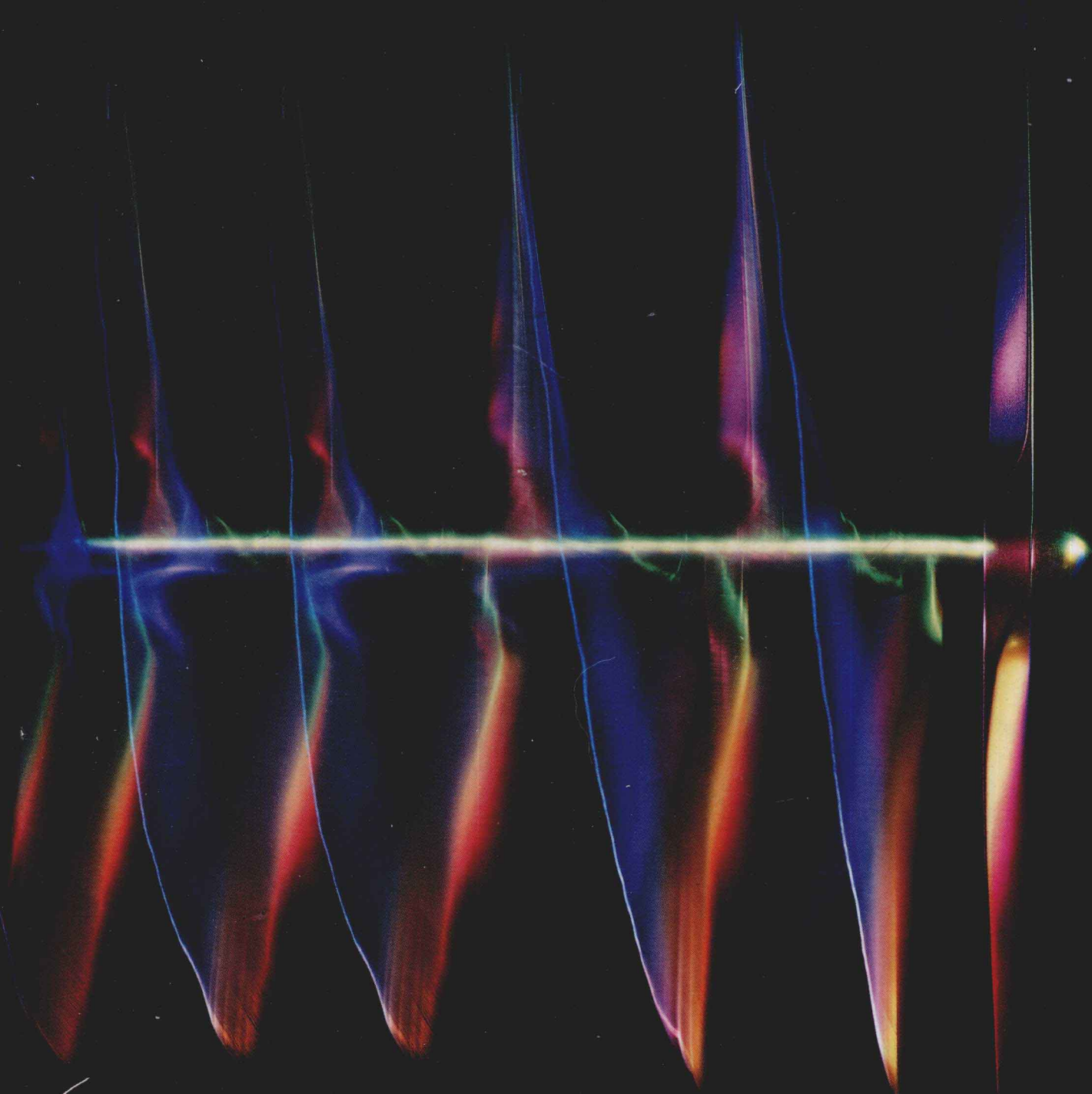


CALCULUS

Larson / Hostetler / Edwards

Fourth Edition



CALCULUS

WITH ANALYTIC GEOMETRY

Fourth Edition

ROLAND E. LARSON

ROBERT P. HOSTETLER

The Pennsylvania State University
The Behrend College

BRUCE H. EDWARDS

University of Florida

with the assistance of DAVID E. HEYD
The Pennsylvania State University
The Behrend College

D. C. HEATH AND COMPANY
Lexington, Massachusetts Toronto

Chapter Opener Photo Credits

Chapter 1: Bettmann Archive.
Chapter 2: Ray Ellis/Photo Researchers.
Chapter 3: Dan Helms/duomo.
Chapter 4: Mark Antman/The Image Works.
Chapter 5: NASA/Johnson Space Center.
Chapter 6: Lionel Delevingne/Stock, Boston.
Chapter 7: Focus on Sports.
Chapter 8: NASA/Johnson Space Center.
Chapter 9: Focus on Sports.
Chapter 10: Kent and Donna Dannen/Photo Researchers.
Chapter 11: NASA/Johnson Space Center.
Chapter 12: Ellis Herwig/Stock, Boston.
Chapter 13: Steven Sutton/duomo.
Chapter 14: Ford Motor Company; bottom: Los Alamos National Laboratory.
Chapter 15: AP/Wide World Photos.
Chapter 16: NASA/Johnson Space Center.
Chapter 17: Cary Wolinsky/Stock, Boston.

