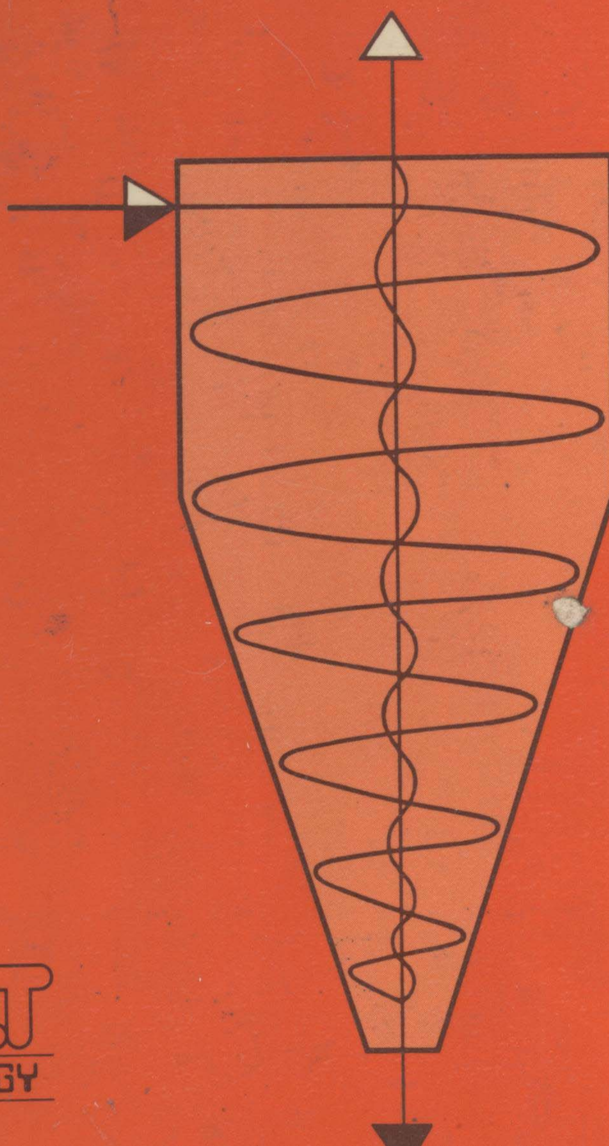
Hydrocyclones

L. Svarovsky



HOLT TECHNOLOGY

HYDROCYCLONES

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Preface

There has been a revival of interest in hydrocyclones recently, particularly in chemical engineering and the oil industry, and this can be attributed to several factors.

Firstly, information about the usefulness of hydrocyclones in applications outside mineral processing has only recently started filtering through to chemical and other process engineers. The traditional applications of hydrocyclones in mineral processing, with the accompanying specialized terminology, the specific problems which are unique to that industry and the type of journals publishing the papers on hydrocyclones have made it difficult for chemical engineers to recognize the hydrocyclone as a more general tool, suitable for the chemical industry too. Furthermore, there are still very few university or polytechnic chemical engineering courses which as much as mention hydrocyclones, let alone teach the basic scale-up and performance characteristics. It has therefore taken almost 40 years, since the last world war, for the communication gap between the two disciplines to be bridged. There are other examples, such as hydrometallurgy and solid–liquid separation, where a real interdisciplinary communication has at last emerged.

The second factor in the recent revival is the new need of the oil industry, particularly in the North Sea, for compact, reliable and simple units capable of separating gas from oil, sand from oil or water from oil, or preferably all of these from each other in one operation. Hydrocyclones seem to fill the bill and interest is still growing in that area.

Thirdly, hydrocyclones have been further developed and general understanding has increased. They can do more today than they could 40, or even only 20, years ago.

This book has grown out of a short course manual. Over the past four years I have given short courses on hydrocyclones under the sponsorship of the Institution of Chemical Engineers, BHRA Fluid Engineering and the Center for Professional Advancement, held in Britain, Holland and the USA, and I therefore gratefully acknowledge the support of the above institutions.

To the best of my knowledge there has been no book written specifically on hydrocyclones since the 1960s when, within the space of six years, there were four books published [1–4]: two in Russian, one in Czech and one in English. Of those, the one by Bradley undoubtedly made most impact and has since been used as a standard reference on the subject. A lot has happened to hydrocyclones since the 1960s and this book attempts to fill the gap.

My aim was not to replace Bradley's book or any of the other three (although they have long been out of print) but to supplement them by concentrating mainly on the developments of the past two decades. Quite naturally, our understanding of the hydrocyclone has progressed significantly since then and this book can therefore be written in a rather more authoritative style, more as a textbook than a detailed report on every piece of research concerning hydrocyclones to date. As a teaching text, it is also more self-contained, starting with chapters on the fundamental principles of particle–fluid interaction and of the grade efficiency concept, both subjects being important for an understanding of the rest of the book.

