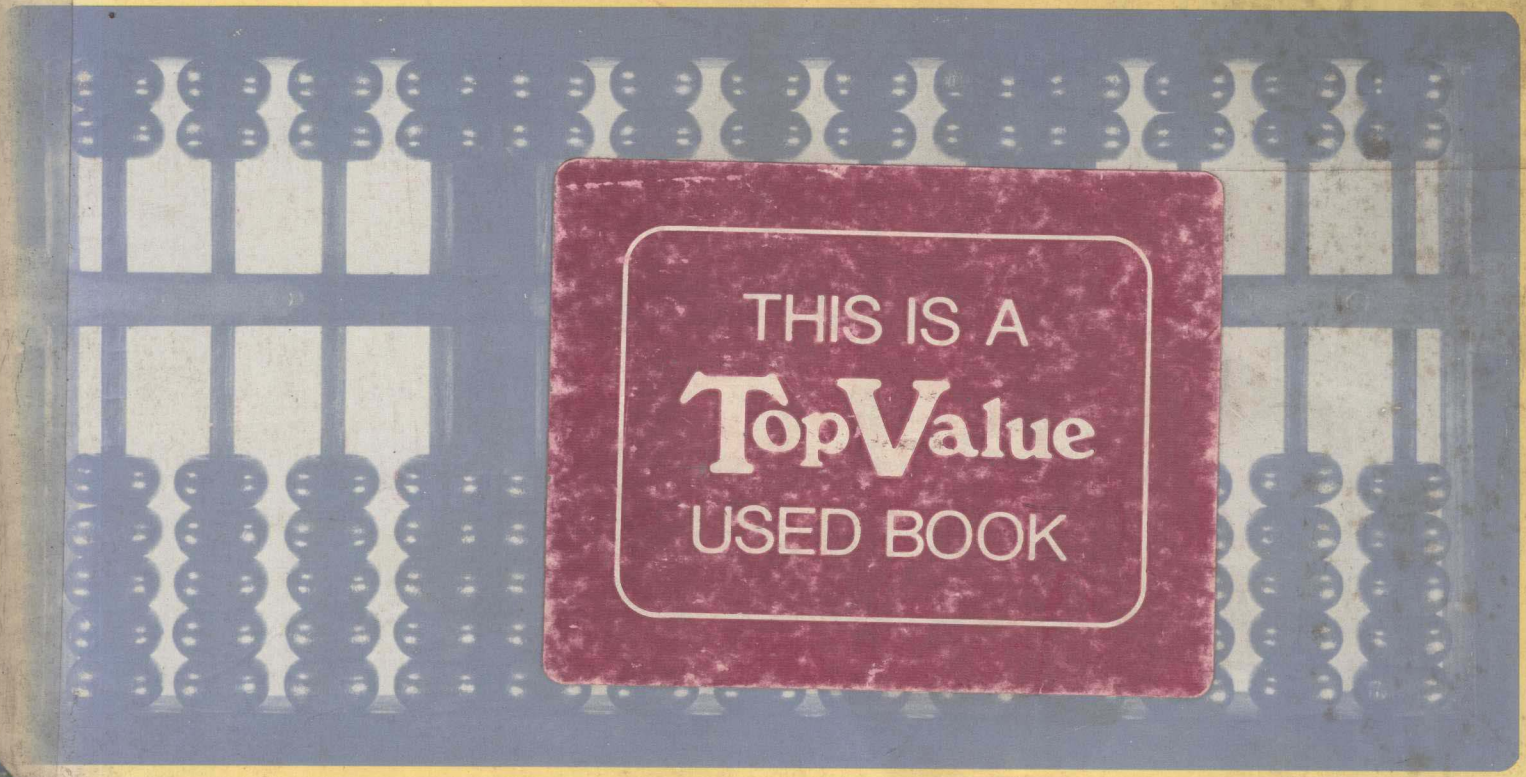
Essential Mathematical Skills



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Essential Mathematical Skills

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Preface

This book is a write-in work-text in arithmetic. The material is suitable for either a skills-oriented or a prealgebra course.

Uses The text is easily used in a traditional lecture course, but it has all the student involvement essential for any of the various individualized study courses from modified lecture to math lab or self-study.

Organization Reliance has been placed on a simple linear organization of the material. Each set of examples is followed by corresponding practice problems, many of which are guided, and located within the text material. There are over 450 of these guided practice problems. This gives the student immediate feedback on his or her understanding of the examples. Frequent questions are interspersed throughout the text to test comprehension of the fine points and special methods. Answers to the practice problems and questions are conveniently located at the bottom of the pages on which they appear.

Goals Sections within chapters begin with stated learning goals. The material in each section is separated and keyed to these goals. Exercises are also keyed to section goals. Students can easily find the material and exercises for each goal.

