Computer and Optically Formed Holographic Optics

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Ivan Cindrich
Sing H. Lee
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Volume 1211

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COMPUTER AND OPTICALLY FORMED HOLOGRAPHIC OPTICS

Volume 1211

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INTRODUCTION

Holographic optical elements, or diffractive optics, provide the optical system designer with an expanded set of approaches to system design. In this technology a diffractive element is constructed in a thin layer of material or surface of a substrate. The underlying nature of the construction process involves formation of fine spatial variations in the transmittance or optical path length of the material. The diffractive element may be constructed on a dedicated substrate surface or on the surface of other devices, and in some configurations a superposition of more than one element in a single thin layer can be realized.

This collection of papers on Computer and Optically Formed Holographic Optics describes design and development activities being carried out at many organizations both here and abroad. The papers cover such topics as theory, design, and performance analysis methods, materials and fabrication techniques, and applied design examples.

The papers on Theory and Design Methods encompass design optimization and performance bounds determination, encoding methods and data formatting for the recording of computer-generated elements, fundamental issues and design choices in the kinoform class of holographic optics (which includes binary optics), computer-aided design methods, and performance analysis approaches.

Fabrication Techniques and Materials papers cover many important aspects of the practical realization of diffractive optics. Fabrication methods, the choice of materials and their properties, methods for construction of optical elements (such as recording of optically formed wavefronts, computer-based point-wise recording approaches, and lithographic mask pattern recording), design parameter versus performance parameter tradeoff analyses, and modeling and analysis for performance prediction are addressed.

The papers in Sessions 3 and 4 on Applications cover the development of diffractive optics for a variety of uses. Wide field-of-view optics for airborne laser communication systems, beamforming for laser-diode arrays, head-up display beam combiners, filtering and beam positioning for the near infrared, real-time formation of optical elements using two-dimensional spatial light modulators of several types, special purpose elements for use in optical interconnects, neural networks, object recognition correlators, and wide-angle beam steering are all topics included in these sessions.

COMPUTER AND OPTICALLY FORMED HOLOGRAPHIC OPTICS

Volume 1211

The papers for the 1990 conference give new insight into and ideas for design, fabrication, and applications of this technology. Regarding future conference considerations, our 1991 conference will include some changes worth noting at this time. It will be held in San Diego, California, at the SPIE mid-year symposium, and preliminary planning includes the possibility of a display room during the symposium that will be dedicated to examples of diffractive optics components and systems. We invite and encourage interested workers and organizations to submit a letter of interest to SPIE, noting the type of diffractive element that would be submitted and necessary support equipment.

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COMPUTER AND OPTICALLY FORMED HOLOGRAPHIC OPTICS

Volume 1211

CONTENTS

(continued)

COMPUTER AND OPTICALLY FORMED HOLOGRAPHIC OPTICS

Volume 1211

COMPUTER AND OPTICALLY FORMED HOLOGRAPHIC OPTICS

Volume 1211

SESSION 1

Theory and Design Methods

Chairs
J. Allen Cox
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Characteristics of diffractive optical elements/digital holograms

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ABSTRACT

A model to design diffractive optical elements is described. It allows a prediction of the theoretical limit of the diffraction efficiency and the development and characterisation of calculation methods.

1. INTRODUCTION

In diffractive optics the calculation, fabrication, and application of diffractive optical elements (DOE) are investigated. Considerations in fields like binary optics or digital and optical holography are closely related to research in diffractive optics. In this paper the diffraction efficiency (η) and signal–to–noise ratio (SNR) of the elements are considered. Both characteristics are of concern in applications of diffractive optical elements.

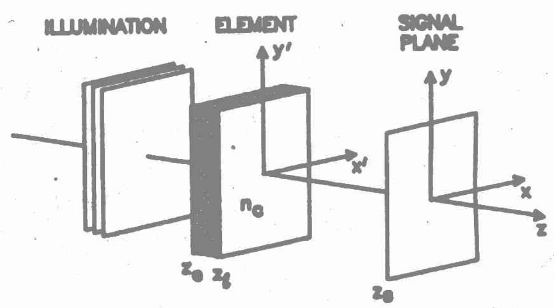


Fig. 1. Illumination of a DOE with notations used.

