World Scientific Series on Directions in Condensed Matter Physics — Vol. 1

DIRECTIONS IN CONDENSED MATTER PHYSICS

Memorial Volume in Honor of Shang-keng Ma

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World Scientific

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Published by

World Scientific Publishing Co Pte Ltd.
P. O. Box 128, Farrer Road, Singapore 9128
242 Cherry Street, Philadelphia PA 19106-1906, USA

Library of Congress Cataloging-in-Publication Data

Directions in condensed matter physics.

1. Condensed matter. 2. Ma, Shang-keng, 1940–1983.
I. Ma, Shang-keng, 1940–1983. II. Grinstein, Geoffrey.
III. Mazenko, Gene.
QC173.4.C65D57 1986 530.4'1 86–18920
ISBN 9971-978-42-3
ISBN 9971-978-58-X (pbk.)

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Printed in Singapore by Fu Loong Lithographer Pte Ltd.

PREFACE

This volume is dedicated to the memory of our colleague, teacher, and friend, Shang-keng Ma, who died at age 43 in November, 1983, after a protracted battle with cancer. The premature death of this kind, gifted, incredibly modest man has left a painful void in the lives of those of us who worked with him.

We met Shang shortly after completing our graduate studies, and soon entered into collaboration with him. In the rare moments not spent fretting over job prospects, most of our contemporaries in physics at the time (we certainly among them) were casting about for a salutary style of living their lives as physicists. Among the many forceful and charismatic personalities of theoretical physics, the young, mild and self-effacing Shang Ma seemed an unlikely role model for this search. It was only in working with him that one began to understand his special, to us irresistibly appealing, approach to doing physics.

When we first started working with Shang there was excitement among condensed matter theorists over the newly-discovered renormalization-group techniques for treating critical phenomena. He had already contributed weightily to the development of these ideas through, for example, his work on the $1/n$ expansion, his elucidation, with Nickel and Fisher, of the effect of long-range interactions, and his invention, with Halperin and Hohenberg, of renormalization-group methods for critical dynamics. Compared to his knowledge and achievements in this field, ours were minimal. This would have been impossible to infer from the tone of our discussions, however. Shang had an instinct for equality and an instinctive sympathy for the underdog; the lowliest student and the most radiant luminary received at his hands the same dignified courtesy. While he could be keenly critical of ideas, evaluating people was distasteful to him. In his presence one felt secure that one was not on trial. One's mere interest in a problem that excited him was sufficient recommendation. Trying to make finer judgments was considered a waste of time, and anyone with enough energy was assumed capable of contributing to the general understanding.

It is hard to overestimate how liberating it was for a young person to work in such an atmosphere. One felt protected from the relentless competition of the world outside. The problem under study was paramount, and there was quiet confidence that the problem could be solved; worry about "being scooped" and "having impact" seemed petty. In the face of Shang's generosity of spirit (his version, having nothing to do with fact, was always that his co-workers were responsible for 100% of any progress achieved, his role being something along the lines of keeping them supplied with sharp pencils), small-minded impulses in his collaborators withered and died.

Much of Shang's early work involved field theoretic pyrotechnics, and many of us who worked with him fairly early on still harbor the suspicion that he had occult powers of diagrammatic analysis. In the most tangled mass of Feynman graphs, Shang somehow knew where the interesting behavior lay. Sometimes he even had difficulty explaining how he knew, though days of labor by the rest of us would usually prove him right. We remember one occasion late in 1975 when the three of us, cloistered in a small office on the first floor of the physics building in La Jolla, were trying to extract the dominant infrared singularity from a disheartening set of diagrams. Though straightforward, the analysis was tedious. We two sat at our desks, methodically depleting the world's proven pulp and paper reserves. Shang sat on a chair in the corner, frowning at the proverbial back of the envelope – in this case a small airmail envelope with little space for mistakes. Every half hour or so he would make an invisible mark on it with a fountain pen. Then he would cross the mark out, looking ever more glum. After some hours of this the two of us with desks and paper had arrived at results. Neither had any confidence in his answer; the two results disagreed violently. We turned to Shang. He gave us his envelope. In the center of the scratchings was a minute algebraic expression, which agreed with one of ours (there seems now to be dispute over precisely whose) and turned out to be correct. There were no supporting calculations. Much impressed by this, we asked him to explain how he had done it. He began in his usual matter-of-fact way, unclear as to what the fuss was about, but could see that we weren't following. He stopped and pondered a bit. Suddenly his face was alight with the key to elucidating the method: "When you think about it a little," he said, "what else could it be?"