Acquisitions Editors: Mary Lu Walsh and Ann Marie Jones

Developmental Editor: Cathy Cantin

Production Editor: Laurie Johnson

Designer: Sally Steele

Production Coordinator: Michael O'Dea

Photo Researcher: Wendy Johnson

Composition: Jonathan Peck Typographers

Technical Art: Folium

Cover: Martucci Studio

Copyright © 1990 by D. C. Heath and Company.

Previous editions copyright © 1986, 1982, 1979 by D. C. Heath and Company.

All rights reserved. No part of this publication may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopy, recording, or any information storage or retrieval system, without permission in writing from the publisher.

Published simultaneously in Canada.

Printed in the United States of America.

International Standard Book Number: 0-669-16406-2

Library of Congress Catalog Card Number: 88-83340

Preface

Calculus with Analytic Geometry, Fourth Edition, is designed for use in a calculus course for students in engineering, economics, life sciences, mathematics, and the physical sciences. In writing the text, we were guided by two primary objectives that have crystallized over many years of teaching calculus. For the student, our objective was to write in a precise, readable manner and to clearly define and demonstrate the basic concepts of calculus. For the instructor, our objective was to design a comprehensive teaching instrument that uses proven pedagogical techniques, thus freeing the instructor to make the most efficient use of classroom time.

Changes in the Fourth Edition

In the Fourth Edition, all examples, theorems, definitions, and prose have been revised—or at least considered for revision. Many new examples, exercises, and applications have been added to the text. The major changes in the Fourth Edition, listed by chapter, are as follows:

- The basic organization of Chapter 1 is the same as in the Third Edition. Section 1.4 contains a new summary of Equations of Lines, and Section 1.5 contains a new summary of the graphs of basic functions.
- Chapter 2 begins with a new introduction to the tangent line problem. Section 2.1 has a new example dealing with the ϵ - δ definition applied to a quadratic function. Section 2.3 begins with a new summary of a strategy for finding limits.
- The “Four-Step Process” terminology has been eliminated from Section 3.1 in favor of simply applying the definition of the derivative. The introduction to the Chain Rule has been rewritten.
- Section 4.3 on the First Derivative Test has been reorganized. The section on business and economics applications has been moved to the end of the chapter.
- At the request of several users of the text, Chapter 5 has been reorganized. In an effort to get to the Fundamental Theorem of Calculus earlier in the

chapter, we moved the section on integration by substitution to Section 5.5. The material on sigma notation was condensed and now appears as the first part of Section 5.2. Numerical integration was moved from Chapter 8 to Section 5.6, and the introduction of the natural logarithmic function was postponed to Chapter 6.

- Chapter 6 has also been reorganized. It now begins with an introduction to the natural logarithmic function. There is a separate section on bases other than e and applications (Section 6.5) and a separate section on growth and decay (Section 6.7).
- In Chapter 7, we moved the material on arc length and surface area to Section 7.4, preceding the section on work.
- The section on numerical integration was moved from Chapter 8 to Chapter 5. In Section 8.2, the tabular method of repeated integration by parts was added to the text.
- At the request of many users of the Third Edition, the introduction to Taylor polynomials, which was in the first section of Chapter 9, has been moved to Section 9.7.
- In Chapter 10, the material on classifying conics was moved from Section 10.3 to Section 10.4.
- In Chapter 11, the section on area and arc length in polar coordinates now occurs before the section on polar equations for conics and Kepler's Laws.
- Although the table of contents for the five sections in Chapter 13 is the same as it was in the Third Edition, this chapter was substantially rewritten—especially the material introducing the arc length function and the material on curvature.
- New material on computer graphics in three dimensions was added to Section 14.1. A new discussion of continuity of a function of three variables was added to Section 14.2. The material on complex zeros of polynomial functions was deleted from Section 14.9, and a new example on least squares regression was added. In Section 14.10, the method of Lagrange multipliers was rewritten to agree with standard presentation.
- In Chapter 15, the introduction to Jacobians was rewritten and moved from the fourth section to the end of the chapter.
- As with Chapter 13, the table of contents for Chapter 16 is the same as in the Third Edition, but the chapter was substantially rewritten. Section 16.1 now formally defines inverse square fields, has five new examples, and introduces divergence of a vector field. In Section 16.2, the introduction to line integrals was rewritten, and two new examples were added. In Section 16.3, Theorem 16.7 is new. In Section 16.5, Examples 2 and 5 were rewritten, and material on Gauss's Law was added.