Figure 1 illustrates a typical DOE situation. The wave leaving the element is $F(x')$. In the plane z_s the diffracted wave is $f(x)$. This wave defines the function of the element and we call it the signal. In the terminology of holography one can say that the information of the signal is stored within the DOE.

In order to calculate a DOE one has to answer the question: How can one find, starting from a specified signal, the corresponding complex diffraction index of the element? Using a formal description this inverse problem can be formulated by

$$n_c(x',z) = \mathcal{T}^{-1}\ \mathcal{W}^{-1} f(x). \tag{1}$$

The operator $\mathcal{W}$ describes the wave propagation from plane z_ℓ to z_s, and $\mathcal{T}$ the influence of the element on the illuminating wave.

Usually paraxial optics is employed to find the operators. Then, the wave propagation can be described by the important Fresnel or Fourier transform case. The operator $\mathcal{T}$ can be treated with the optical path method and the result is the amplitude and phase transmission of the element.[1] Thus, $\mathcal{T}^{-1}$ and $\mathcal{W}^{-1}$ are known and the calculation problem seems to be solved. But one has to satisfy constraints concerning n_c and therefore $F(x')$. One is interested in elements without absorption, i.e., $|F(x')| = 1$ and further, most fabrication techniques require a quantization of the phase transmission.

Because in general the wave $F(x')$ resulting from the inverse wave propagation does not satisfy the constraints one has to consider transformation methods (indicated by the operator $\mathcal{O}$).

$$G(x') = \mathcal{O} F(x'), \tag{2}$$

where $G(x')$ satisfies the constraints. Decomposing $G(x')$ into

$$G(x') = F(x') + C(x'), \tag{3}$$

the diffracted wave $g(x)$ is given by

$$g(x) = f(x) + c(x). \tag{4}$$

Thus, the operator $\mathcal{O}$ causes the noise $c(x)$ which in general leads to a signal error.

The requirement for an errorfree signal located in a signal window F is

$$g(x) = \alpha\, f(x), \quad x \in \mathsf{F} \tag{5a}$$

for complex valued signals and

$$|g(x)|^2 = \alpha^2\, |f(x)|^2, \quad x \in \mathsf{F} \tag{5b}$$

for intensity signals. These equations do not define $g(x)$ completely. Thus, there are some freedoms in the signal plane:

1. the freedom of the amplitude outside of F (AF), i.e.

$$c(x) = 0, \quad x \in \mathsf{F}, \tag{6a}$$

2. the freedom of the scale factor α within F (SF), i.e.

$$g(x) = \alpha\, f(x), \quad x \in \mathsf{F}, \tag{6b}$$

3. the freedom of the phase within F (PF) for intensity signals, i.e.

$$|f(x) + c(x)|^2 = |f(x)|^2, \quad x \in \mathsf{F}. \tag{6c}$$

These freedoms can be utilised to satisfy the constraints. What kind of freedom is available in different applications?

The AF has only to be restricted when a convolution with the signal is performed, e.g., in filter applications and in case of continuous signals (convolution with a sinc–function). Then it is necessary to introduce a gap between signal and noise to have space for the convolution. The SF is always available. It is of fundamental importance to achieve high diffraction efficiency. The PF is available for intensity signals, but it is necessary to distinguish between discrete and continuous signals. In the latter case the PF is restricted when one want to avoid speckle.[2,3]

The basic constraint in diffractive optics is $|G(x')| = 1$. In what follows it is discussed how to utilise the freedoms to satisfy this constraint. We restrict ourselves to Fourier type elements. Thus, the coordinate x' is replaced by the spatial frequency u and $\mathcal{W}$ is the Fourier transform $\mathcal{F}$. The generalisation to Fresnel type elements is straightforward since the tranforms only differ by a quadratic phase factor, disregarding a quadrativ signal phase.