Like the courses on which it is based, the book attempts to strike a balance between the essential basic theory and the practical aspects of hydrocyclone design, operation, installation, selection, scale-up and specific applications. In brief, its aim is to bring the reader up to date on the principles and industrial practices of hydrocyclone technology, and to bring together information presently scattered in the literature of mineral processing, chemical engineering and water treatment.

The 10 chapters are structured around specific aspects of hydrocyclone technology so that the reader can select any chapter he or she might want and go directly to it. Thus, for example, readers interested in design and scale-up may go to Chapters 7 and 8, those wishing to test cyclones might read Chapters 2 and 6, those interested in fundamentals may want to look at Chapters 1, 3 and 4, and readers seeking general information on the principles, cyclone types and applications will find most of what they need to know in Chapters 3, 5 and 10.

It is expected that the book will be useful to engineering students interested in solid–liquid or solid–solid separation, and to equipment user industries as well as to equipment manufacturers specializing in the same operations. More specifically, the student readership may include college, polytechnic and university students studying for HNC, HND, College Diploma, BSc (or BTech, BEng) and MSc (or MEng) degree courses. The most relevant courses are Chemical (or Process) Engineering, Minerals Engineering and Processing, Fuel Technology, Petrochemicals, and Metallurgy. It is also expected that the book will become a manual or a required text in a number of specialized post-experience courses.

The target industries include mineral processing, mining, chemical engineering, petrochemicals, oil production, power generation, waste water and effluent treatment, food processing, pharmaceuticals, textiles and dyestuffs, hydrometallurgy, and any other industries dealing with particulate slurries. The general industrial readership then may include Process Engineers, Plant Superintendents, Quality Control Engineers, Research and Development or Production personnel as well as Technicians in training.

My own route to hydrocyclones has been via gas cleaning in my early days in Prague, and via solid–liquid separation later on in Bradford. It was a natural transition for me to extend my much earlier and continuing interest in the gas cyclone to its younger but close relative, the hydrocyclone. I started to research the subject in 1974 when I bought my first commercial hydrocyclone. Since then I have tested many other makes of hydrocyclones donated by different manufacturers, and I am grateful to Richard Mozley Ltd, Van Tongeren UK Ltd, Vickerys Ltd, Dorr–Oliver Company Ltd and Dutch State Mines for their support in this way.

At Bradford, a continuous stream of both undergraduate and postgraduate students have worked with me and researched various aspects of hydrocyclone operation and testing. I would therefore like to mention the following names (in chronological order), and apologize for any omissions: A. R. Tyas, B. J. Gallagher, D. K. Furniss, A. S. Bavishi, K. G. Jones, A. S. Grewal, G. Cassells, W. N. Yeo, G. Park, N. Waite, S. G. Chopra, A. B. Mullally, A. Heyes, K. Gibson, B. S. Marasinghe, R. de Andrade Medronho and S. Bassett. Most of the contributions and test examples in this book are based on the work of the above-mentioned students.

Apart from my own first-hand experience, I have also drawn on the experience of many industrial friends, too numerous to mention, who either through attending one of my short courses or through a consultancy have shared with me their successes or failures when trying to use hydrocyclones.

Special acknowledgements, however, are due to several prominent personalities in the field of hydrocyclone technology with whom I have had the benefit of making personal acquaintance and tapping their wealth of knowledge: Bryant Fitch of Carnegie–Mellon University, Professor H. Trawinski of AKW, Professor K. Rietema of Eindhoven University, Richard Mozley of Richard Mozley Ltd, Mr S. K. de Kok, consultant, Martin Thew of Southampton University, Dave Osborne of Kilborn Engineering (B.C.) Ltd, and Thomas Bier of Dorr–Oliver Inc.

Finally, my sincere thanks are also due to my wife Jitka who has always been there, ready to help and support me in my efforts.

L. Svarovsky

List of Symbols

N.B. All units in SI.

