is no canonical way to define a preferred basepoint. This is well-known for spin structures, which are identified to an affine space over $H^1(M;\mathbb{Z}_2)$. Once a preferred framing on M is fixed, the set of all framings can be identified to the set of homotopy classes of maps from M to SO(3), which is rather easily recognized to be a central extension of $H^1(M;\mathbb{Z}_2)$ by $\mathbb{Z}$; however the dependence on the choice of the basepoint is somewhat intriguing. The situation is more complicated for the case of combings: if we fix a framing on M then the set of all combings can be identified to the set of homotopy classes of maps $v : M \to S^2$; if h denotes the preferred generator of $H^2(S^2;\mathbb{Z})$ we can associate to v the element $g(v) = v^*(h) \in H^2(M;\mathbb{Z})$, and the map g is surjective. Moreover, for $k \in H^2(M;\mathbb{Z})$ and $v, v' \in g^{-1}(k)$, the difference between v and v' can be described as a "Hopf number", which lies in $\mathbb{Z}$ when k is a torsion element, and in $\mathbb{Z}_{2d}$ when k is not a torsion element and d is the greatest integer divisor of k. So, if we fix a basepoint in each of the fibres of g, then we can identify the fibre with the appropriate $\mathbb{Z}$ or $\mathbb{Z}_{2d}$, but the choice of these basepoints, just as the choice of the framing from which we have started, cannot be made canonical.

To our point of view, heuristically speaking, it follows from the facts remarked in the previous paragraph that combinatorial realizations of $\mathcal{M}_{\mathrm{comb}}$ or $\mathcal{M}_{\mathrm{fram}}$ or $\mathcal{M}_{\mathrm{spin}}$ must be supported by classes of objects capable of capturing deep topological properties of 3-manifolds. We have devised such a class of objects by introducing *branched standard spines*. This notion has been obtained as a combination of the two essentially classical concepts of *standard spine* and of *branched surface*, and this makes our definition a rather natural one.

The interest into effective combinatorial presentations was increased in recent years by the development of the theory of quantum invariants (see e.g. [56]). On one hand the existence and structure of these invariants has been predicted, starting from Witten's interpretation of the Jones polynomial, from general principles in quantum field theory [61]; but on the other hand an effective and rigorous construction of the invariants has only been given, according to the current opinion, via combinatorial presentations such as surgery with the Kirby calculus (for the Reshetikhin-Turaev-Witten invariants), or spines and triangulations with the appropriate moves (for the Turaev-Viro invariants). Our work was partially influenced and motivated by this consideration, and we show in Chapter 8 that our combinatorial presentation of spin manifolds is suitable for an effective implementation and computation of the spin-refined version of the Turaev-Viro invariants.

In the last few years new models for invariants have been proposed (see e.g. [23], [29], [33]) which are based on ideas similar to those used for the quantum invariants (i.e. representations of certain classes of diagrams into algebraic objects, typically Hopf algebras with extra structures), but do not strictly fall in the quantum category (as axiomatized for instance in [56]). The invariants constructed by G. Kupenberg [33] are in their most general form invariants of framed and combed manifolds, and moreover their definition does not immediately yield a computation recipe; therefore it is very natural ask if our combinatorial presentations support an effective implementation of the invariants. We discuss this question in greater detail in Chapter 9, where we also mention other fields in which our work might have applications. In particular, after remarking that our combed calculus dually represents the set of homotopy classes of oriented plane distributions on 3-manifolds, we give some hints on the possible relations with the theory of foliations and contact structures (recall that every homotopy class of oriented plane fields contains both distributions and contact structures).

1.2 Branched standard spines and an outline of the construction

The authors who mainly contributed to developing the theory of standard spines are Casler [13], Matveev [37] and Piergallini [45]; we also remind the reader that on this theory is based the combinatorial realization of manifolds with boundary given in [6]. We warn the reader that our definition of *standard* spine includes the requirement that the strata of the natural stratification should be cells; this cellularity condition is in our opinion essential to base a *local* calculus on spines, and we will point out its importance in various instances. We will use the term *simple* for spines in which the cellularity condition is dropped.

The notion of branched surface was originally introduced in [60] for the study of hyperbolic attractors (see also [11]). Later on, in a number of papers (e.g. [17], [19] and the references quoted therein) branched surfaces have been viewed as codimension 1 objects capable of supporting and generalizing classical notions about surfaces, in particular incompressibility.