"Anything else!" we thought, but were so awed by the mystic timbre of his question that we stood there dumbly, pretending all was clear. Shang seemed pleased. A moment later we were interrupted, and never did get a chance to press him for a less ancient explanation.

As his career progressed, Shang came to rely less on formal field-theoretic calculations and more on simpler physical arguments. One could already sense this trend in the mid-70's, when much of his work still involved diagrammatic analysis. (This was a marvellously productive period for Shang. Between the years 1972 and 1979 he developed (with Halperin and Hohenberg) the dynamical renormalization group mentioned earlier, and was the first (together with Mazenko) to incorporate mode-coupling terms in this framework. He discovered, in superconductors (with Halperin and Lubensky), the first example of the important phenomenon now known as "fluctuation-induced first-order phase transitions," produced (with Imry) the classic paper on random fields, and invented the Monte Carlo renormalization group. His book on critical phenomena, published in 1976, remains a standard reference in the field.)

We caught a wonderful glimpse of Shang's gradually shifting focus on the morning following the incident recounted above. Having assumed our places we two were having difficulty concentrating on the task at hand. (In fact, we recall embarking on a less-than-scholarly debate over which group performed the song "Give Me Just a Little More Time.") Shang sat motionless in his corner with a half sheet of paper. So far as we remember he wrote nothing, again looking more and more miserable as the morning wore on. We stopped for lunch. It was clear that Shang, who usually seemed so phlegmatic, was dejected. We asked what the problem was. "Oh," he said, "I've wasted the entire morning. I couldn't even do the simplest calculation."

"What were you trying to calculate?"

"Oh nothing. I was just trying to estimate the sign of some quantity."

We tried to control ourselves, but the idea of trying to "estimate" a sign struck us as so funny that we were soon choking with the effort. A pale smile broke through the gloom in Shang's expression, and at once we were all howling with laughter that made conversation elsewhere in the canteen impossible. (Later that afternoon a relieved Shang told us that the discovery of a missing factor of -1 had improved his estimate, and that the sign was looking more encouraging.) "Estimating the sign" became a standing joke among us, but "estimate" continued to replace "calculate" in Shang's subsequent descriptions of the physics he was doing. There was no diminution of his mathematical facility. More and more often, however, the questions he found most absorbing were those for which an insight into the physics purchased more than a calculation. This evolution persisted right up until his death, and is discernible in his later work, notably in his invention of an extremely original coincidence counting method for computing the entropy of complex systems, and his important contributions to the understanding of the random field Ising model. His book on statistical mechanics (written in Chinese and only recently translated into English) is full of ingenious physical arguments.

The sense of security one had working with Shang was not entirely a result of his largeness of spirit. It derived in part from one's experience that when the problem in question turned nasty Shang would find a window through which it looked benign. His way of seeing a problem was seldom the same as other people's, and seldom constant for four days running. He was ruthless about discarding an approach in which flaws began to appear, no matter how much prior effort it represented. He was forever turning things around, standing them on their ears, trying to make them yodel. In a collaboration with Shang one risked being uprooted every morning. Overnight the whole program might have been scrapped and a more promising one devised. Those of us more sentimental about our pasts than he were often left with a feeling of weightlessness — struggling for balance, straining to keep up, starting from scratch *again*. Eventually a satisfactory course would be found and we would have a few hours of peaceful calculating. Then off we would go again, destination only vaguely known.