2. TREATMENT OF CONSTRAINT $|G(u)| = 1$

In order to treat the constraint $|G(u)| = 1$ it is helpful to consider the energy of $G(u)$ and the normalised $F(u)$. Using Parseval's theorem one obtains

$$< |F(u)|^2 > = < |f(x)|^2 > = \eta_0 < 1. \tag{7}$$

< > indicates the average value and η_0 the energy of the signal. Moreover, η_0 is the diffraction efficiency of the ROACH of $F(\mathrm{u})$, that is a DOE which controls amplitude and phase.[4] The total energy of $G(\mathrm{u})$ and $g(\mathrm{x})$ is

$$< |G(\mathrm{u})|^2> = < |g(\mathrm{x})|^2> = 1. \tag{8}$$

The energy within the signal window, i.e the diffraction efficiency of the DOE, is

$$< |g(\mathrm{x})|^2>_{F} = \eta. \tag{9}$$

We are interested in the theoretical limit of η, when a signal is specified, and in the dependence of the actual calculation method on η. In order to answer these questions it is helpful to define the projection method.

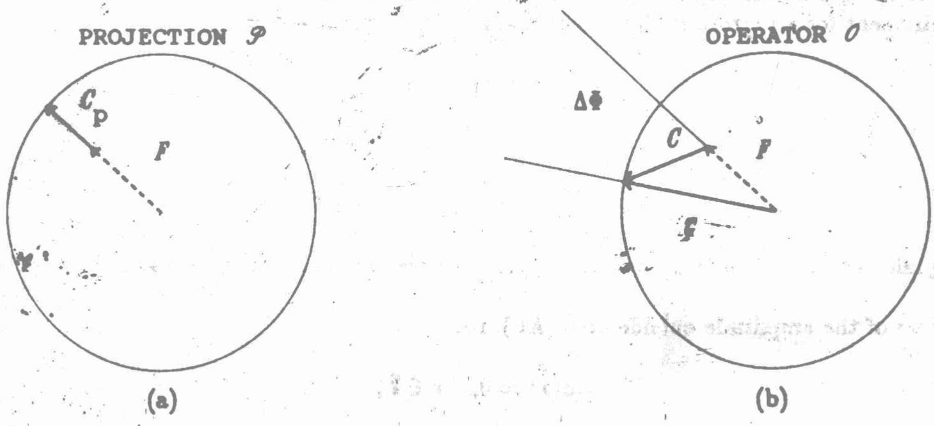


Fig. 2. Comparison of projection (a) with general methods (b) to satisfy the constraint of constant modulus.

The projection method introduces the term $C(\mathrm{u})$ with minimal energy. Figure 2(a) illustrates how the constraint of constant modulus can be satisfied by the projection method. It does not change the phase of $F(\mathrm{u})$ and results in

$$G(\mathrm{u}) = \mathcal{P}F(\mathrm{u}) = \exp\left[i\Phi(\mathrm{u})\right], \tag{10}$$

where $\arg\left[F(\mathrm{u})\right] = \Phi(\mathrm{u})$ and $\mathcal{P}$ indicates the projection operator, i.e. a special choice of $\mathcal{O}$. Figure 3 shows the effect of the projection method on a one dimensional testsignal with a constant phase. A drastically disturbed signal is the result. The freedoms are not utilized. Obviously, it is necessary to use other methods.

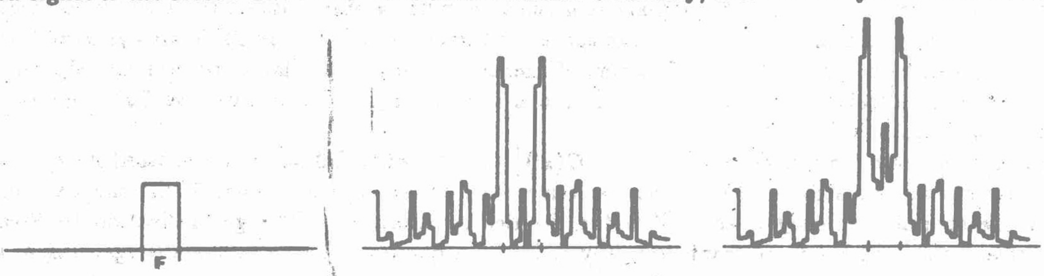


Fig. 3. Illustration of the effect of the projection method on a testsignal with a constant phase. From left to right the signal $f(\mathrm{x})$, the noise $|c(\mathrm{x})|$ and $|g(\mathrm{x})| = |f(\mathrm{x}) + c(\mathrm{x})|$ are shown.

