

WORLD SCIENTIFIC SERIES ON  
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# DYNAMICS AND CONTROL OF HYBRID MECHANICAL SYSTEMS

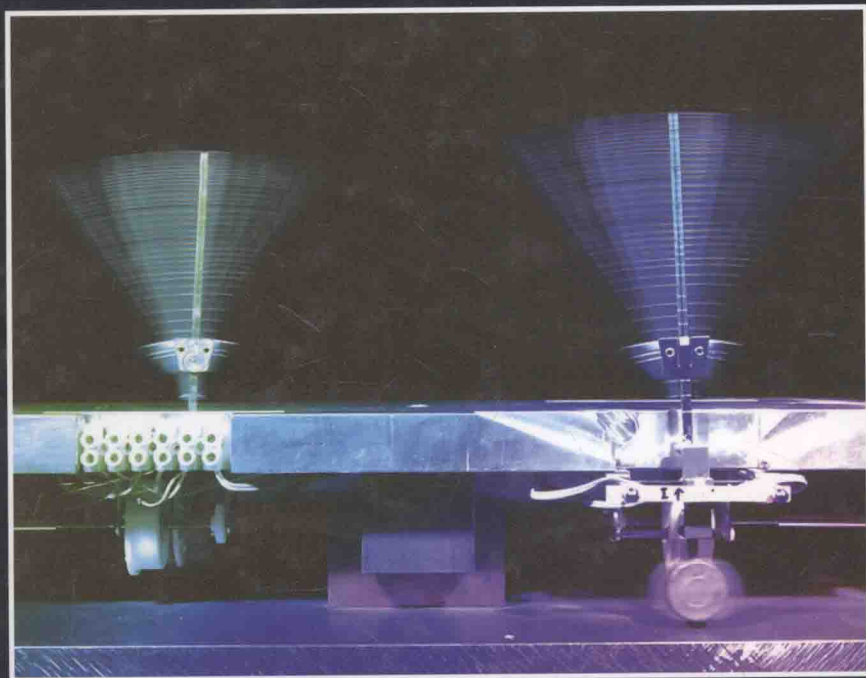
edited by

**Gennady Leonov**

**Henk Nijmeijer**

**Alexander Pogromsky**

**Alexander Fradkov**



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 World Scientific

*Published by*

World Scientific Publishing Co. Pte. Ltd.

5 Toh Tuck Link, Singapore 596224

*USA office:* 27 Warren Street, Suite 401-402, Hackensack, NJ 07601

*UK office:* 57 Shelton Street, Covent Garden, London WC2H 9HE

**British Library Cataloguing-in-Publication Data**

A catalogue record for this book is available from the British Library.

**DYNAMICS AND CONTROL OF HYBRID MECHANICAL SYSTEMS**

**World Scientific Series on Nonlinear Science, Series B — Vol. 14**

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ISBN-13 978-981-4282-31-4

ISBN-10 981-4282-31-6

Printed by FulIsland Offset Printing (S) Pte Ltd. Singapore

**DYNAMICS AND CONTROL OF  
HYBRID MECHANICAL SYSTEMS**

## WORLD SCIENTIFIC SERIES ON NONLINEAR SCIENCE

Editor: Leon O. Chua  
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*Dedicated to Ilya Izrailevich Blekhman  
on the occasion of his 80th birthday*

# Preface

The book is based on the material presented at a mini-symposium “Dynamics and Control of Hybrid Mechanical Systems” at the 6th European Nonlinear Dynamics Conference (ENOC) held in St Petersburg, Russia, in 2008. In turn, the abovementioned mini-symposium was based on results of a similarly entitled Dutch-Russian research project funded by the Dutch organization for Pure Research (now) and the Russian Foundation for Basic Research. The project “Dynamics and Control of Hybrid Mechanical Systems” (DyCoHyMS) ran over the period 2006–2008 and turned out to be quite successful in terms of cooperation and scientific output. This is partly reflected in this book. A number of other related contributions were included into the volume and it now contains results of several international and interdisciplinary collaborations in the field, and reflects state-of-the-art scientific and technological development in the area of hybrid mechanical systems.

The papers in this volume aim to provide a better understanding of the dynamics and control of a large class of hybrid dynamical systems that are described by possibly different models in different state space domains. They not only cover important aspects and tools for hybrid systems analysis and control, but also a number of experimental realizations. Special attention is given to synchronization — a universal phenomenon in nonlinear science that gained tremendous significance since its discovery by Huijgens in the 17th century, see chapter 1 for an introduction to the observations of Huijgens regarding the phase synchronization of pendulum-clocks. Possible applications of the results introduced in the book include control of mobile robots, control of CD/DVD players, flexible manufacturing lines, and complex networks of interacting agents or robots.

It is our honor and pleasure to dedicate this book to Ilya Izrailevich

Blekhman, on the occasion of his 80th birthday celebrated in 2008. Professor Blekhman is one of the most profound thinkers and contributors in the area of nonlinear oscillations and synchronization in the XXth and XXIth century. His biography, for the first time published in English in such detail, follows below. We take the chance to wish Ilya Izrailevich Blekhman good health and new scientific achievements.