A branched standard spine P of a (compact, connected and oriented) 3-manifold N with non-empty boundary is a standard spine of N endowed with an oriented branching, which roughly speaking means that pointwise an oriented tangent plane to P is well-defined, and the local behaviour can be described by certain very natural models. To every such a P we can associate a vector field on N (well-defined up to a certain type of homotopy) which is positively transversal to P and has a prescribed local behaviour on a neighbourhood of ∂N; namely, the field is tangent to ∂N only along finitely many simple curves of *apparent contour*, along which the boundary is "concave" with respect to the flow. This theory specializes to the closed case by requiring that N should be bounded by S^2 and the field should have the simplest possible behaviour near the boundary. This allows to extend the field to the closed manifold M obtained by capping off the boundary sphere of N, and the field on M turns out to be well-defined up to homotopy. Since we can prove that all combings on M are obtained via this procedure from some P, we have the reconstruction map. The moves of the calculus for combings are essentially branched versions of the usual moves of the Matveev-Piergallini calculus for standard spines. However we want to emphasize that the results known for the non-branched case do not imply the fact that the combing moves generate the equivalence relation induced by the reconstruction map, and our proof that this indeed happens is based on an independent argument.

As we already mentioned, the natural realm in which the foundations of the theory of branched standard spines are placed is the category of 3-manifolds with boundary, of which the closed case is a specialization. The same is true case also for the core of the combing calculus, which is first established in the case with boundary and then specialized. Moreover the proof is divided into two steps; at first we only use general position arguments to derive a weaker version of the calculus, in which spines may not be standard and therefore moves are non-local; later a harder work allows us to recover the standard context and hence locality.

In the closed case we show how to complete the calculus for combings by means of a local move, which we call *combinatorial Pontrjagin move*, whose topological counterpart allows to obtain from each other any two combings on the same manifold. This leads therefore to a new combinatorial realization of (connected, oriented) closed 3-manifolds.

The combinatorial realization of combings allows us to deduce a similar one for framings, starting with the following (easy) remarks: the first vector of a framing on a closed manifold M is a combing, and a combing v on M extends to a framing if and only if the Euler class $\mathcal{E}(v) \in H^2(M;\mathbb{Z})$ of the plane field complementary to v is null. Our next step is to construct for a branched standard spine P of M a preferred cocycle c_P which represents the Euler class of the combing carried by P. Then we show how to explicitly associate to an integral 1-cochain whose coboundary is c_P a well-defined framing, and we prove that two such cochains define the same framing if and only if the reduction modulo 2 of their difference is a $\mathbb{Z}_2$-coboundary. Therefore the objects of the combinatorial realization of framed manifolds are pairs (P, x), where x is a mod-2 1-cochain on P which lifts to an integral 1-cochain $\tilde{x}$ with $\delta\tilde{x} = c_P$, and x is viewed up to $\mathbb{Z}_2$-coboundaries. The moves of the framing calculus are obtained by enhancing to such pairs the moves of the combing calculus, making sure that whenever (P, x) and (P', x') are related by a move then they define the same framing.

A similar scheme works for spin structures. Such a structure on a manifold M can be viewed as a framing of M on the singular set $S(P)$ of a spine P of M, where the framing extends to the whole of P and is viewed up to homotopy on $S(P)$. Given a certain branched spine P and the corresponding combing v, the second Stiefel-Whitney class of the plane field complementary to v is just the reduction modulo 2 of $\mathcal{E}(v)$, therefore it coincides with $w_2(M)$ and so it is null; moreover $H^1(P;\mathbb{Z}_2) = H^1(M;\mathbb{Z}_2)$, and it easily follows that every spin structure has a representative whose first vector is v. To every mod-2 1-cochain x on P whose coboundary is the reduction modulo 2 of c_P we can associate a spin structure which depends only on the class of x modulo $\mathbb{Z}_2$-coboundaries. Therefore spin 3-manifolds are encoded by such pairs (P, x), and the moves to take into account are the same as in the case of framings (which is coherent with the fact that each framing determines a spin structure), together with suitable enhancements of the combinatorial Pontrjagin move of the closed calculus.