Occasionally Shang's thinking about a problem was so unconventional as to be hard to follow. Most of us who worked with him had the experience of dismissing as nonsense a particularly alien-sounding idea of his, only to rediscover it for ourselves after weeks of work. Shang seemed unconcerned about being misunderstood in this way. He was decidedly lacking in evangelistic fervor. It seemed sufficient for him to have convinced himself of the validity of a point of view. Others would eventually come around if his reasoning were sound.

One encountered a similar self-sufficiency when one tried to explain things to him. He was always supportive, and open to new ideas, but when one grew to know him one could perceive a sharp distinction between ideas that appealed to him and ones that simply did not. One was sometimes drawn to undignified excesses of salesmanship in trying to promote a favorite thought from the latter category to the former. Occasionally one sank so low as to dredge up a scrap of evidence to show that one of the great men in the field subscribed to the doctrine one was espousing. This never produced any response but a shy laugh, though it may from time to time have stiffened Shang's resistance. At such moments one could feel the fierceness of his intellectual independence. He had to understand everything in his own way. Lore which did not make sense to him left him suspicious, no matter how common its acceptance or illustrious its origins. He was not the least intimidated by the views of celebrated physicists. Physics was not a spectator sport to Shang, and he saw the hero worship which so strangely pervades the profession as an impediment to his participation.

While not indifferent to others' opinions of his abilities, Shang refused to grant much importance to such judgments. He never hesitated to ask a question that might further his understanding, even if that question exposed a gap in his knowledge. We have all been in a group where someone uses the latest "buzz" word and everyone agrees knowingly that "buzz" is just great. There is a tacit conspiracy that no one will demand to know what "buzz" means. In such situations Shang would ask, without aggression, what the rest of us lacked the confidence to ask: "What is 'buzz,' anyhow?" (Once, as a session chairman at a March meeting of the American Physical Society he delighted the crowd by asking the author of a paper with a five-letter acronym in its title what the initials stood for.) One then experienced the suspense of discovering whether anyone could in fact explain "buzz." It was on such occasions that we loved Shang best, for his honesty and courage.

Quiet courage remained a part of Shang's make-up to the end of his life. He continued to carry out his teaching duties at UCSD despite the severe pain and debilitation of his disease. Scant days before his death he consulted a doctor who pronounced him gravely ill, at which point he felt that he had satisfactorily discharged his responsibilities and could stop teaching. It was characteristic of Shang's independence of mind that this was the first time he had seen a doctor in many months. Having concluded that contemporary medicine could do nothing further for him, he had terminated conventional medical treatment to try to cope with his illness in his own way.

Shang-keng Ma adhered as determinedly as anyone we have met to the vision which draws so many of us to theoretical physics in the first place: the image of sitting in a quiet spot with pencil, paper, and perhaps a friend, and thinking hard about a problem until one understands something about it. Through the years that intensely personal vision tends to get lost in a swarm of job applications, priority squabbles, funding anxieties, jet lags, learned committees, lustful thoughts of honors and prizes, and the rest of the ogres in the punishing road of "career advancement." In hindsight it seems comically naive; one forgets its original power. Shang managed never to take his eyes off that youthful ideal. He paid dues, like the rest of us, to the establishment which supported him, but always remained slightly aloof from it, refusing to let the less noble of its values clutter his path. For all his mildness he was absolutely uncompromising on this point. There was nothing of the huckster in Shang. The idea of promoting himself or his work was utterly foreign to him. He sacrified to these principles some recognition for his scientific accomplishments. We feel certain that the sacrifice was made consciously and without regret. For those of us lucky enough to have known him, he leaves behind more than a significant body of research. He leaves behind a standard against which we measure our conduct as physicists and as human beings. Despite our sadness in being unable to call Shang, to gossip with him and to hear about his latest ideas, there is something comforting in the picture we carry of his gentle, sympathetic smile – something that helps us remember why we are doing what we do.