Features

Order of Topics The seventeen chapters readily adapt to either semester or quarter systems. In each system both differentiation *and* integration can be introduced in the first course of the sequence. There is some flexibility in the order and depth in which the chapters can be covered. For instance, much of the precalculus material in Chapter 1 can be used as individual review. The ϵ - δ discussion of limits in Chapter 2 can be given minimal coverage. Sections 7.6 and 7.7 can be covered later in the course. Chapter 9 can be covered at

any time after Chapter 8. The coverage of Section 12.7 can be delayed until just before Section 15.8.

Definitions and Theorems Special care has been taken to state the definitions and theorems simply, without sacrificing accuracy.

Proofs We have chosen to include only those proofs that we have found to be both instructive and within the grasp of a beginning calculus student. Moreover, in presenting proofs, we have found that extensive detail often obscures rather than illuminates. For this reason, many of the proofs are presented in outline form, with an emphasis on the essence of the argument. (See the proof of the Product Rule in Section 3.4.) In some cases, we have included a more complete discussion of proofs in Appendix A. (See the proof of the Chain Rule in Section 3.5.)

Graphics The Fourth Edition has over 2325 figures. Of these, over 1080 are in the examples and exposition, over 650 are in the exercise sets, and over 590 are in the odd-numbered answers. The new art program in the Fourth Edition was computer generated for accuracy. Designed with additional colors used systematically for clarity, this improved art program will help students better visualize mathematical concepts, particularly in the presentation of complex, three-dimensional material. For example, axes and primary graphs are brown, while planes are blue and primary three-dimensional surfaces are red.

Enhanced Presentation As a new feature, the Fourth Edition has been designed with a functional use of four colors that strengthens the text as a pedagogical tool. Each color is used consistently to aid both reading and reference. For example, all theorems and definitions are highlighted by brown boxes, and equation side comments are given in red.

Exercises Over 1000 new problems have been added, so the text now contains nearly 8200 exercises. The exercises are graded, progressing from skill-development problems to more challenging problems involving applications and proofs. Many exercise sets begin with a group of exercises that provides the graphs of functions involved. Review exercises are included at the end of each chapter.

Examples The text contains nearly 1000 examples, each titled for easy reference. Many of the examples include red side comments that clarify the steps of the solution.

Computer/Calculator Exercises Another new feature in the Fourth Edition is the addition of exercises involving the use of a computer or graphics calculator. See, for instance, Exercises 69 and 70 in Section 1.3 or Exercises 79–82 in Section 1.6.

Numerical Methods With the increasing power and accessibility of computers, numerical techniques are becoming more widely used. This edition reflects this trend by introducing numerical integration earlier in the text, and by adding a tabular method of integration by parts. Calculators or computers are useful in these areas as well as with topics such as limits, Newton's Method, and Taylor polynomials.

Chapter Introductions As a new feature in the Fourth Edition, each chapter begins with a chapter overview and a special motivational application.

Applications We chose applications that offer variety and integrity, and that require a minimal knowledge of other fields. The Fourth Edition contains over 2000 different applications.

Summaries Many sections have summaries that identify core ideas and procedures—see Sections 3.4, 4.10, 6.7, 9.6, and 13.5. In some instances, an entire section summarizes the preceding topics—see Sections 4.6 and 8.1.

Historical Notes Throughout the text, we include several short biographical notes about prominent mathematicians. These are designed to help students gain an appreciation of both the people involved in the development of calculus and the nature of the problems that calculus was designed to solve.

Remarks The text contains many special instructional notes to students in the form of “Remarks.” These notes appear after definitions, theorems, or examples and are designed to give additional insight, help avoid common errors, or describe generalizations.

Supplements

For Students

- The *Study and Solutions Guide* by David E. Heyd contains detailed solutions to several representative problems from each exercise set. In the text, these exercises are identified in the exercise sets by blue numbers. The solutions to these exercises are given in greater detail than the examples in the text, with special care taken to show the algebra involved. In addition, the *Study and Solutions Guide* contains a review of algebra and a set of true-false questions for each chapter in the text.
- *The Algebra of Calculus* by Eric Braude provides additional calculus review. Keyed to the text, it offers a review of calculus topics, examples with solutions, related problems and their answers, and exercises with answers in the back of the book.