Illustrations The illustrations are practical and functional. They appear only when a visual presentation will actually enhance the learning process.

Examples Easy-to-read explanations and illustrative problems with numerous step-by-step examples are provided. The guided practice problems and questions are designed to reinforce the examples. There are over 1000 such examples, practice problems, and questions.

Statement Problems The solution of statement problems receives attention in several sections. Most exercise sets contain numerous statement problems involving everyday or applied situations.

Metric System The chapter on measurement stresses the use of the common units of the metric system.

Calculator All material on the calculator is reserved to the last chapter and incorporates applications in consumer math. Computations done on the calculator are indicated by the symbol .

Exercises There are over 4000 graded problems in the exercises ranging from skill-building to real-world statement problems. The even-numbered problems match the odd-numbered problems in type and level of difficulty. Problems in the exercise sets are grouped according to learning goals. An extensive set of practice-review problems supplements each chapter.

Tests Each chapter has a practice test. Four alternative forms of these tests and two alternative forms of a final examination are provided in the Instructor's Manual.

Answers The answer section contains answers to all exercises except every fourth problem (problem numbers divisible by 4). Answers to all practice-review problems and all in-text test problems are also included.

Supplements An Instructor's Manual is provided containing four alternate forms of each chapter test and two alternate forms of a final examination. Answers to those problems without answers in the text and to all the alternate forms of tests are also given.

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1/Whole Numbers: Addition and Subtraction

When we think of three books, three dollars, three apples, we are thinking of the number idea **three**. If we write 3, we are using a symbol for the number idea **three**. This symbol is called a **numeral**.

> Symbols which represent numbers—such as 3, 12, 1000, ||| —are called **numerals**.

The basic numerals for our number system are the **digits**:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

We use the digits to form other numerals such as 10,296.

1-1 / Whole Numbers

GOALS

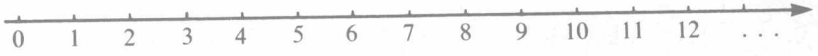
- ☐ Identify whole numbers and locate them on the number line.
- ☐☐ Use the order symbols, $<$ and $>$, with whole numbers.
- ☐☐☐ Know the place name of a digit in a whole number.

☐ The **whole numbers** consist of 0 and all of the numbers we use for counting:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, ...

The 3 dots mean "and so on."

To picture the whole numbers we use a **number line** where each whole number has a point of the line assigned to it.



Examples What whole number is indicated by point P on each number line?

1.

P is 7.

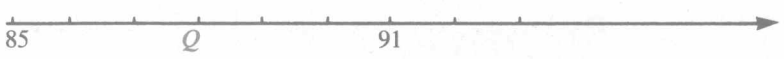
2.

P is 23.

PRACTICE PROBLEMS Identify point Q.

1.

Q is _____.

2. 

Q is _____.

☐☐ We say that the whole numbers are **ordered**. One whole number may be **less than** or **greater than** another whole number.

> Wilma has \$90 in her savings account. Charlie has \$98 in his account.
>
> Wilma's savings are **less than** Charlie's savings.
>
> Charlie's savings are **greater than** Wilma's savings.

How do we decide if a whole number is less than or greater than another? We can use the number line.

Find 90 and 98 on the number line.

It is always true that:

1. The number on the left is less than the number on the right,

   90 is less than 98

2. The number on the right is greater than the number on the left,

   98 is greater than 90

These two statements mean the same thing. They are just two different ways of saying it. We call them **equivalent** statements.

Examples Write a "less than" and a "greater than" statement for 27 and 34.

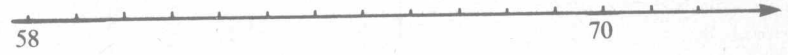
3. 27 **less than** 34
4. 34 **greater than** 27

ANSWERS **Practice Problems:** **1.** 4 **2.** 88

PRACTICE PROBLEMS Write "less than" and "greater than" statements for these number pairs and locate them on the number line.

3. 14, 6

4. 60, 72



We use symbols for less than and greater than.

less than $<$

greater than $>$

Note that the point of the symbol is always toward the smaller number.

$27 < 34$ means "27 less than 34"

$34 > 27$ means "34 greater than 27"

QUESTION **1.** What whole number is less than every other whole number? _____

There is no greatest whole number.

Examples Enter the correct order symbol for each pair of whole numbers.

5. 17 34
17 < 34

6. 81 79
81 > 79

7. 110 125
110 < 125

PRACTICE PROBLEMS Enter the correct order symbol for each pair of whole numbers.