These methods can be characterised by

$$G(\mathrm{u}) = \mathcal{O}F(\mathrm{u}) = \exp\left[i\{\Phi(\mathrm{u}) + \Delta\Phi(\mathrm{u})\}\right]. \tag{11}$$

They cause a change $\Delta\Phi(u)$ of the phase $\Phi(u)$ in the element plane, as illustrated in Fig. 2(b). To get an idea how to choose $\mathcal{O}$ and therefore how to change the phase it is important to examine what kind of freedoms can be utilized to satisfy the constraint.

2.1. Exclusive use of scale factor or phase freedom

The scale factor α is not useful to satisfy the constraint $|G(u)| = \alpha\,|F(u)| = 1$. The exclusive use of the phase freedom leads to the introduction of a phase factor in the signal plane, i.e.

$$g(x) = f(x)\exp[i\delta(x)]. \tag{12}$$

Thus, the energy in the signal plane is not changed which is in contradiction to the change in the element plane (cf. eqs. (7) and (8)). In conclusion PF or SF alone are not useful to satisfy the constraint.

2.2. Exclusive use of the amplitude freedom

Using AF alone the aim is to introduce a $C(u)$ which results in a $c(x)$ spatially separated from the signal. Thus, there is no change of the energy within the signal window, i.e.

$$< |g(x)|^2 >_F = < |f(x)|^2 > = \eta = \eta_0. \tag{13}$$

The diffraction efficiency of the DOE is equal η_0. Consideration of the energy in the whole signal plane and the Parseval's theorem leads to a necessary condition on the energy of $C(u)$, viz.

$$< |C(u)|^2 > = 1 - < |F(u)|^2 >. \tag{14}$$

All methods using AF alone introduce a $C(u)$ of this energy. It is possible to directly introduce a method which is able to satisfy the constraint given by condition (14).

One possibility to fulfil this condition is to proceed in a pointwise fashion. Then, the equation

$$|C(u)|^2 = 1 - |F(u)|^2 \tag{15a}$$

follows. The modulus of C(u) is

$$|C(u)| = \sqrt{1 - |F(u)|^2}. \tag{15b}$$

Interpreting eq. (15a) with the help of Pythagoras' theorem one can conclude that the change of the phase is $\Delta\Phi(u) \in \{ -\pi/2, \pi/2 \}$ (cf. eq. (11)) as illustrated in Fig. 4. The distribution of the change of the phase can be described by a binary phase grating $\exp[i\Delta\Phi(u)]$. In conclusion one obtains

$$C(u) = \sqrt{1 - |F(u)|^2}\,\exp[i\Phi(u)]\,\exp[i\Delta\Phi(u)]. \tag{15c}$$

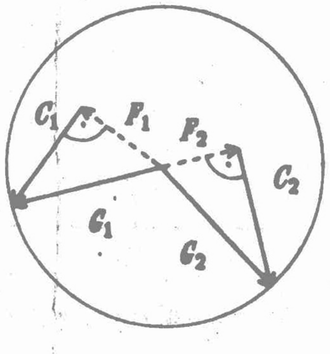


Fig.4. Illustration of $\Delta\Phi$ according to eq. (15a).