For Instructors

- A *Complete Solutions Guide* by Dianna L. Zook is available in three volumes. This guide contains brief solutions to every exercise in the text, including exercises requiring proofs.
- An *Instructor's Guide* by Ann R. Kraus contains, for each chapter in the text, sample tests, concept reviews in the form of true-false questions, and suggestions for classroom instruction.
- A package of color transparencies of figures from the text is available.

For Students and Instructors

- The *Calculus Applications in Engineering and Science* supplement contains solved examples and exercises (with answers to the odd-numbered ones) covering applications in engineering, physics, chemistry, biology, and other fields.

Technology

Software

A library of software products for this text is available for the IBM-PC, Macintosh, and Apple II. To make the most of the software products described below, instructors or students may choose to solve appropriate exercises from the text and those provided in a User Manual as well as analyze functions of the user's own selection. All of the following products can be used for classroom demonstration and have been class tested.

- *Math Lab Calculus*, The Math Lab (IBM-PC, Apple II)
Offering both two-dimensional and three-dimensional computer graphics and numerical computations, the package consists of one disk and a User Manual with 80 lab assignments. These assignments are presented in worksheet format and are keyed to the text. Additionally, all computer/calculator exercises in the text can be solved using this software.
- *Math Utilities* (updated version 4.0), Bridge Software (IBM-PC)
This is a program of three powerful graphing packages: CURVES, SURFS (Surfaces), and DIFFS (Differential Equations). Each package includes one disk and a User Manual. These enhanced graphing packages allow the user to save graphs and offer labelling and annotating capabilities for creating classroom handouts.
- *BestGrapher*, George Best (IBM-PC, Macintosh)
This software graphs both two- and three-dimensional figures as well as offers some features that help teach concepts through simulation, such as rotation of a curve about an axis. A lab package that accompanies the software provides assignments in a worksheet format keyed to the text.
- *Computer Activities for Calculus*, Technology Training Associates (IBM-PC)
This package includes three disks and a User Manual. Each easy-to-use disk has two units of tutorials for self-study and the Grapher, a graphing tool for two-dimensional figures that is designed to handle a wide variety of elementary functions.
- *TrueBASIC CALCULUS*, *TrueBASIC* (IBM-PC) and *TrueBASIC MULTI-VARIATE CALCULUS: MACFUNCTION* (Macintosh)
Each package includes one disk and a User Manual. *TrueBASIC CALCULUS* performs numerical routines and two-dimensional graphing for topics in the first two semesters of calculus. *MACFUNCTION* is a tool for plotting and examining three-dimensional graphs of functions.
- *Workout for Calculus*, Fourth Edition, Joseph Mazur (IBM-PC)
This tutorial software offers exercises with on-line hints and guidance for student self-study. Keyed on screen to the text, the software generates problems and then adjusts the difficulty of the exercises with sensitivity to the student's level of accomplishment.
- *Derive*, Soft Warehouse (IBM-PC)
Special discounts on this computer algebra software are available upon adoption of the text.

IBM-PC is a registered trademark of International Business Machines Corporation. Apple and Macintosh are the registered trademarks of Apple Computer, Inc. TrueBASIC is the registered trademark of TrueBASIC, Inc.

- *Calculus*, Broderbund (Macintosh)

Special discounts on this tutorial software are available upon adoption of the text.

Videotapes

- *Calculus Videotapes*, Video Tutorial Services, Inc.

Interactive videotapes combine thorough coverage of major topics with state-of-the-art computer graphics. Keyed to the text, these videotapes will help students practice, learn, and review calculus. A *Study Guide* and an *Instructor's Guide* are also available.

Calculators

- *HP28S*, *HP48SX*, *Casio FX7000G*, *Casio FX7500G*

Special discounts on these calculators are available upon adoption of the text.

Computerized Testing

- *HeathTest Plus for Calculus* (IBM-PC, Apple II, Macintosh)

Instructors can produce chapter tests, mid-terms, and final exams easily and accurately. Instructors can also edit existing questions or add new ones as desired, or preview questions on screen and add them to a test with a single keystroke. The software supports graphics and offers both multiple-choice and open-ended questions. A User Manual and a printed test item file are available.

- *HeathTest for Calculus* (IBM-PC)

This is an algorithm-based program that generates tests, quizzes, or worksheets in a multiple-choice format. A User Manual with printed test items accompanies the software.

Acknowledgments

We would like to thank the many people who have helped us at various stages of this project during the past seventeen years. Their encouragement, criticisms, and suggestions have been invaluable to us.