5. a. 9 4
b. 16 32

6. a. 27 29
b. 5 0

7. a. 108 65
b. 18 22

ANSWERS **Practice Problem:** **3.** 6 less than 14, 14 greater than 6

4. 60 less than 72, 72 greater than 60

Question: **1.** 0

Practice Problems: **5. a.** $9 > 4$ **6. a.** $27 < 29$ **7. a.** $108 > 65$
b. $16 < 32$ **b.** $5 > 0$ **b.** $18 < 22$

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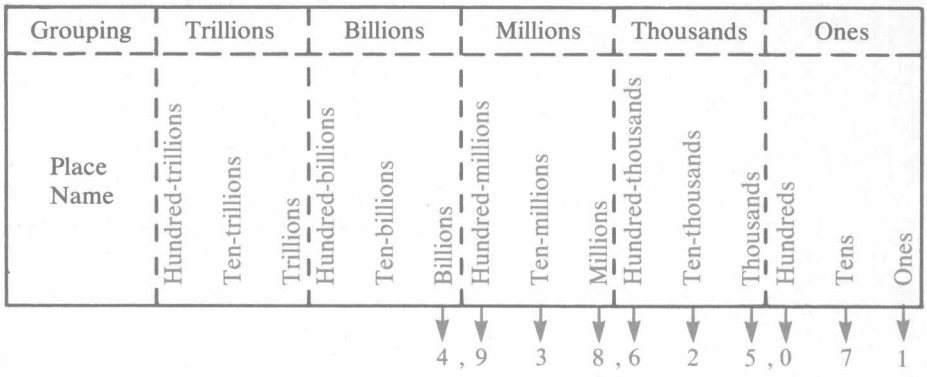
▢▢▢ The positions the digits occupy in a whole number are called **places**. The digits of a large whole number are separated into groupings of three places each by commas. Each place has a **place name**.



Examples The table shows the place names and groupings for the 10 digit whole number 4,938,625,071. The place names and groupings for some selected digits from this whole number are listed below.

	Digit	*Place name*	*Grouping*
8.	6	hundred-thousands	thousands
9.	3	ten-millions	millions
10.	7	tens	ones
11.	4	billions	billions

PRACTICE PROBLEMS As in the examples, provide the place name and grouping for these digits taken from 4,938,625,071.

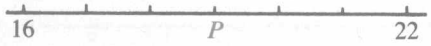
	Digit	*Place name*	*Grouping*
8.	0		
9.	9		
10.	2		
11.	8		

1-1 / Exercises

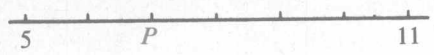
▢ Identify whole numbers and locate them on the number line.

What whole number does P represent on each number line?

1.



ANSWERS **Practice Problems:** **8.** hundreds, ones **9.** hundred-millions, millions **10.** ten-thousands, thousands **11.** millions, millions

2. 

3.

4.

5.

6.

7.

8.

☐☐ Use the order symbols, $<$ and $>$, with whole numbers.

Enter the correct order symbol for each pair of whole numbers.

9. 3 9

10. 12 4

11. 21 16

12. 10 18

13. 0 5

14. 26 13

15. 4 19

16. 101 98

17. 200 199

18. 1001 1005

19. (nonzero whole number) 0

20. 0 (nonzero whole number)

☐☐☐ Know the place name of a digit in a whole number.

Write the place name and grouping of the indicated digit.

	Place name	*Grouping*
21. 1,018,417 ↑	_____	_____
22. 691,176 ↑	_____	_____
23. 250,376 ↑	_____	_____
24. 37,156 ↑	_____	_____
25. 6,893,201 ↑	_____	_____
26. 10,004,000,000 ↑	_____	_____
27. 807,416,125 ↑	_____	_____

		Place name	*Grouping*
28.	9,409,876 ↑	_____	_____
29.	6 ↑	_____	_____
30.	78 ↑	_____	_____

1-2 / Expanded Form and Word Form

GOALS

- ☐ Write the expanded form for a whole number.
- ☐☐ Write the word form for a whole number.

☐ The place a digit occupies in a whole number tells us the numerical value the digit contributes to the number.

124 means $\boxed{1}$ hundred + $\boxed{2}$ tens + $\boxed{4}$ ones

or

$$124 = 100 + 20 + 4$$

This is the **expanded form** for 124.

Examples Write the expanded form for these whole numbers.

1. 29,356 means 2 ten-thousands + 9 thousands + 3 hundreds + 5 tens + 6 ones

$$29{,}356 = 20{,}000 + 9000 + 300 + 50 + 6$$

2. 406 means 4 hundreds + 0 tens + 6 ones

$406 = 400 + 6$ (no tens)

A 0 in a place means no numerical value is needed for that place in expanded form.

3. $467{,}032 = 400{,}000 + 60{,}000 + 7000 + 30 + 2$ (no hundreds)

4. $964{,}008{,}087 = 900{,}000{,}000 + 60{,}000{,}000 + 4{,}000{,}000 + 8000 + 80 + 7$

PRACTICE PROBLEMS Write the expanded form for these whole numbers.