Due to convolutions with different orders of the grating the resulting $c(x)$ can be spatially separated from the signal. Figure 5 shows an example for the one dimensional testsignal. It should be compared with Fig. 2. The center of the noise is now separated from the signal. The method according to eq. (15c) is a generalised version of the synthetic coefficient method.[5,6]

The pointwise solution of condition (14) is one possibility. An alternative method is to search for an integral solution. The complex error diffusion concept has been suggested to this end.[7] Another method is an iterative procedure. It can be described by

$$Q_{IT} = [\mathcal{X}\mathcal{P}]^n. \tag{16}$$

It consists of the projection operator $\mathcal{P}$ and an operator $\mathcal{X}$ which eliminates the signal error caused by $\mathcal{P}$ in each cycle: n indicates the number of iterations. Dependent on the choice of the operartor $\mathcal{X}$ different names for the iteration are used, e.g. iterative Fourier transform algorithm, projection algorithm and fixpoint iteration. Various sciencieets have contributed to its theory.[8-14] In digital holography the method was introduced to use a combination of PF and SF.[8,9] In the following it is shown that the method is a powerful tool to utilise any kind of freedom.

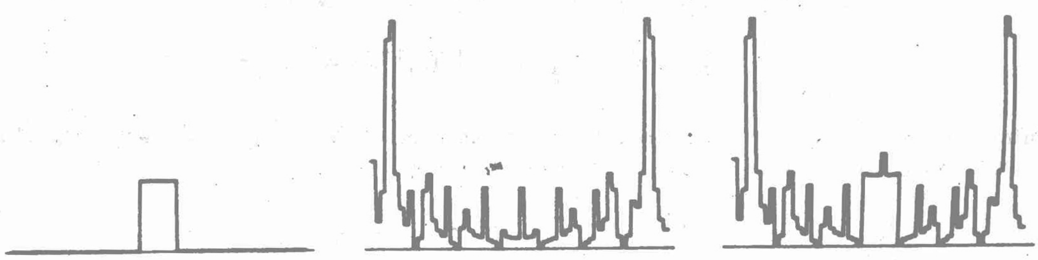


Fig. 5. The testsignal $f(x)$, the noise $c(x)$ and the diffraction pattern $g(x)$ obtained by the method according to eq. (15c).

Figure 6(a) shows an example of the iterative use of the AF. In Fig. 6(b) a restricted AF was used to introduce a gap between signal and noise. In both cases the result is a $c(x)$ which is almost sero within F.

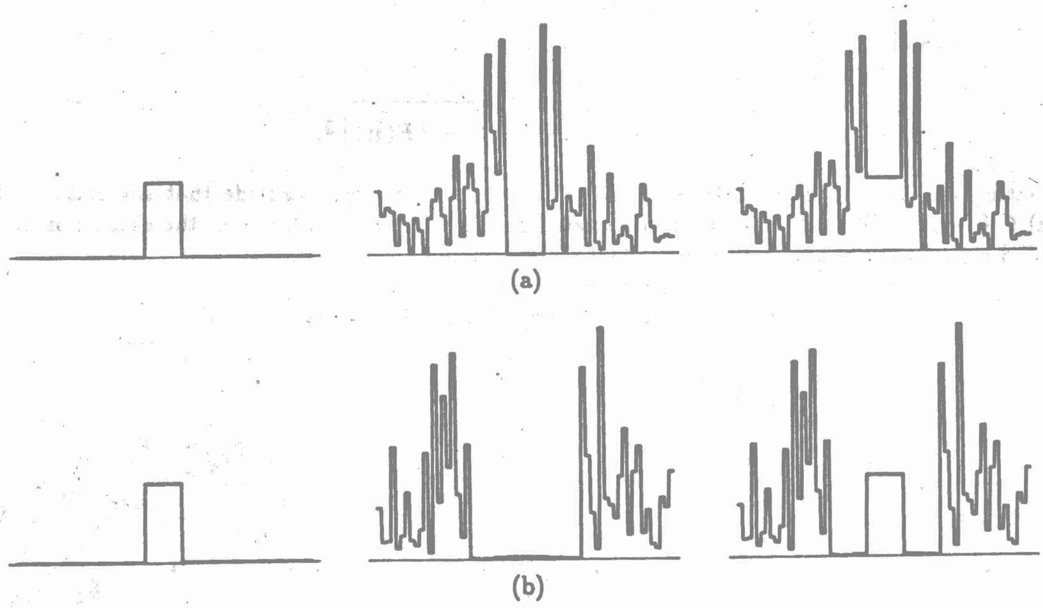


Fig. 6. Distribution of $f(x)$, $|c(x)|$, and $|g(x)|$ obtained by iterative use of AF (a) and restricted AF (b).

