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Graham G. Ross

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Graham G. Ross

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EDITOR'S FOREWORD

The problem of communicating in a coherent fashion recent developments in the most exciting and active fields of physics seems particularly pressing today. The enormous growth in the number of physicists has tended to make the familiar channels of communication considerably less effective. It has become increasingly difficult for experts in a given field to keep up with the current literature; the novice can only be confused. What is needed is both a consistent account of a field and the presentation of a definite "point of view" concerning it. Formal monographs cannot meet such a need in a rapidly developing field, and, perhaps more important, the review article seems to have fallen into disfavor. Indeed, it would seem that the people most actively engaged in developing a given field are the people least likely to write at length about it.

FRONTIERS IN PHYSICS has been conceived in an effort to improve the situation in several ways. Leading physicists today frequently give a series of lectures, a graduate seminar, or a graduate course in their special fields of interest. Such lectures serve to summarize the present status of a rapidly developing field and may well constitute the only coherent account available at the time. Often, notes on lectures exist (prepared by the lecturer himself, by graduate students, or by postdoctoral fellows) and are distributed in mimeographed form on a limited basis. One of the principal purposes of the FRONTIERS IN PHYSICS series is to make such notes available to a wider audience of physicists.

It should be emphasized that lecture notes are necessarily rough and informal, both in style and content; and those in

the series will prove no exception. This is as it should be. The point of the series is to offer new, rapid, more informal, and, it is hoped, more effective ways for physicists to teach one another. The point is lost if only elegant notes qualify.

The publication of collections of reprints of recent articles in very active fields of physics will improve communication. Such collections are themselves useful to people working in the field. The value of the reprints will, however be enhanced if the collection is accompanied by an introduction of moderate length which will serve to tie the collection together and, necessarily, constitute a brief survey of the present status of the field. Again, it is appropriate that such an introduction be informal, in keeping with the active character of the field.

The informal monograph, representing an intermediate step between lecture notes and formal monographs, offers an author the opportunity to present his views of a field which has developed to the point where a summation might prove extraordinarily fruitful but a formal monograph might not be feasible or desirable.

Contemporary classics constitute a particularly valuable approach to the teaching and learning of physics today. Here one thinks of fields that lie at the heart of much of present-day research, but whose essentials are by now well understood, such as quantum electrodynamics or magnetic resonance. In such fields some of the best pedagogical material is not readily available, either because it consists of papers long out of print or lectures that have never been published.

The above words, written in 1961, continue to be timely. The search for a grand unified theory of the strong, weak, electromagnetic, and gravitational interactions, based on non-abelian gauge theories and their supersymmetric generalizations, is one of the great intellectual endeavors of our time. In this volume, based on his lectures at Oxford over the last few years, Dr. Ross describes in some detail the ideas which have been explored and their application to cosmology. I share his hope that his book will instruct new workers in the field, taking them from Maxwell's equations and the ideas of symmetries to the latest grand unified models. It is a pleasure to welcome Dr. Ross to the ranks of contributors to FRONTIERS IN PHYSICS.

Urbana, Illinois
August, 1984

David Pines

PREFACE

The last fifteen years have seen remarkable advances in our knowledge and understanding of the fundamental forces of nature. This has led to the development of successful theories for the strong and weak interactions to complement Quantum Electrodynamics, the theory of the electromagnetic force. The similarity of these theories, which are all based on local gauge field theories, has led to the development of Grand Unified theories in which the strong, electromagnetic and weak interactions are different aspects of a single underlying gauge theory.

These notes, based on lectures presented at Oxford over the last few years, introduce the ideas of local gauge field theories and the construction of realistic models. The perturbative structure of the theory is discussed, paying particular attention to the calculation of higher order radiative corrections, a topic not usually covered in introductory texts, but of central importance in the extension of the standard model. We then turn to the construction of

Grand Unified theories and their phenomenological implications. Since the subject is still under development, emphasis is placed on a discussion of the various ideas that have been explored, such as technicolour and supersymmetry, rather than a review of specific models. However the material is covered in sufficient detail to enable the reader to follow each step.

Finally we review the interplay between cosmology and grand unification, an area of intense activity in which the field theory ideas of Grand Unification have shed new light on such problems as the production of a baryon number excess. A bibliography is included to augment the discussion in the text and to provide source material to the reader for further research.

I am grateful to my colleagues at Oxford, at the Rutherford Appleton Laboratory, at Cal. Tech. and at CERN for sharing their insights with me. Several people have helped by reading parts of the manuscript, in particular Guy Coughlan, Graham Blair and Stephen Wilkinson. A special vote of thanks is due to Pierre Ramond for enlightening discussions and detailed comments on the manuscript. Finally, I would like to thank Linda Clarke for cheerfully coping with the typing of this text.

G. Ross,
Oxford 1984.

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1. INTRODUCTION

The development in our understanding of the fundamental forces rests to a large extent on our understanding of the underlying symmetries of nature. Coupled to this has been the development of quantum field theories, in which the fundamental states are described by fields whose interactions obey the underlying symmetries. Taken together these ideas have led to remarkably successful field theories for the strong, weak and electromagnetic interactions. They have also led to a class of theory in which the strong, weak and electromagnetic interactions are combined in a "Grand Unified" theory, each force being just a different facet of the same underlying interactions. The unification has been extended to include the fourth force, gravity, in the "Super Grand Unified" theories. In this introductory chapter we will review the conceptual basis for these unified field theories.

1.1 Symmetries

In 1905 Einstein identified the invariance group of space and time, a development that led to Einstein's theory of

gravity. Later new symmetries, such as isospin, acting on internal quantum numbers, and having nothing to do with space and time, were identified. These symmetries are not exact and govern only the strong interactions. Their identification allowed for a classification and simplification of strong interaction phenomena. Other symmetries, such as parity, charge conjugation and strangeness were found to be violated only by the weak interactions. It seemed that different interactions were governed by different symmetries and unification of the fundamental forces did not appear likely.

Initially it was thought the new internal symmetries should be global symmetries, not depending on the position in space and time. However it has long been known that the laws of electromagnetism possess another local symmetry in which charge is locally conserved, meaning that charged fields have a phase that varies freely from point to point in space and time. Indeed, Maxwells equations may be derived starting only with this local gauge symmetry. In 1954 Yang and Mills and Shaw proposed that local symmetries be extended from this $U(1)$ group of phase rotations to non-Abelian symmetries, of the type that had been proposed as symmetries for the strong interactions. However, this generalisation led to the prediction that there should be new vector bosons, like the photon, which should be massless, apparently quite in contradiction with observation. Subsequent developments led to an understanding of how approximate symmetries, with massive vector bosons, may have a fundamental basis. This rests on the idea of "spontaneous" symmetry breaking in which the Hamiltonian and commutation relations of a quantum system may possess an exact symmetry, but this symmetry may turn out not to be a symmetry of the vacuum, and so the physical state need not manifestly exhibit the symmetry.