STRENGTH OF MATERIALS

PART I
Elementary Theory and Problems

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THIRD EDITION

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PREFACE TO THE THIRD EDITION

In the preparation of the third edition of this book a considerable number of new problems were added, and answers to many of the old problems inserted. The book was expanded by the addition of two new chapters; namely, Chapter VIII which deals with bending of beams in a plane which is not a plane of symmetry, and Chapter XII on the bending of curved bars. In Chapter VIII the notion of shear center, which is of great practical importance in the case of thin walled structures, is introduced. In Chapter XII is presented the material on curved bars which previously appeared in the second volume of this book. That material has been entirely rewritten and new material added. It is hoped with these major changes, as well as the innumerable minor changes throughout the entire text, that the volume will be not only more complete, but also more satisfactory as a textbook in elementary courses in strength of materials. The author wishes to thank Professor James M. Gere of Stanford University, who assisted in revising the volume and in reading the proofs.

S. Timoshenko

Stanford University
March 25, 1955

PREFACE TO THE SECOND EDITION

In preparing the second edition of this volume, an effort has been made to adapt the book to the teaching requirements of our engineering schools.

With this in view, a portion of the material of a more advanced character which was contained in the previous edition of this volume has been removed and will be included in the new edition of the second volume. At the same time, some portions of the book, which were only briefly discussed in the first edition, have been expanded with the intention of making the book easier to read for the beginner. For this reason, chapter II, dealing with combined stresses, has been entirely rewritten. Also, the portion of the book dealing with shearing force and bending moment diagrams has been expanded, and a considerable amount of material has been added to the discussion of deflection curves by the integration method. A discussion of column theory and its application has been included in chapter VIII, since this subject is usually required in undergraduate courses of strength of materials. Several additions have been made to chapter X dealing with the application of strain energy methods to the solution of statically indetermined problems. In various parts of the book there are many new problems which may be useful for class and home work.

Several changes in the notations have been made to conform to the requirements of American Standard Symbols for Mechanics of Solid Bodies recently adopted by The American Society of Mechanical Engineers.

It is hoped that with the changes made the book will be found more satisfactory for teaching the undergraduate course of strength of materials and that it will furnish a better foundation for the study of the more advanced material discussed in the second volume.

S. Timoshenko

Palo Alto, California
June 13, 1940

PREFACE TO THE FIRST EDITION

At the present time, a decided change is taking place in the attitude of designers towards the application of analytical methods in the solution of engineering problems. Design is no longer based principally upon empirical formulas. The importance of analytical methods combined with laboratory experiments in the solution of technical problems is becoming generally accepted.

Types of machines and structures are changing very rapidly, especially in the new fields of industry, and usually time does not permit the accumulation of the necessary empirical data. The size and cost of structures are constantly increasing, which consequently creates a severe demand for greater reliability in structures. The economical factor in design under the present conditions of competition is becoming of growing importance. The construction must be sufficiently strong and reliable, and yet it must be designed with the greatest possible saving in material. Under such conditions, the problem of a designer becomes extremely difficult. Reduction in weight involves an increase in working stresses, which can be safely allowed only on a basis of careful analysis of stress distribution in the structure and experimental investigation of the mechanical properties of the materials employed.

It is the aim of this book to present problems such that the student's attention will be focussed on the practical applications of the subject. If this is attained, and results, in some measure, in increased correlation between the studies of strength of materials and engineering design, an important forward step will have been made.

The book is divided into two volumes. The first volume contains principally material which is usually covered in required courses of strength of materials in our engineering schools. The more advanced portions of the subject are of interest chiefly to graduate students and research engineers, and are incorporated in the second volume of the book. This contains also the new developments of practical importance in the field of strength of materials.

In writing the first volume of strength of materials, attention was given to simplifying all derivations as much as possible so that a student with the usual preparation in mathematics will be able to read it without difficulty. For example, in deriving the theory of the deflection curve, the *area moment method* was extensively used. In this manner, a considerable simplification was made in deriving the deflections of beams for various loading and supporting conditions. In discussing statically indeterminate systems, the *method of superposition* was applied, which proves very useful in treating such problems as continuous beams and frames. For explaining combined stresses and deriving principal stresses, use was made of the *Mohr's circle*, which represents a substantial simplification in the presentation of this portion of the theory.