Fourth Edition Focus Group Homer F. Bechtell, University of New Hampshire; K. Elayn Gay, University of New Orleans; Hideaki Kaneko, Old Dominion University; Judith A. Palagallo, University of Akron; John Tweed, Old Dominion University

Fourth Edition Reviewers Keith Bergeron, United States Air Force Academy; Jorge Cossio, Miami-Dade Community College; Rosario Diprizio, Oakton Community College; Ali Hajjafar, University of Akron; Ransom Van B. Lynch, Phillips Exeter Academy; Bennet Manvel, Colorado State University; Duff A. Muir, United States Air Force Academy; Charlotte J. Newsom, Tidewater Community College; Terry J. Newton, United States Air Force Academy; Wayne J. Peeples, University of Texas; Jorge A. Perez, LaGuardia Community College; Barry J. Sarnacki, United States Air Force Academy; George W. Schultz, St. Petersburg Junior College; Frank Soler, De Anza College; Michael Steuer, Nassau Community College; John Tweed, Old Dominion University; Jay Wiestling, Palomar College; August J. Zarcone, College of Dupage; Li Fong, Johnson County Community College

Third Edition Reviewers Dennis Albér, Palm Beach Junior College; Garret J. Etgen, University of Houston; William R. Fuller, Purdue University; Timothy J. Kearns, Boston College; Norbert Lerner, State University of New York at Cortland; Robert L. Maynard, Tidewater Community College; Barbara L. Osofsky, Rutgers University; Jean E. Rubin, Purdue University; Lawrence A. Trivieri, Mohawk Valley Community College; J. Philip Smith, Southern Connecticut State University

Second Edition Reviewers Harry L. Baldwin, Jr., San Diego City College; Phillip A. Ferguson, Fresno City College; Thomas M. Green, Contra Costa College; Arnold J. Insel, Illinois State University; William J. Keane, Boston College; David C. Lantz, Colgate University; Richard E. Shermoen, Washburn University; Thomas W. Shilgalis, Illinois State University; Florence A. Warfel, University of Pittsburgh

First Edition Reviewers Paul W. Davis, Worcester Polytechnic Institute; Eric R. Immel, Georgia Institute of Technology; Frank T. Kocher, Jr., Pennsylvania State University; Joseph F.

Krebs, Boston College; Maurice L. Monahan, South Dakota State University; Robert A. Nowlan, Southern Connecticut State University; N. James Schoonmaker, University of Vermont; Bert K. Waits, Ohio State University

We would also like to thank users of the Third Edition who answered a questionnaire concerning changes they wanted in the new edition. They are George Anderson, Rhode Island College; Frank P. Battles, Massachusetts Maritime Academy; Derek I. Bloomfield, Orange County Community College; Karen J. Edwards, Paul Smith's College; Theodore Hanley, State University of New York at Utica/Rome; Peter Herron, Suffolk County Community College; Ann M. Joyce, Chestnut Hill College; Arthur Kaufman, College of Staten Island; Alan Levine, Franklin and Marshall College; James Magliano, Union County College; Frank Morgan, Castleton State College; Raymond Pluta, Castleton State College; M. Susan Richman, Pennsylvania State University; Carmen Vlad, Pace University; Christopher White, Castleton State College.

A special thanks to all the people at D. C. Heath and Company who worked with us in the development of the Fourth Edition, especially Ann Marie Jones and Mary Lu Walsh, Mathematics Acquisitions Editors; Cathy Cantin, Developmental Editor; Laurie Johnson, Production Editor; Sally Steele, Designer; Carolyn Johnson, Editorial Assistant; Mike O'Dea, Production Manager; and Wendy Johnson, Photo Researcher.

Several other people also worked on this project: David E. Heyd wrote the *Study and Solutions Guide* and solved the exercises, Dianna L. Zook wrote the *Complete Solutions Manual*, Ann R. Kraus wrote the *Instructor's Guide*, Timothy R. Larson prepared the art, Linda L. Kifer proofread the galleys, Linda M. Bollinger proofread the galleys and typed the supplements, Helen Medley solved the exercises and performed an accuracy check for the text, and Kathleen Evanoff, Randall Hammond, and Paula Sibeto solved the exercises and assisted with the production of the supplement package.

A special note of thanks goes to the over 500,000 students who have used earlier editions of the text.

On a personal level, we are grateful to our wives, Deanna Gilbert Larson, Eloise Hostetler, and Consuelo Edwards, for their love, patience, and support. Also, a special thanks goes to R. Scott O'Neil.

If you have suggestions for improving this text, please feel free to write to us. Over the past seventeen years we have received many useful comments from both instructors and students, and we value these very much.

Roland E. Larson, Robert P. Hostetler, Bruce H. Edwards

Calculus

To accommodate the different methods of teaching calculus, D. C. Heath also offers the two texts described below. Each has its own supplements package. The following is a brief discussion of how each book differs from CALCULUS WITH ANALYTIC GEOMETRY, Fourth Edition.