In finding for this book a theme appropriate to his memory, we were guided by the idea that Shang-keng Ma was interested in essentially all areas of theoretical physics. He would give any subject a fair hearing in selecting those bits of fields that most appealed to him. Since the bulk (though not all) of his work was in condensed matter physics, we decided to try to assemble a collection of review-style articles in active, emerging areas of condensed matter theory. Feeling that we would achieve the most stimulating and topical book by encouraging the authors to write about subjects which intrigue them most at present, we made no attempt to constrain the distinguished contributors as to topic. We told them only that we wanted the volume to be irresistible to a graduate student looking for an exciting area in which to begin research. We are delighted by all of the contributions, and grateful to the authors for their considerable efforts and patience. In the final analysis we feel pleased with this collection because we can imagine Shang wanting to pick it up and read it.

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PERCOLATION

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1. Introduction

The mathematical problem of *percolation* is very easily defined: consider an infinite periodic lattice of sites (or bonds). Each site (or bond) is occupied (with probability p) or empty (with probability $1-p$). Nearest neighbor occupied sites (bonds) form connected *clusters*. As p increases from zero, the average size of these clusters increases. For the infinite system, this average size diverges at the *percolation threshold*, p_c. Above p_c the system contains an *infinite* cluster, which connects between its edges.

Near p_c, the only length scale relevant for describing the properties of the system is the *connectedness length*, ξ, defined as the root mean square distance between pairs of sites (or bonds) which belong to the same finite cluster. Since ξ diverges to infinity at p_c, there remains no measuring unit with which to scale the dependence of various properties on lengths. As a result, the system looks qualitatively the same when looked at with different magnifications. This phenomenon, called *self-similarity*, is responsible for various anomalous properties of dilute systems near their percolation threshold. Since these properties arise at *all* length scales, they do not depend on many local details (e.g., the lattice structure), which become *irrelevant* to the large scale behavior. This yields many *universal* quantities, which depend only on the *dimensionality* of the system. It is thus sufficient to find these quantities on simple models, and then apply them to a wide variety of realistic situations.

The existence of the infinite cluster only above p_c is similar to the appearance of an order parameter in a phase transition. The divergence of the connectedness length at p_c also resembles that of the correlation length near critical points. Indeed, all the theoretical modern tools of critical phenomena can be used to describe this geometrical transition. Some of these are reviewed below, in Secs. 2 and 3. These sections form an attempt to present the theory in a didactic way, with a strong emphasis on the *geometry*.

Obviously, percolation theory is relevant to many physical dilute or alloyed systems. It is impossible to do justice to all the interesting applications in this review. Instead, I chose to present two specific examples, which exhibit the strong influence of the self-similarity geometry on the physics. Section 4 discussed dilute magnets, and Sec. 5 reviews some electrical conductivity and diffusion phenomena. Naturally, the choice of the examples included here was affected by my personal taste. This fact, and the attempt to be didactic, must have caused the absence of many relevant references, and I apologise to their authors. Several excellent recent reviews contain much more complete lists of references, and I refer the readers to those for more details (Essam, 1980; Stauffer, 1979, 1985; Deutscher, Zallen and Adler, 1983; Stinchcombe, 1986 and references therein).

2. Fractal Geometry and Scaling

2.1. *Self-similarity and fractal dimensionality*

It is easy to produce by computer a percolation picture. One simply runs through each lattice site (or bond), and decides to occupy it (or not) using random numbers between 0 and 1. There exist various algorithms to identify connected clusters, and some of these were reviewed in detail by Stauffer (1979, 1985). Given a finite sample realization, one can perform various measurements of geometrical properties of the clusters, and collect data on their statistics.

