THE

LOGIC OF CHANCE

AN ESSAY

ON THE FOUNDATIONS AND PROVINCE OF THE THEORY OF PROBABILITY,

WITH ESPECIAL REFERENCE TO ITS LOGICAL BEARINGS AND ITS APPLICATION TO

MORAL AND SOCIAL SCIENCE, AND TO STATISTICS,

BY

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"So careful of the type she seems
So careless of the single life."

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PREFACE TO FIRST EDITION.

ANY work on Probability by a Cambridge man will be so likely to have its scope and its general treatment of the subject prejudged, that it may be well to state at the outset that the following Essay is in no sense mathematical. Not only, to quote a common but often delusive assurance, will 'no knowledge of mathematics beyond the simple rules of Arithmetic' be required to understand these pages, but it is not intended that any such knowledge should be acquired by the process of reading them. Of the two or three occasions on which algebraical formulæ occur they will not be found to form any essential part of the text.

The science of Probability occupies at present a somewhat anomalous position. It is impossible, I think, not to observe in it some of the marks and consequent disadvantages of a *sectional* study. By a small body of ardent students it has been cultivated with great assiduity, and the results they have obtained will always be reckoned among the most extraordinary products of mathematical genius. But by the general body of thinking men its principles seem to be regarded with indifference or suspicion. Such persons may admire the ingenuity displayed, and be struck with the profundity of many of the calculations, but there seems to

them, if I may so express it, an *unreality* about the whole treatment of the subject. To many persons the mention of Probability suggests little else than the notion of a set of rules, very ingenious and profound rules no doubt, with which mathematicians amuse themselves by setting and solving puzzles.

It must be admitted that some ground has been given for such an opinion. The examples commonly selected by writers on the subject, though very well adapted to illustrate its rules, are for the most part of a special and peculiar character, such as those relating to dice and cards. When they have searched for illustrations drawn from the practical business of life, they have very generally, but unfortunately, hit upon just the sort of instances which, as I shall endeavour to show hereafter, are among the very worst that could be chosen for the purpose. It is scarcely possible for any unprejudiced person to read what has been written about the credibility of witnesses by eminent writers, without his experiencing an invincible distrust of the principles which they adopt. To say that the rules of evidence sometimes given by such writers are broken in practice, would scarcely be correct; for the rules are of such a kind as generally to defy any attempt to appeal to them in practice.

This supposed want of harmony between Probability and other branches of Philosophy is perfectly erroneous. It arises from the belief that Probability is a branch of mathematics trying to intrude itself on to ground which does not altogether belong to it. I shall endeavour to show that this belief is unfounded. To answer correctly the sort of questions to which the science introduces us does generally demand some knowledge of mathematics, often a great knowledge, but the discussion of the fundamental principles on which the rules are based does not necessarily require any such

qualification. Questions might arise in other sciences, in Geology, for example, which could only be answered by the aid of arithmetical calculations. In such a case any one would admit that the arithmetic was extraneous and accidental. However many questions of this kind there might be here, those persons who do not care to work out special results for themselves might still have an accurate knowledge of the principles of the science, and even considerable acquaintance with the details of it. The same holds true in Probability; its connection with mathematics, though certainly far closer than that of most other sciences, is still of much the same kind. It is principally when we wish to work out results for ourselves that mathematical knowledge is required; without such knowledge the student may still have a firm grasp of the principles and even see his way to many of the derivative results.

The opinion that Probability, instead of being a branch of the general science of evidence which happens to make much use of mathematics, *is* a portion of mathematics, erroneous as it is, has yet been very disadvantageous to the science in several ways. Students of Philosophy in general have thence conceived a prejudice against Probability, which has for the most part deterred them from examining it. As soon as a subject comes to be considered 'mathematical' its claims seem generally, by the mass of readers, to be either on the one hand scouted or at least courteously rejected, or on the other to be blindly accepted with all their assumed consequences. Of impartial and liberal criticism it obtains little or nothing.

The consequences of this state of things have been, I think, disastrous to the students themselves of Probability. No science can safely be abandoned entirely to its own devotees. Its details of course can only be studied by those who

make it their special occupation, but its general principles are sure to be cramped if it is not exposed occasionally to the free criticism of those whose main culture has been of a more general character. Probability has been very much abandoned to mathematicians, who as mathematicians have generally been unwilling to treat it thoroughly. They have worked out its results, it is true, with wonderful acuteness, and the greatest ingenuity has been shown in solving various problems that arose, and deducing subordinate rules. And this was all that they could in fairness be expected to do. Any subject which has been discussed by such men as Laplace and Poisson, and on which they have exhausted all their powers of analysis, could not fail to be profoundly treated, so far as it fell within their province. But from this province the real principles of the science have generally been excluded, or so meagrely discussed that they had better have been omitted altogether. Treating the subject as mathematicians such writers have naturally taken it up at the point where their mathematics would best come into play, and that of course has not been at the foundations. In the works of most writers upon the subject we should search in vain for anything like a critical discussion of the fundamental principles upon which its rules rest, the class of enquiries to which it is most properly applicable, or the relation it bears to Logic and the general rules of inductive evidence.