Calculus with Analytic Geometry, Alternate Fourth Edition with Late Trigonometry
Larson/Hostetler

This text, with late trigonometry, is also designed for a three-semester course. Additionally, the text offers a different treatment of the following topics: limits, applications of integration, exponential and logarithmic functions, and vectors.

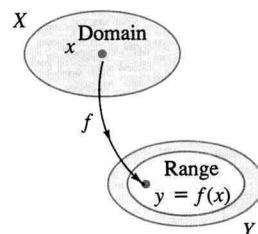
Calculus with Analytic Geometry, Third Edition, Part I
Larson/Hostetler

This single-variable text is designed for a two-semester course. All six trigonometric functions are reviewed in Chapter 1, then used throughout the text.

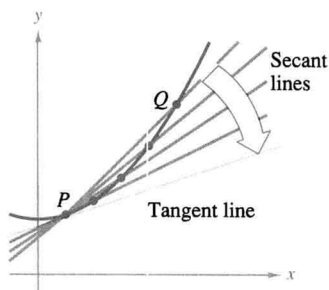
Contents

1 The Cartesian Plane and Functions 1

1.1	Real Numbers and the Real Line	1
1.2	The Cartesian Plane	10
1.3	Graphs of Equations	18
1.4	Lines in the Plane	27
1.5	Functions	36
1.6	Review of Trigonometric Functions	47
	Review Exercises	61



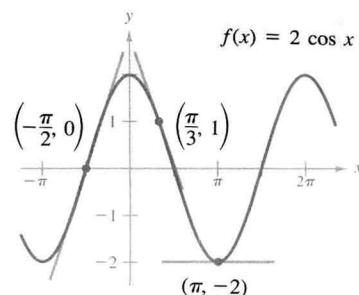
2 Limits and Their Properties 64



2.1	An Introduction to Limits	65
2.2	Properties of Limits	75
2.3	Techniques for Evaluating Limits	83
2.4	Continuity and One-Sided Limits	90
2.5	Infinite Limits	101
	Review Exercises	108

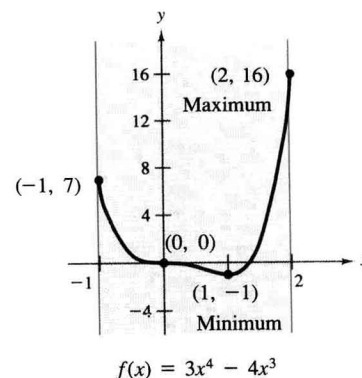
3 Differentiation 110

3.1	The Derivative and the Tangent Line Problem	111
3.2	Velocity, Acceleration, and Other Rates of Change	121
3.3	Differentiation Rules for Constant Multiples, Sums, Powers, Sines, and Cosines	129
3.4	Differentiation Rules for Products, Quotients, Secants, and Tangents	138
3.5	The Chain Rule	147
3.6	Implicit Differentiation	156
3.7	Related Rates	164
	Review Exercises	173

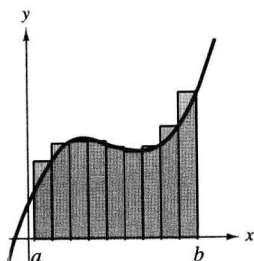


4 Applications of Differentiation 176

4.1	Extrema on an Interval	177
4.2	Rolle's Theorem and the Mean Value Theorem	185
4.3	Increasing and Decreasing Functions and the First Derivative Test	190
4.4	Concavity and the Second Derivative Test	199
4.5	Limits at Infinity	206
4.6	A Summary of Curve Sketching	215
4.7	Optimization Problems	224
4.8	Newton's Method	233
4.9	Differentials	241
4.10	Business and Economics Applications	248
	Review Exercises	256



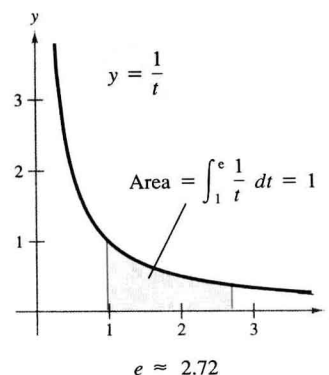
5 Integration 258



5.1	Antiderivatives and Indefinite Integration	259
5.2	Area	269
5.3	Riemann Sums and the Definite Integral	281
5.4	The Fundamental Theorem of Calculus	291
5.5	Integration by Substitution	300
5.6	Numerical Integration	312
	Review Exercises	321

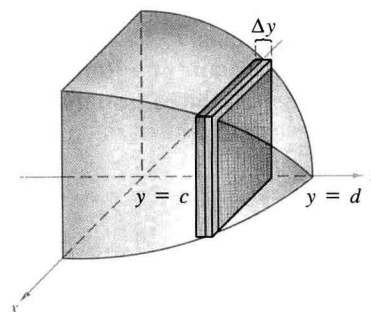
6 Logarithmic, Exponential, and Other Transcendental Functions 324