Using these methods of simplifying the presentation, the author was able to condense the material and to discuss some problems of a more advanced character. For example, in discussing torsion, the twist of rectangular bars and of rolled sections, such as angles, channels, and I beams, is considered. The deformation and stress in helical springs are discussed in detail. In the theory of bending, the case of non-symmetrical cross sections is discussed, the *center of twist* is defined and explained, and the effect of shearing force on the deflection of beams is considered. The general theory of the bending of beams, the materials of which do not follow Hooke's law, is given and is applied in the bending of beams beyond the yielding point. The bending of reinforced concrete beams is given consideration. In discussing combinations of direct and bending stress, the effect of deflections on the bending moment is considered, and the limitation of the method of superposition is explained. In treating combined bending and torsion, the cases of rectangular and elliptical cross sections are discussed, and applications in the design of crankshafts are given. Considerable space in the book is devoted to methods for solving elasticity problems based on the consideration of the strain energy of elastic bodies. These methods are applied in discussing statically indeterminate systems. The stresses produced by impact are also discussed. All these problems of a more advanced character are printed in small type, and may be omitted during the first reading of the book.

The book is illustrated with a number of problems to which solutions are presented. In many cases, the problems are chosen so as to widen the field covered by the text and to illustrate the application of the theory in the solution of design problems. It is hoped that these problems will be of interest for teaching purposes, and also useful for designers.

The author takes this opportunity of thanking his friends who have assisted him by suggestions, reading of manuscript and proofs, particularly Messrs. W. M. Coates and L. H. Donnell, teachers of mathematics and mechanics in the Engineering College of the University of Michigan, and Mr. F. L. Everett of the Department of Engineering Research of the University of Michigan. He is indebted also to Mr. F. C. Wilharm for the preparation of drawings, to Mrs. E. D. Webster for the typing of the manuscript, and to the Van Nostrand Company for its care in the publication of the book.

S. Timoshenko

Ann Arbor, Michigan
May 1, 1930

NOTATIONS

α.......... Angle, coefficient of thermal expansion, numerical coefficient
β.......... Angle, numerical coefficient
γ.......... Shearing strain, weight per unit volume
Δ.......... Unit volume expansion, distance
δ.......... Total elongation, total deflection, distance
ϵ.......... Unit strain
ϵ_x, ϵ_y, ϵ_z..... Unit strains in x, y and z directions
θ.......... Angle, angle of twist per unit length of shaft
μ.......... Poisson's ratio
σ.......... Unit normal stress
σ_1, σ_2....... Principal stresses
σ_n.......... Unit normal stress on plane perpendicular to the direction n
σ_x, σ_y, σ_z..... Unit normal stresses on planes perpendicular to the x, y and z axes
σ_U.......... Ultimate stress
σ_W.......... Working stress
$\sigma_{Y.P.}$ .... Yield point stress
τ.......... Unit shear stress
τ_{xy}, τ_{yz}, τ_{zx}.. Unit shear stresses on planes perpendicular to the x, y and z axes, and parallel to the y, z and x axes
τ_W.......... Working stress in shear
$\tau_{Y.P.}$ .... Yield point stress in shear
φ.......... Angle
ω.......... Angular velocity
A.......... Cross sectional area
a, b, c, d..... Distances
C.......... Torsional rigidity, constant of integration
D, d........ Diameters
E.......... Modulus of elasticity

G	Modulus of elasticity in shear
H	Horizontal force, horsepower
h	Height, thickness
I_p	Polar moment of inertia of a plane area
I_y, I_z	Moments of inertia of a plane area with respect to the y and z axes
I_{yz}	Product of inertia of a plane area with respect to the y and z axes
K	Bulk modulus of elasticity
k	Spring constant, numerical factor
k_y, k_z	Radii of gyration of a plane area with respect to the y and z axes
l	Length, span
M	Bending moment
M_t	Torque
n	Factor of safety, revolutions per minute, normal to a plane
P, Q	Concentrated forces
p	Pressure, steel ratio for reinforced concrete beams
q	Load per unit length, pressure
R	Reaction, force, radius
r	Radius, radius of curvature
S	Axial force in a bar
t	Temperature, thickness
U	Strain energy
u	Deflection, distance
V	Volume, shearing force
v	Velocity, deflection, distance
W	Total load, weight
w	Weight per unit length, strain energy per unit volume
w_1	Strain energy per unit weight
X, Y, Z	Axial forces in bars, unknown reactions
x, y, z	Rectangular coordinates
Z	Section modulus