*Gennady Leonov,  
Henk Nijmeijer,  
Alexander Pogromsky,  
Alexander Fradkov*



# Biography: Ilya Izrailevich Blekhman

Ilya Izrailevich Blekhman, a leading Russian expert in the theory of non-linear oscillations, the dynamics of machines and vibration technology was born on 29 November 1928 in Kharkov (now in Ukraine) and completed five years of secondary school in Leningrad (now Saint Petersburg) before World War II. During the blockade of the city in 1942 he was evacuated to the Urals, to Sverdlovsk (now Ekaterinburg) and, after passing two intermediate examinations as an external candidate, graduated from school with a gold medal award. Between 1945 and 1947 he studied at the Mechanics Department of the Urals Polytechnic Institute and, at the same time, at the Mechanics and Mathematics Faculty of the Urals State University as an external student. In 1947 he entered the Physics and Mathematics Faculty of Leningrad Polytechnic Institute, graduating with distinction as a research engineer in technical mechanics in 1951. Among the famous physicists, mathematicians and mechanics researchers whose lectures he attended during his student years were G.Yu. Dzhanelidze, A.F. Ioffe, A.I. Lurie, I.M. Malkin, B.V. Numerov, V.I. Smirnov and G.M. Fikhtengol'ts.

Blekhman began his research and engineering career in 1949 (still as a student) at the All-Union Scientific Research and Design Institute for Mechanical Processing of Mineral Resources (now Mekhanobr-Tekhnika Corporation), which he subsequently set up and is still working for the last 60 years. Currently he directs a department in the area of mechanics and applied mathematics. Since December 1996, he has been director of the laboratory of vibration mechanics run jointly by the Mekhanobr-Tekhnika Corporation and the Institute of Problems in Mechanical Engineering of the Russian Academy of Sciences. He defended his candidate (Ph.D.) dissertation in 1955 under the supervision of Prof. A.I. Lurie and his doctoral thesis in 1962. He got the title of professor in 1969.

Blekhman has promoted and developed several new areas in the theory of nonlinear oscillations, in applied mechanics, and in the foundations of vibration processes and mechanical engineering. Among his most important achievements are:

- the development (together with G.Yu. Dzhanelidze) of the theory of vibrational motion — the process of “directed” slow changes, which occur under the effect of rapid “undirected actions”;
- the discovery of self-synchronization of rotating bodies (rotors) and the creation of a theory to explain this phenomenon;
- the determination of an extremum condition for stability, which extends the classical Lagrange-Dirichlet theory of the stability of equilibrium positions to synchronized rotations of weakly interacting bodies;
- the discovery and investigation of a class of nonlinear mechanical systems which, during vibration, acquire the “average potential” property: slower motions in those systems correspond to the motions of a certain average potential system, although the original system is highly non-conservative;
- the application of the classical Laval self-balancing principle to multi-rotor and nonlinear systems;
- the discovery (jointly with colleagues) and theoretical justification of the development of gravitational gas lift flows, which is useful for the efficient exploitation of gas lift flows, which has facilitated plans for the efficient exploitation of the energy and raw materials from the Pacific Ocean and which also contributed to the prevention of some ecological disasters;
- the development and validation of a general mechanical-mathematical approach to study of the effect of vibration on nonlinear mechanical systems (vibration mechanics and vibrorheology);
- introduction (jointly with K.A. Lurie) of a new principle in material science: the idea of dynamic materials and composites.

Many of these results have been presented in reference publications and textbooks. He is an author or co-author of more than 150 scientific papers, 2 scientific registered discoveries, 40 inventions and patents and 8 books, including “Vibrational Transportation” (with G.Yu. Dzhanelidze, Moscow: Nauka, 1964, in Russian), “Synchronization of dynamical systems” (Moscow: Nauka, 1971, in Russian), “Synchronization in science

and technology” (New-York: ASME Press, 1988), “Vibrational mechanics” (Singapore: World Scientific, 2000; in Russian: Moscow, Fizmatlit, 1994). He is a co-author, with A. D. Myshkis and Ya. G. Panovko, of a methodological book “Applied Mathematics: Subject, Logic and Features” (published in Russia in 1976, 1991 and 2005), which describes methodological features of applied mathematics as a science and which has received wide recognition. The discovery, theoretical justification and description of the synchronization of rotating bodies and the development of gravitational gas lift flows have been registered as scientific discoveries.

Blekhman has made some important contributions in engineering designs, notably, the construction of new vibration machines for enriching mineral resources-crushers, mills, sifters, flotation units, concentrators and so on. He and his successors have taken out many patents, and have sold licenses to leading firms in the USA, England and Germany.

Blekhman has established a leading Russian research school in the area of the theory of vibration processes and machines; seven Doctor of Sciences and about 40 Candidate of Sciences (Ph.D.) degrees have been awarded under his supervision. He is a member of panels which judge doctoral dissertations at the Mekhanobr-Tekhnika Corporation, Institute of Problems in Mechanical Engineering and the St. Petersburg Marine Technical University, as well as a member of a number of research councils of the Russian Academy of Sciences. He has given several courses of lectures for post-graduates and undergraduates in various higher technical teaching institutes in St. Petersburg and has lectured at MIT and Worcester Polytechnic in the USA, at polytechnic universities in Great Britain, Germany, the Netherlands, Denmark and Poland and at a number of other universities and institutes in other countries. Blekhman has been a member of Russian National Committees on theoretical and applied mechanics and (since 1965) on the theory of machines and mechanisms; in 1990 he was elected a full member of the Russian Engineering Academy. Blekhman was a member of editorial boards of the journals *Applied Mathematics and Mechanics* and *Mechanics of Solids* for over more than 50 years.

In 1998 Blekhman was awarded the Prize of the Russian Government in Science and Technology, in 1999 he received the Alexander von Humbolt Prize (Germany), and in 2000 the Al-Khoresmi Prize (Iran). In 2001 he received the honorary title “Honorary Machine Builder of Russian Federation”, in 2003 he received the honorary title “Distinguished Reasercher of the Russian Federation”. In 2009, Blekhman was awarded the Tchebyshev Prize in the area of mathematics and mechanics.

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