Figure 1a shows a computer simulation of site percolation on the triangular lattice, at its threshold concentration $p_c = \frac{1}{2}$. The sites on the largest cluster, which connect between the boundaries of the finite sample, are emphasized by showing the bonds which connect them. Figure 1b then shows a coarse graining of the same picture: sites on the lattice were grouped into cells of three sites (forming triangles). Each cell was then replaced by an occupied single new site if a majority of its sites was occupied, and by an empty new site otherwise. Qualitatively, Fig. 1b cannot be distinguished from a piece of Fig. 1a. It is impossible to tell from the picture at what level of coarse graining, or magnification, the two pictures were taken. This is a qualitative manifestation of *self-similarity.*

Quantitatively, we may verify self-similarity by several measurements. First, we may choose a point on the large cluster (e.g., at the center of the picture) and count the sites on the cluster within a box of linear size L. For large L, the average of this number approaches an *asymptotic* power law,

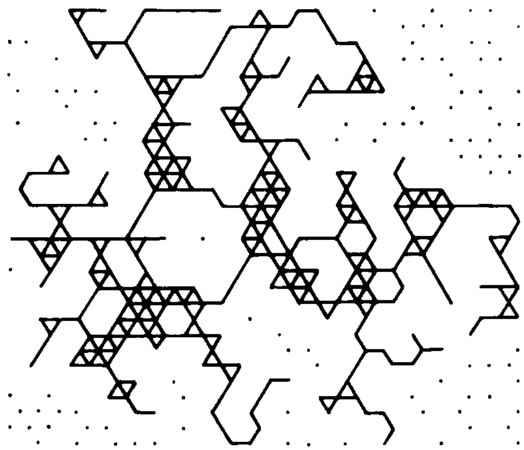
$$M(L) \approx \bar{A} L^D \quad . \tag{2.1}$$

This behavior is to be contrasted with that of a homogeneous system, in which $M(L) = \rho L^d$, where d is the Euclidean dimensionality ($d = 2$ in Fig. 1) and ρ is the uniform density. The exponent D, which is found to be close to 1.9 at $d = 2$, to 2.5 at $d = 3$, and equal to 4 for $d > 6$, is called the *fractal dimensionality* (Mandelbrot, 1982) of the cluster. Power laws like (2.1) are expected whenever there exists no other length by which L can be scaled.

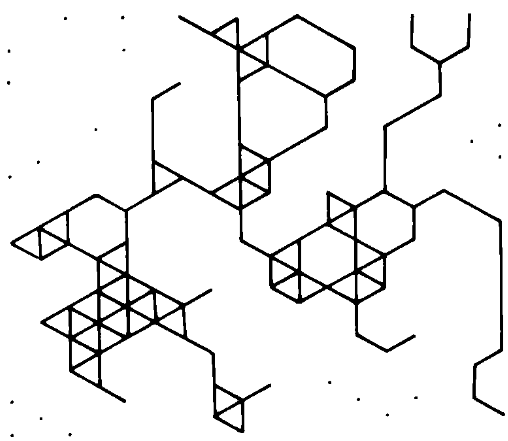
An alternative measurement of D arises from a comparison of the number of sites on the largest clusters of Figs. 1a and 1b. Since the unit of length was changed by a factor $b = \sqrt{3}$, the length L in Fig. 1a becomes L/b in Fig. 1b. If there exists no other parameter on which $M(L)$ depends, then $M(L)$ must be a *homogeneous function,* of the form

$$M(L) = b^D M(L/b) \quad , \tag{2.2}$$

The power law $M(L) \sim L^D$ is the only function independent of b which satisfies (2.2).



(a)



(b)

Fig. 1. Site percolation on a triangular lattice, at $p_c = \frac{1}{2}$. The sites on the largest connected cluster are emphasized by the connecting bonds. (a) Original simulation. (b) Coarse-grained version, with triangular cells of three sites being occupied by a majority rule.

We note that the relation (2.1) applies only to the *average*. Measurements around individual different central points may result in *fluctuations* around the average, and in variations in the amplitude A. The mean square deviation turns