CONTENTS

PART I

CHAPTER I

TENSION AND COMPRESSION WITHIN THE ELASTIC LIMIT

1. Elasticity.—A material body consists of small particles, or molecules, between which forces are acting. These molecular forces resist the change in the shape of the body which external forces tend to produce. Under the action of external forces the particles of the body are displaced and the displacements continue until equilibrium is established between the external and internal forces. The body is then in a *state of strain*. During deformation the external forces acting upon the body do work, and this work is transformed completely or partially into *potential energy of strain*. A watch spring is an example of such an accumulation of potential energy in a strained body. If the forces which produced the deformation of the body are now gradually diminished, the body will return wholly or partly to its original shape and during this reversed deformation the potential energy of strain which was accumulated in the body may be recovered in the form of external work.

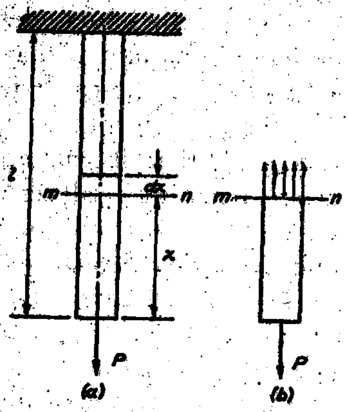

FIG. 1.

Consider, for instance, a prismatic bar loaded at the end as shown in Fig. 1.[1] Under the action of this load a certain elongation of the bar will take place. The point of application of the load will then move in a downward direction and positive work will be done by the load during this

[1] It is assumed that the load is acting along the axis of the bar, i.e., along the line passing through the centroids of the cross sections.

motion. When the load is diminished, the elongation of the bar diminishes also, the loaded end of the bar moves upward and the potential energy of strain will be transformed into the work of moving the load in the upward direction.

The property by which a body returns to its original shape after removal of the load is called *elasticity*. The body is *perfectly elastic* if it recovers its original shape completely after unloading; it is *partially elastic* if the deformation produced by the external forces does not disappear completely after unloading. In the case of a perfectly elastic body the work done by the external forces during deformation is completely transformed into potential energy of strain.[2] In the case of a partially elastic body, part of the work done by the external forces during deformation is dissipated in the form of heat, which is developed in the body during the non-elastic deformation. Experiments show that such structural materials as steel, wood and stone may be considered as perfectly elastic within certain limits, depending upon the properties of the material. Assuming that the external forces acting upon the structure are known, it is a fundamental problem for the designer to establish the proportions of the members of the structure such that it will approach the condition of a perfectly elastic body under all service conditions. Only in this way can we be certain of continuous reliable service from the structure and avoid any *permanent set* in its members.

2. Hooke's Law.—By direct experiment with the extension of prismatic bars (Fig. 1) it has been established for many structural materials that within certain limits the elongation of the bar is proportional to the tensile force. This simple linear relationship between the force and the elongation which it produces was first formulated by the English scientist Robert Hooke[3] in 1678 and bears his name. Using the notation:

P = force producing extension of bar,
l = length of bar,

[2] The small temperature changes which usually accompany elastic deformation and the corresponding heat exchange with the surroundings are neglected in this consideration (see Part II).

[3] Robert Hooke, *De Potentia restitutiva*, London, 1678.

A = cross-sectional area of bar,
δ = total elongation of bar,
E = elastic constant of the material, called the *Modulus of Elasticity*,

Hooke's experimental law may be given by the following equation:

$$\delta = \frac{Pl}{AE}. \tag{1}$$

The elongation of the bar is proportional to the tensile force and to the length of the bar and inversely proportional to the cross-sectional area and to the modulus of elasticity. In making tensile tests precautions are usually taken to ensure central application of the tensile force. In Fig. 2 is shown a method of fixing the ends of a circular tensile test specimen in a tensile test machine. In this manner any bending of the bar will be prevented. Excluding from consideration those portions of the bar in the vicinity of the applied forces,[4] it may be assumed that during tension all longitudinal fibers of the prismatic bar have the same elongation and that cross sections of the bar originally plane and perpendicular to the axis of the bar remain so after extension.