6.1	The Natural Logarithmic Function and Differentiation	325
6.2	The Natural Logarithmic Function and Integration	334
6.3	Inverse Functions	342
6.4	Exponential Functions: Differentiation and Integration	350
6.5	Bases Other Than e and Applications	357
6.6	Growth and Decay	365
6.7	Inverse Trigonometric Functions and Differentiation	371
6.8	Inverse Trigonometric Functions: Integration and Completing the Square	380
6.9	Hyperbolic Functions	389
	Review Exercises	399



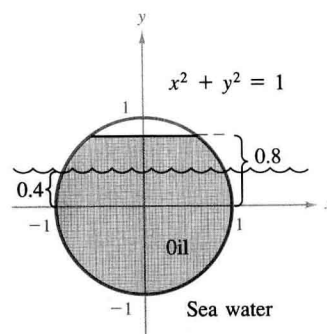
7 Applications of Integration 402

7.1	Area of a Region Between Two Curves	403
7.2	Volume: The Disc Method	411
7.3	Volume: The Shell Method	422
7.4	Arc Length and Surfaces of Revolution	429
7.5	Work	438
7.6	Fluid Pressure and Fluid Force	446
7.7	Moments, Centers of Mass, and Centroids	452
	Review Exercises	462



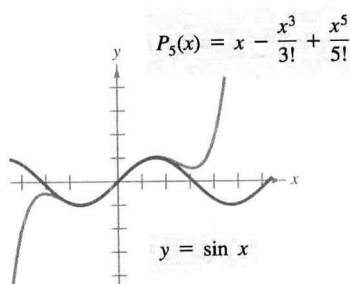
8 Integration Techniques, L'Hôpital's Rule, and Improper Integrals 464

8.1	Basic Integration Formulas	465
8.2	Integration by Parts	472
8.3	Trigonometric Integrals	482
8.4	Trigonometric Substitution	492
8.5	Partial Fractions	502
8.6	Integration by Tables and Other Integration Techniques	512
8.7	Indeterminate Forms and L'Hôpital's Rule	519
8.8	Improper Integrals	528
	Review Exercises	538



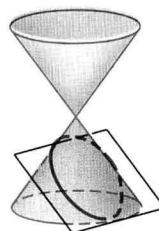
9 Infinite Series 540

9.1	Sequences	541
9.2	Series and Convergence	554
9.3	The Integral Test and p -Series	563
9.4	Comparisons of Series	568
9.5	Alternating Series	574
9.6	The Ratio and Root Tests	581
9.7	Taylor Polynomials and Approximations	587
9.8	Power Series	598
9.9	Representation of Functions by Power Series	606
9.10	Taylor and Maclaurin Series	613
	Review Exercises	624

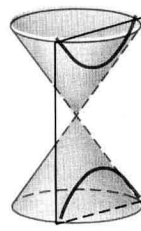


10 Conic Sections 626

- 10.1 Parabolas 627
- 10.2 Ellipses 635
- 10.3 Hyperbolas 644
- 10.4 Rotation and the General Second-Degree Equation 652
- Review Exercises 658



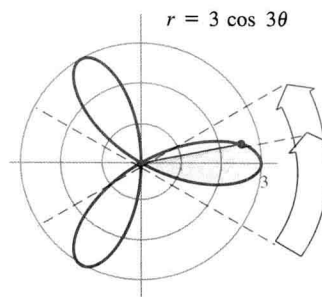
Ellipse



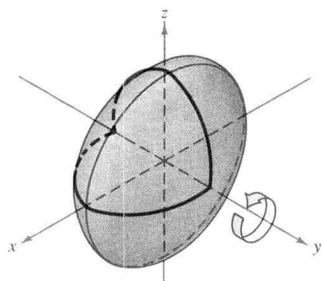
Hyperbola

11 Plane Curves, Parametric Equations, and Polar Coordinates 660

- 11.1 Plane Curves and Parametric Equations 661
- 11.2 Parametric Equations and Calculus 669
- 11.3 Polar Coordinates and Polar Graphs 676
- 11.4 Tangent Lines and Curve Sketching in Polar Coordinates 683
- 11.5 Area and Arc Length in Polar Coordinates 690
- 11.6 Polar Equations for Conics and Kepler's Laws 698
- Review Exercises 706



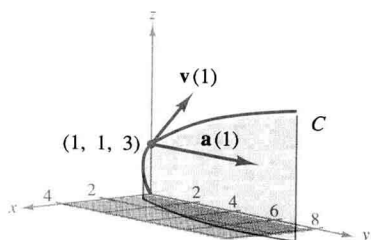
12 Vectors and the Geometry of Space 708