Any method which only employs the AF suffers from a $c(\mathbf{x})$ with high energy when η_0 is small. The question is: How much energy of $c(\mathbf{x})$ can be used to increase the diffraction efficiency η? In section 2.3. this question is answered.

2.3. Combination of amplitude and scale factor freedom

In order to obtain a diffraction efficiency larger than η_0 it is necessary to use SF in addition to AF. Then, $g(\mathbf{x})$ is equal to $\alpha f(\mathbf{x})$ within F and

$$\eta = \alpha^2 \eta_0. \tag{17}$$

The upper limit of α determines the upper limit of η.

The distribution $g(\mathbf{x})$ in the whole signal plane is

$$g(\mathbf{x}) = \alpha f(\mathbf{x}) + c'(\mathbf{x}); \;\; c'(\mathbf{x}) = 0, \mathbf{x} \in \mathrm{F}. \tag{18a}$$

$c'(\mathbf{x})$ describes the use of the AF. $G(\mathbf{u})$ is

$$G(\mathbf{u}) = \alpha F(\mathbf{u}) + C'(\mathbf{u}). \tag{18b}$$

Application of the law of conservation of energy leads to a necessary condition for the energy of $C'(\mathbf{u})$, vis.

$$< |C'(\mathbf{u})|^2 > = 1 - \alpha^2 < |F(\mathbf{u})|^2 >. \tag{19a}$$

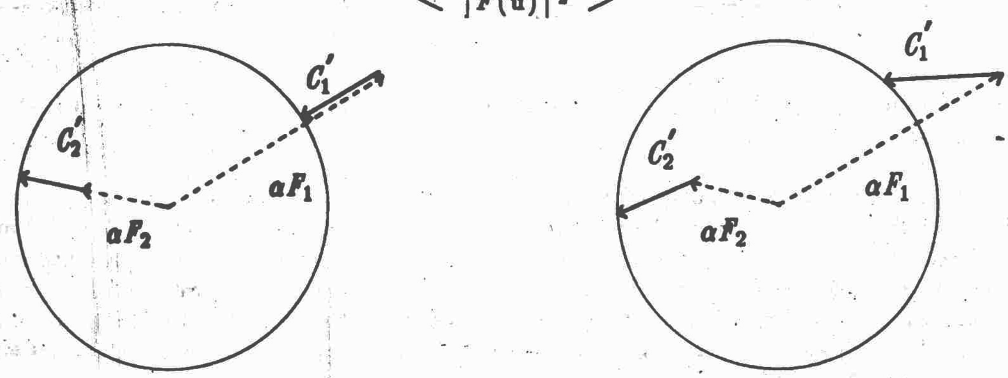
It is similar to condition (14), but now the factor α appears. As a consequence it is not possible to satisfy condition (19a) pointwise. Thus, iterative methods are the ones to use.

What is possible to learn from eq. (19a) about the upper limit of α? In case one would know the minimum of the left side of eq. (19a), the upper limit of α is obtained. Fortunately, this minimum is known. Figure 7(a) sketches the situation for two values of αF. $C'(\mathbf{u})$ serves to introduce the modulus 1. The shortest way to do it is indicated in this figure. The result is

$$< |C'(\mathbf{u})|^2 > \; \geq \text{minimum} = < (1 - \alpha \, |F(\mathbf{u})|)^2 >. \tag{19b}$$

Inserting this value into condition (19a) and solving the equation for α leads to the upper limit

$$\alpha \leq \frac{< |F(\mathbf{u})| >}{< |F(\mathbf{u})|^2 >}. \tag{19c}$$



(a) (b)

Fig. 7. Illustration to obtain an upper limit of the scale factor α.