1.3 Graphic encoding

Another element of our work is a graphic encoding of the objects of the representations. This is conceptually subordinate to the intrinsic theory of branched spines, but not secondary if one has in mind the effectiveness (and possibly an actual computer implementation) of the combinatorial realizations. To a branched standard spine we associate a planar graph by embedding in 3-space a neighbourhood of the singular set of the spine, in such a way that along the singular set the tangent plane is "almost horizontal", and then projecting on the horizontal plane. The (oriented) branching allows to define this operation uniquely, and implies that to every graph with some simple extra structures there corresponds a unique branched standard spine, and moreover the decoding procedure is completely explicit and effective.

From a given graph it is very easy to explicitly reconstruct the 2-cells of the corresponding spine, and also to find the preferred cochain which represents the Euler class of the combing carried by the spine. Moreover the graph itself corresponds to the singular set of the spine, so a 1-cochain on the spine can be viewed just as a colouring of the edges of the graph. These facts imply that the various elements which interact in the combinatorial realization of spin and framed 3-manifolds can be very easily translated in terms of the graphs.

We also want to note that on one hand the graphs still carry in a rather transparent way all the information on the topological situation, and on the other hand they allow a formal control of the various manipulations (see also [7]). In Chapter 10 we will show how to explicitly carry out the computation of some homological invariants using the graphs only.

1.4 Statements of representation theorems

In this section we will describe in a completely self-contained way, i.e. without referring to the notions described in the previous sections, our *combinatorial realizations* of $\mathcal{M}$, $\mathcal{M}_{\mathsf{comb}}$, $\mathcal{M}_{\mathsf{fram}}$ and $\mathcal{M}_{\mathsf{spin}}$. As already pointed out, the reader should refer to the rest of this work to understand correctly the *reconstruction* as a two-step process, which takes from a graph to a branched standard spine and then to a 3-manifold with extra structure.

We start by defining the combinatorial objects of our realizations. Before doing this we adopt a useful convention for the description of graphs. We will be dealing in the sequel with abstract finite graphs in which every vertex has a neighbourhood with

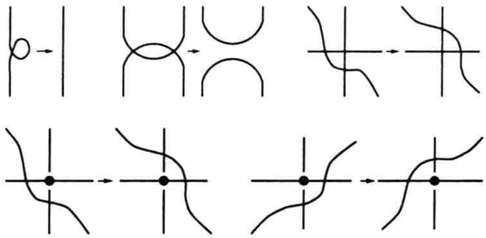


Figure 1.1: Reidemeister-type moves

a prescribed embedding in the plane. Our convention is that *we will always describe the whole graph by a generic immersion in the plane, marking by solid dots the genuine vertices, to distinguish them from the double points of the immersion.* Equivalently, we will refer to planar graphs and view them up to planar isotopy and the Reidemeister-type moves of Figure 1.1, which express the fact that the vertices which are not marked by solid dots are actually fake vertices.

Axioms for normal o-graphs. We will denote by $\mathcal{N}$ the set of finite connected quadrivalent graphs Γ with the following properties:

N1. In Γ there is at least one vertex, and every vertex has a neighbourhood which is embedded in the plane as a normal crossing, where the embedding is viewed up to planar isotopy. Moreover two opposite germs of edges incident to the vertex are marked as being over the other two, as in link projections;

N2. Each edge has a direction, and the directions of opposite edges match through the vertices.

Note that, if one chooses to view o-graphs as genuinely planar graphs with fake crossings, then one has to be careful and remember that non-marked crossings do not break edges.

The set of normal o-graphs will be used to represent manifolds with boundary, and the following subset will correspond to closed manifolds:

Axioms for closed normal o-graphs. We will denote by $\mathcal{G}$ the set of elements Γ of $\mathcal{N}$ which satisfy also:

C1. If one removes the vertices and joins the edges which are opposite to each other, the result is a unique (oriented) circuit;

C2. The trivalent graph obtained from Γ by the rules of Figure 1.2 is connected.

C3. Consider the disjoint union of oriented circuits obtained from Γ by the rules of Figure 1.3. Then the number of these circuits is exactly one more than the number of vertices of Γ.

Since the same objects will be used to encode combed closed manifolds and closed manifolds without extra structures, we set $\mathcal{G}_{\text{comb}} = \mathcal{G}$. Before proceeding we note that, given an o-graph Γ and one of the oriented circuits γ of Figure 1.3, we can associate