1. $17{,}048 = 10{,}000 +$ _____ + _____ $+ 8$

2. $5409 =$ _____ $+ 400 +$ _____.

3. $4{,}921{,}652 =$ _____ $+ 900{,}000 +$ _____ $+ 1000 +$ _____ $+$ _____ $+$ _____.

4. $149{,}005{,}023 =$ _____ + _____ + _____ $+ 5000 +$ _____ $+ 3$

ANSWERS **Practice Problems:** **1.** $10{,}000 + 7000 + 40 + 8$ **2.** $5000 + 400 + 9$
3. $4{,}000{,}000 + 900{,}000 + 20{,}000 + 1000 + 600 + 50 + 2$
4. $100{,}000{,}000 + 40{,}000{,}000 + 9{,}000{,}000 + 5000 + 20 + 3$

☐☐ Whole numbers can be expressed in word form as well as in numeral form. The word form is used when speaking, in writing checks and receipts, in legal documents, and whenever it is desired to emphasize a number.

Word Forms Using a Hyphen

Most two-digit whole numbers from 21 through 99 such as

twenty-six, fifty-two, eighty-seven, ...

Place names such as

ten-thousand, hundred-million, ...

Examples Here are some word forms for three-digit whole numbers.

5. 203 two hundred three

Don't use "and." We use it later for the decimal point.

6. 526 five hundred twenty-six

7. 812 eight hundred twelve

PRACTICE PROBLEMS Try writing word forms for these.

5. 106 _____

6. 718 _____

7. 251 _____

For larger whole numbers we state the three-digit number in each grouping—remember groupings are separated by commas—and also the name of the grouping. Separate with commas just as in the whole number.

260, 064

two hundred sixty thousand, sixty-four

grouping name

omit grouping name for ones

703,514,832 is read or stated as seven hundred three million, five hundred fourteen thousand, eight hundred thirty-two.

Examples

8. 97,042,004 ninety-seven million, forty-two thousand, four

9. 207,096 two hundred seven thousand, ninety-six

10. 50,501 fifty thousand, five hundred one

ANSWERS **Practice Problems:** **5.** one hundred six **6.** seven hundred eighteen **7.** two hundred fifty-one

PRACTICE PROBLEMS Write the word forms.

8. 4014 _____

9. 70,005 _____

10. 346,735 _____

1-2 / Exercises

☐ Write the expanded form for a whole number

Write expanded forms.

1. 672 = _____ + _____ + _____

2. 239 = _____ + _____ + _____

3. 6947 = _____ + _____ + _____ + _____

4. 8684 = _____ + _____ + _____ + _____

5. 90,730 = _____ + _____ + _____

6. 19,203 = _____ + _____ + _____ + _____

7. 385,100 = _____ + _____ + _____ + _____

8. 507,031 = _____ + _____ + _____ + _____

9. 14,040,090 = _____ + _____ + _____ + _____

10. 30,404,500 = _____ + _____ + _____ + _____

Write whole numbers for expanded forms.

Example Problems

8 thousands + 7 hundreds + 2 tens + 6 ones = 8726
4 hundreds + 0 tens + 6 ones = 406
9000 + 300 + 20 + 1 = 9321
40,000 + 100 + 80 + 2 = 40,182

no thousands

11. 10,000 + 9000 + 200 + 30 + 3 =

12. 30,000 + 8000 + 500 + 4 =

ANSWERS **Practice Problems:** **8.** four thousand, fourteen **9.** seventy thousand, five **10.** three hundred forty-six thousand, seven hundred thirty-five

13. 3 thousands + 5 hundreds + 4 tens + 6 ones =

14. 7 thousands + 6 hundreds + 5 tens + 4 ones =

15. 500,000 + 50,000 + 7000 + 30 + 1 =

16. 100,000 + 40,000 + 4000 + 40 =

17. 7 hundreds + 5 tens + 1 one =

18. 3 hundreds + 7 tens + 5 ones =

19. What is the whole number which contains 2 ten-thousands, 5 hundreds, 4 tens, and 2 ones?

20. What is the whole number which contains 8 ten-thousands, 9 hundreds, 9 tens, and 5 ones?

▢▢ Write the word form for a whole number.

Write word forms.

21. 3931

22. 6153

23. 21,457,000

24. 42,238,000

25. 1,426,000,010

26. 2,090,901,000

Write the word form for the number in each sentence.

27. New York State has an area of 47,831 square miles.

28. Pennsylvania has an area of 44,966 square miles.

29. Mount St. Helens in Washington is 9677 feet high.

30. Mount Rainier in Washington is 14,410 feet high.

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