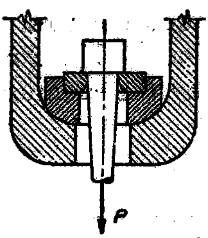

FIG. 2.

In discussing the magnitude of internal forces let us imagine the bar cut into two parts by a cross section *mn* and let us consider the equilibrium of the lower portion of the bar (Fig. 1*b*). At the lower end of this portion the tensile force P is applied.

On the upper end the forces represent the action of the particles of the upper portion of the strained bar on the particles of the lower portion. These forces are continuously distributed over the cross section. Familiar examples of such a continuous distribution of forces over a surface are hydrostatic pressure and steam pressure. In handling such continuously distributed forces *the intensity of force*, i.e., the force per unit area, is of great importance. In the present case of axial tension, in

[4] The more complicated stress distribution near the points of application of the forces is discussed in Part II.

which all fibers have the same elongation, the distribution of forces over the cross section *mn* will be *uniform.* The resultant of these forces will pass through the centroid of the cross section and will act along the axis of the bar. Taking into account that the sum of these forces, from the condition of equilibrium (Fig. 1*b*), must be equal to P and denoting the force per unit of cross-sectional area by σ, we obtain

$$\sigma = \frac{P}{A}. \tag{2}$$

This force per unit area is called *unit tensile stress* or simply *stress.* In this book, force is measured in pounds and area in square inches, so that stress is measured in pounds per square inch. The elongation of the bar per unit length is determined by the equation

$$\epsilon = \frac{\delta}{l} \tag{3}$$

and is called the *unit elongation* or the *tensile strain.* Using eqs. (1), (2) and (3), Hooke's law may also be written in the following form:

$$E = \frac{\sigma}{\epsilon} \tag{4}$$

and we see that *modulus of elasticity is equal to unit stress divided by unit strain* and may be easily calculated provided the stress and corresponding unit elongation are found from a tensile test. The unit elongation ϵ is a pure number representing the ratio of two lengths (see eq. 3); therefore, from eq. (4) it may be concluded that modulus of elasticity is measured in the same units as stress σ, i.e., in pounds per square inch. Average values of the modulus E for several materials are given in the first column of Table 1.[5]

Eqs. (1)–(4) may be used also in the case of compression of prismatic bars. Then, δ denotes the total longitudinal contraction, ϵ the *compressive strain* and σ the *compressive stress.*

[5] More details on the mechanical properties of materials are given in Part II.

TABLE 1: MECHANICAL PROPERTIES OF MATERIALS

Material	E lb/in.^{2}	Yield Point lb/in.^{2}	Ultimate Strength lb/in.^{2}
Structural carbon steel 0.15 to 0.25% carbon	30×10^6	30×10^3–40×10^3	55×10^3–65×10^3
Nickel steel 3 to 3.5% nickel	29×10^6	40×10^3–50×10^3	78×10^3–100×10^3
Duraluminum	10×10^6	35×10^3–45×10^3	54×10^3–65×10^3
Copper, cold rolled	16×10^6		28×10^3–40×10^3
Glass	10×10^6		3.5×10^3
Pine, with the grain	1.5×10^6		8×10^3–20×10^3
Concrete, in compression	4×10^6		3×10^3

For most structural materials the modulus of elasticity for compression is the same as for tension. In calculations, tensile stress and tensile strain are considered as positive, and compressive stress and strain as negative.

Problems

1. Determine the total elongation of a steel bar 25 in. long, if the tensile stress is equal to 15×10^3 lb per sq in.

Answer. $\delta = \frac{1}{80}$ in.

2. Determine the tensile force on a cylindrical steel bar of 1 in. diameter, if the unit elongation is equal to 0.7×10^{-3}.

Solution. The tensile stress in the bar, from eq. (4), is

$$\sigma = \epsilon \cdot E = 21 \times 10^3 \text{ lb per sq in.}$$

The tensile force, from eq. (2), is

$$P = \sigma \cdot A = 21 \times 10^3 \times \frac{\pi}{4} = 16{,}500 \text{ lb.}$$

3. What is the ratio of the moduli of elasticity of the materials of two bars of the same size if under the action of equal tensile forces the unit elongations of the bars are in the ratio $1:\frac{15}{8}$? Determine these elongations if one of the bars is of steel, the other of copper, and the tensile stress is 10,000 lb per sq in.