a	acceleration due to an external field ($m\,s^{-2}$)
a'	capital cost constant, eqn (8.14) (ms^{-2})
A	cross-section area of a particle (m^2)
AI	abrasion index defined in eqn (6.25)
b'	capital cost exponent, eqn (8.14) (–)
c	solids concentration, fraction by volume (–)
c_i	initial concentration in a tank, eqn (9.10) ($kg\,m^{-3}$)
c_o	slurry concentration in a tank at time t, eqn(9.10) ($kg\,m^{-3}$)
c_u	underflow concentration by volume (–)
c_w	solids concentration, fraction by mass (–)
C_e	cost of electricity per unit of energy (£ J^{-1})
C_D	particle drag coefficient (–)
d	separation density ($kg\,m^{-3}$)
d_{75}	separation density at 75% recovery ($kg\,m^{-3}$)
d_{50}	separation density at 50%, i.e. cut density ($kg\,m^{-3}$)
d_{25}	separation density at 25% recovery ($kg\,m^{-3}$)
D	cyclone diameter (inside diameter) (m)
D^*	diameter of the centre of inlet from cyclone axis (m)
D_a	diameter of air core (m)
D_o	outlet (overflow) diameter (m)
D_i	inlet diameter (or equivalent by area) (m)
D_u	underflow diameter (m)
E_N	Newton efficiency defined in eqns (2.17) or (2.18) (–)
E_O	Ogawa efficiency, eqn (2.19) (–)
E_P	sorting sharpness index (E_{cart} Probable) ($kg\,m^{-3}$)
E_S	separation efficiency defined in eqn (2.21) (–)
E_T	total efficiency defined in eqn (2.1) (–)
E'_T	reduced total efficiency, eqn (2.2) (–)
Eu	Euler number, eqn (1.15)
f	function
f'	investment multiplier, eqn (8.13) (–)
F	cumulative percentage of solids in the feed (%)
F_c	cumulative percentage in the underflow (coarse product) (%)
F_D	drag force on particle (N)
F_f	cumulative percentage in the overflow (fine product) (%)
F_W	additional drag force due to wall (N)
Fr	Froude number, eqn (1.17) (–)
g	gravity acceleration ($m\,s^{-2}$)
G	rate of particle generation ($kg\,s^{-1}$)

$G(x)$	grade efficiency function (curve) (–)
G'	reduced grade efficiency (–)
G_i	grade efficiency of stage i (–)
G_s	grade efficiency of a separator (hydrocyclone) (–)
H	time worked per year, eqn (8.13) (s year^{-1})
$H(25/75)$	sharpness index, eqn (2.16) (–)
k	constant
k_d	factor defined by eqn (4.22)
K	constant
K_1,K_2	constants in eqn (6.14)
K_p	constant in eqn (6.1) (–)
l	length of vortex finder (m)
L	total length of cyclone (m)
m	particle mass, also exponent (kg)
M	mass flowrate of solids in feed (kg s^{-1})
M_c	mass flowrate of solids in underflow (kg s^{-1})
M_f	mass flowrate of feed suspension (kg s^{-1})
M_s	mass flowrate of solubles in feed, eqn (9.15) (kg s^{-1})
M_u	mass flowrate of underflow suspension (kg s^{-1})
n	exponent (–)
n_p	exponent in eqn (6.1) (–)
N	number of cyclones (integer)
N_o	number of cyclones (continuous variable)
O	overflow volumetric flowrate (m^3 s^{-1})
p	pressure (N m^{-2})
P_T	total penetration, eqn (2.4) (–)
q	feed flowrate to a multiple pass system (m^3 s^{-1})
Q	feed volumetric flowrate to a hydrocyclone (m^3 s^{-1})
r	radius (m)
R	cyclone radius (m)
R_f	underflow-to-throughput ratio (flow ratio), eqn (2.3) (–)
R_{fi}	flow ratio of stage i (–)
R_w	fraction of feed water entering the underflow (–)
Re	cyclone Reynolds number, eqn (1.14) (–)
Re_P	particle Reynolds number, eqn (1.2) (–)
s	composite function, eqn (9.13) (s^{-1})
S	water split (underflow to overflow) (–)
S_i	water split of stage i (–)
Stk	Stokes number, eqn (1.13) (–)
Stk_{50}	Stokes number corresponding to cut size x_{50} (–)
Stk'_{50}	Stokes number corresponding to reduced cut size x'_{50} (–)
t	time as variable, also thickness of vortex finder wall (s), (m)
T	residence time (s)
u	particle/fluid relative velocity (m s^{-1})
u_h	hindered settling velocity (m s^{-1})
u_r	radial velocity of a particle (m s^{-1})
u_t	terminal settling velocity of a particle (m s^{-1})
U	underflow volumetric flowrate (m^3 s^{-1})
v	fluid velocity (m s^{-1})
v_r	radial fluid velocity (m s^{-1})
v_t	tangential fluid velocity (m s^{-1})
v_z	axial fluid velocity (m s^{-1})