- 12.1 Vectors in the Plane 709
- 12.2 Space Coordinates and Vectors in Space 720
- 12.3 The Dot Product of Two Vectors 727
- 12.4 The Cross Product of Two Vectors in Space 736
- 12.5 Lines and Planes in Space 744
- 12.6 Surfaces in Space 753
- 12.7 Cylindrical and Spherical Coordinates 763
- Review Exercises 770

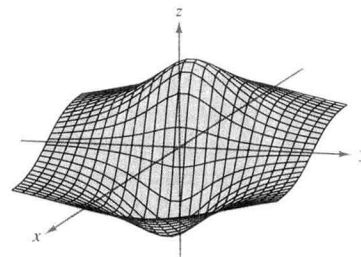
13 Vector-Valued Functions 772

- 13.1 Vector-Valued Functions 773
- 13.2 Differentiation and Integration of Vector-Valued Functions 779
- 13.3 Velocity and Acceleration 787
- 13.4 Tangent Vectors and Normal Vectors 796
- 13.5 Arc Length and Curvature 804
- Review Exercises 816

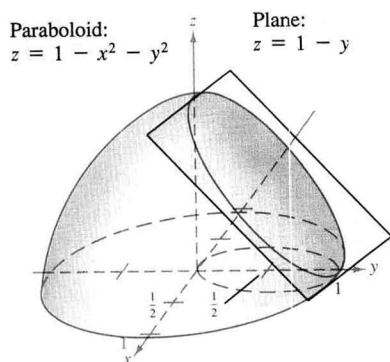


14 Functions of Several Variables 818

14.1	Introduction to Functions of Several Variables	819
14.2	Limits and Continuity	831
14.3	Partial Derivatives	840
14.4	Differentials	849
14.5	Chain Rules for Functions of Several Variables	856
14.6	Directional Derivatives and Gradients	865
14.7	Tangent Planes and Normal Lines	875
14.8	Extrema of Functions of Two Variables	883
14.9	Applications of Extrema of Functions of Two Variables	891
14.10	Lagrange Multipliers	898
	Review Exercises	906



15 Multiple Integration 908

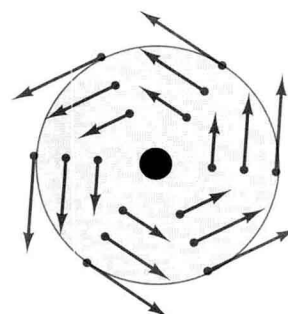


15.1	Iterated Integrals and Area in the Plane	909
15.2	Double Integrals and Volume	916
15.3	Change of Variables: Polar Coordinates	926
15.4	Center of Mass and Moments of Inertia	932
15.5	Surface Area	938
15.6	Triple Integrals and Applications	944
15.7	Triple Integrals in Cylindrical and Spherical Coordinates	954
15.8	Change of Variables: Jacobians	961
	Review Exercises	969

16 Vector Analysis 970

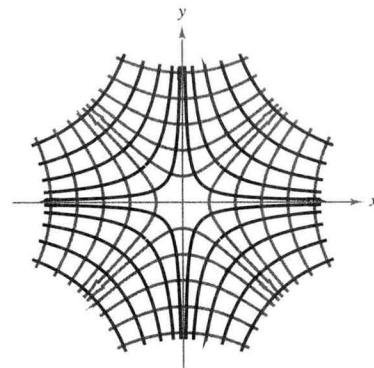
16.1	Vector Fields	971
16.2	Line Integrals	982
16.3	Conservative Vector Fields and Independence of Path	994
16.4	Green's Theorem	1003
16.5	Surface Integrals	1012
16.6	Divergence Theorem	1021
16.7	Stokes's Theorem	1028
	Review Exercises	1033

Velocity field



17 Differential Equations 1036

17.1	Definitions and Basic Concepts	1037
17.2	Separation of Variables in First-Order Equations	1041
17.3	Exact First-Order Equations	1049
17.4	First-Order Linear Differential Equations	1056
17.5	Second-Order Homogeneous Linear Equations	1064
17.6	Second-Order Nonhomogeneous Linear Equations	1071
17.7	Series Solutions of Differential Equations	1078
	Review Exercises	1082



APPENDIX A	Proofs of Selected Theorems	A1
------------	-----------------------------	----

APPENDIX B	Basic Differentiation Rules for Elementary Functions	A13
------------	------------------------------------------------------	-----

APPENDIX C	Integration Tables	A14
------------	--------------------	-----

ANSWERS to Odd-Numbered Exercises	A21
-----------------------------------	-----

INDEX	A99
-------	-----

Calculus Multi-Media Library

Coupons and information regarding software, videotapes, and graphing calculators follow the index at the end of this book.