This want of precision as to ultimate principles is perfectly compatible here, as it is in the departments of Morals and Politics, with a general agreement on processes and results. But it is, to say the least, unphilosophical, and denotes a state of things in which positive error is always liable to arise whenever the process of controversy forces us to appeal to the foundations of the science.

With regard to the remarks in the last few paragraphs, prominent exceptions must be made in the case of two recent works at least[1]. The first of these is Professor de Morgan's *Formal Logic*. He has there given an investigation into the foundations of Probability as conceived by him, and nothing can be more complete and precise than his statement of principles, and his deductions from them. If I could at all agree with these principles there would have been no necessity for the following essay, as I could not hope to add anything to their foundation, and should be far indeed from rivalling his lucid statement of them. But in his scheme Probability is regarded very much from the Conceptualist point of view; as stated in the preface, he considers that Probability is concerned with formal inferences in which the premises are entertained with a conviction short of absolute certainty. With this view I cannot agree. As I have entered into criticism of some points of his scheme in one of the following chapters, and shall have occasion frequently to refer to his work, I need say no more about it here. The other work to which I refer is the profound *Laws of Thought* of the late Professor Boole, to which somewhat similar remarks may in part be applied. Owing however to his peculiar treatment of the subject, I have scarcely anywhere come into contact with any of his expressed opinions.

The view of the province of Probability adopted in this Essay differs so radically from that of most other writers on the subject, and especially from that of those just referred to, that I have thought it better, as regards details, to avoid all criticism of the opinions of others, except where conflict was

[1] I am here speaking, of course, of those only who have expressly treated of the foundations of the science. Mr Todhunter's admirable work on the *History of the Theory of Probability* being, as the name denotes, mainly historical, such enquiries have not directly fallen within his province.

unavoidable. With regard to that radical difference itself Bacon's remark applies, behind which I must shelter myself from any change of presumption.—"Quod ad universalem istam reprehensionem attinet, certissimum vere est rem reputanti, eam et magis probabilem esse et magis modestam, quam si facta fuisset ex parte."

Almost the only writer who seems to me to have expressed a just view of the nature and foundation of the rules of Probability is Mr Mill, in his *System of Logic*[1]. His treatment of the subject is however very brief, and a considerable portion of the space which he has devoted to it is occupied by the discussion of one or two special examples. There are moreover some errors, as it seems to me, in what he has written, which will be referred to in some of the following chapters.

The reference to the work just mentioned will serve to convey a general idea of the view of Probability adopted in this Essay. With what may be called the Material view of Logic as opposed to the Formal or Conceptualist,—with that which regards it as taking cognisance of laws of things and not of the laws of our own minds in thinking about things,—I am in entire accordance. Of the province of Logic, regarded from this point of view, and under its widest aspect, Probability may, in my opinion, be considered to be a portion. The principal objects of this Essay are to ascertain how great a portion it comprises, where we are to draw the boundary between it and the contiguous branches of the general science

[1] This remark, and that at the commencement of the last paragraph, having been misunderstood, I ought to say that the only sense in which originality is claimed for this Essay is in the thorough working out of the Material view of Logic as applied to Probability. I have given a pretty full discussion of the general principles of this view in the tenth chapter, and have there pointed out some of the peculiarities to which it leads.

of evidence, what are the ultimate foundations upon which its rules rest, what the nature of the evidence they are capable of affording, and to what class of subjects they may most fitly be applied. That the science of Probability, on this view of it, contains something more important than the results of a system of mathematical assumptions, is obvious. I am convinced moreover that it can and ought to be rendered both interesting and intelligible to ordinary readers who have any taste for philosophy. In other words, if the large and growing body of readers who can find pleasure in the study of books like Mill's *Logic* and Whewell's *Inductive Sciences*, turn with aversion from a work on Probability, the cause in the latter case must lie either in the view of the subject or in the manner and style of the book.

I take this opportunity of thanking several friends, amongst whom I must especially mention Mr Todhunter, of St John's College, and Mr H. Sidgwick, of Trinity College, for the trouble they have kindly taken in looking over the proof-sheets, whilst this work was passing through the Press. To the former in particular my thanks are due for thus adding to the obligations which I, as an old pupil, already owed him, by taking an amount of trouble, in making suggestions and corrections for the benefit of another, which few would care to take for anything but a work of their own. His extensive knowledge of the subject, and his extremely accurate judgment, render the service he has thus afforded me of the greatest possible value.

Gonville and Caius College,
September, 1866.

PREFACE TO SECOND EDITION.

THE principal reason for designating this volume a second edition consists in the fact that the greater portion of what may be termed the first edition is incorporated into it. Besides various omissions (principally where the former treatment has since seemed to me needlessly prolix), I have added new matter, not much inferior in amount to the whole of the original work. In addition, moreover, to these alterations in the matter, the general arrangement of the subject as regards the successive chapters has been completely changed; the former arrangement having been (as it now seems to me) justly objected to as deficient and awkward in method.

After saying this, it ought to be explained whether any change of general view or results will be found in the present treatment.

The general view of Probability adopted is quite unchanged, further reading and reflection having only confirmed me in the conviction that this is the soundest and most fruitful way of regarding the subject. It is the more necessary to say this, as to a cursory reader it might seem

otherwise; owing to my having endeavoured to avoid the needlessly polemical tone which, as is often the case with those who are making their first essay in writing upon any subject, was doubtless too prominent in the former edition. I have not thought it necessary, of course, except in one or two cases, to indicate points of detail which it has seemed necessary to correct.

A number of new discussions have been introduced upon topics which were but little or not at all treated before. The principal of these refer to the nature and physical origin of Laws of Error (Ch. II.); the general view of Logic, and consequently of Probability, termed the Material view, adopted here (Ch. X.); a brief history and criticism of the various opinions held on the subject of Modality (Ch. XII.); the logical principles underlying the method of Least Squares (Ch. XIII.); and the practices of Insurance and Gambling, so far as the principles involved in them are concerned (Ch. XV.). The Chapter on the Credibility of Extraordinary Stories is also mainly new; this was the portion of the former work which has since seemed to me the least satisfactory, but owing to the extreme intricacy of the subject I am far from feeling thoroughly satisfied with it even now.

I have again to thank several friends for the assistance they have so kindly afforded. Amongst these I must prominently mention Mr C. J. Monro, late fellow of Trinity. It is only the truth to say that I have derived more assistance from his suggestions and criticisms than has been consciously obtained from all other external sources together. Much of this

criticism has been given privately in letters, and notes on the proof-sheets; but one of the most elaborate of his discussions of the subject was communicated to the Cambridge Philosophical Society some years ago; as it was not published, however, I am unfortunately unable to refer the reader to it. I ought to add that he is not in any way committed to any of my opinions upon the subject, from some of which in fact he more or less dissents. I am also much indebted to Mr J. W. L. Glaisher, also of Trinity College, for many hints and references to various publications upon the subject of Least Squares, and for careful criticism (given in the midst of much other labour) of the chapter in which that subject is treated.

I need not add that, like every one else who has had to discuss the subject of Probability during the last ten years, I have made constant use of Mr Todhunter's History.

I may take this opportunity of adding that a considerable portion of the tenth chapter has recently appeared in the January number of *Mind*, and that the substance of several chapters, especially in the more logical parts, has formed part of my ordinary lectures in Cambridge; the foundation and logical treatment of Probability being now expressly included in the Schedule of Subjects for the Moral Sciences Tripos.

March, 1876.

PREFACE TO THIRD EDITION

THE present edition has been revised throughout, and in fact rewritten. Three chapters are new, viz. the fifth (On the conception of Randomness) and the eighteenth and nineteenth (On the nature, and on the employment, of Averages). The eighth, tenth, eleventh, and fifteenth chapters have been recast, and much new matter added, and numerous alterations made in the remaining portions[1]. On the other hand three chapters of the last edition have been nearly or entirely omitted.

These alterations do not imply any appreciable change of view on my part as to the foundations and province of Probability. Some of them are of course due to the necessary changes involved in the attempt to write up to date upon a subject which has not been stationary during the last eleven years. For instance the greatly increased interest now taken in what may be called the Theory of Statistics has rendered it desirable to go much more fully into the Nature and treatment of Laws of Error. The omissions are mainly

[1] I have indicated the new chapters and sections by printing them in italics in the Table of Contents.

due to a wish to avoid increasing the bulk of this volume more than is actually necessary, and to a feeling that the portions treating specially of Inductive Logic (which occupied some space in the last edition) would be more suitable to a regular work on that subject. I am at present engaged on such a work.

The publications which I have had occasion to notice have mostly appeared in various scientific journals. The principal authors of these have been Mr F. Galton and Mr F. Y. Edgeworth: to the latter of whom I am also personally much obliged for many discussions, oral and written, and for his kindness in looking through the proof-sheets. His published articles are too numerous for separate mention here, but I may say generally, in addition to the obligations specially noticed, that I have been considerably indebted to them in writing the last two chapters. Two authors of works of a somewhat more substantial character, viz. Prof. Lexis and Von Kries, only came under my notice unfortunately after this work was already in the printer's hands. With the latter of these authors I find myself in closer agreement than with most others, in respect of his general conception and treatment of Probability.

December, 1887.

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[1] Chapters and sections which are nearly or entirely new are printed in *italics*.

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