V	tank volume (m^3)
w	wash water in the system underflow ($m^3 s^{-1}$)
W	wash liquid flowrate ($m^3 s^{-1}$)
x	particle size, ore size, variable in eqn (9.15) (m)
x_{75}	particle size at 75% efficiency (m)
x_{50}	particle size at 50% efficiency (equiprobable size) (m)
x_{25}	particle size at 25% efficiency (m)
x_a	analytical cut size, eqn (2.14) (m)
x_g	mass geometric mean (m)
x_i	intersection cut size (m)
x'_{50}	reduced cut size (m)
y	mass fraction of solids in feed (–)
y_o	mass fraction of solids in overflow (–)
y_u	mass fraction of solids in underflow (–)
Y	number of depreciation years (years)
z	axial direction
α	flow pattern constant in eqn (4.8), also factor in eqn (6.15) (–)
Δd	difference between cut density and medium density ($kg m^{-3}$)
$\Delta \rho$	density difference between solids and liquid ($kg m^{-3}$)
Δp	static pressure drop ($N m^{-2}$)
ε	voidage (porosity), also turbulent viscosity (–) ($m^2 s^{-1}$)
Φ	function defined in eqn (2.20) (–)
λ	dimensionless parameter, eqn (4.20), also composite function, eqn (9.11) (s^{-1})
μ	liquid viscosity ($N s m^{-2}$)
ρ	fluid density ($kg m^{-3}$)
ρ_s	solids density ($kg m^{-3}$)
σ_g	geometric standard deviation of particle size distribution (–)
σ_{gs}	geometric standard deviation of grade efficiency curve (–)
τ	modified particle relaxation time, eqn (1.9) (s)
τ^*	particle relaxation time, eqn (1.7) (s)
θ	included angle of the cone (°)

Contents

Introduction

The principle and basic design of conventional hydrocyclones is almost 100 years old (first patented in 1891, [5]), but they had not found significant application in industry until after the Second World War. First in mineral processing and mining, but lately also increasingly in the chemical industry, petrochemicals, power generation, textile industry, metalworking industry and many others, hydrocyclones are now well established and their application is still widening.

Applications of hydrocyclones in industry (see Chapter 10 for more details and specific cases) fall into several broad categories of two-phase separation with the liquid being the suspending medium: liquid clarification, slurry thickening, solids washing, degassing of liquids, solids classification or sorting according to density or particle shape. Specially adapted hydrocyclones can also be successfully applied to separation of two immiscible liquids. Recent developments in this field show that such cyclones can separate oil from water, dewater light oils, and produce highly concentrated samples of a lighter dispersed phase.

Each application listed above has its particular requirements and goals, and it calls for changes in design and operation of the cyclone to make the cyclone most suitable for each case. It is therefore necessary, when discussing performance, design and operation of hydrocyclones in the different chapters of this book, to refer to the above-mentioned categories of application. In principle, however, each hydrocyclone separates particles of the dispersed phase (usually heavy) from the liquid (continuous phase) on the basis of density difference between the phases, and the separation depends heavily on particle size.

The principle of operation of hydrocyclones is fully described in Chapter 3 and shown in Fig. 3.1; the hydrocyclone is a static separator based on centrifugal separation in a vortex generated within a cono-cylindrical body. The feed flow, usually entering tangentially into the cyclone, is divided into *underflow* which carries most of the solids (or at least the coarser fraction) still suspended in some liquid, and the *overflow* which contains most of the liquid and some fine solids. The separation is size dependent and the characteristic size of the separation is referred to as *cut size*, described in more detail in Section 2.1.9.

The diameters of individual cyclones range from 10 mm to 2.5 m; cut sizes for most solids range from 2 to 250 μm; flowrates (capacities) of single units range from 0.1 to 7200 ($m^3 h^{-1}$). The operating pressure drops vary from 0.34 to 6 bar, with smaller units usually operated at higher pressures than the large ones. The underflow solids concentrations that can be achieved with hydrocyclones rarely exceed 45 to 50% by volume, depending on the size and design of the unit, operating conditions and the nature of the solids being separated.

The relative merits of hydrocyclones can be summarized as follows:

1. They are extremely versatile in application in that they can be used to clarify liquids, concentrate slurries, classify solids, wash solids, separate two immiscible liquids, degas liquids or sort solids according to density or shape.
2. They are simple, cheap to purchase, install, and run, requiring little in the way of maintenance and support structures.
3. They are small relative to other separators, thus saving space and also giving low residence times, which gives them an advantage in terms of the speed of control over the sedimentation classifiers, for example.
4. The existence of high shear forces in the flow is an advantage in classification of solids because it breaks any agglomerates, and also in treatment of thixotropic and Bingham plastic slurries.

The main disadvantages of hydrocyclones are that:

1. They are somewhat inflexible once installed and operated, giving low turndown ratios, due to the strong dependence of their separation performance on flowrate and feed concentration. They are also inflexible due to their general sensitivity to fluctuations in feed flowrate and solids concentration.
2. There are limitations on their separation performance in terms of sharpness of cut, range of operating cut size, de-watering performance or clarification power. Some of these characteristics may be improved in multi-stage arrangements, but at additional costs of power and investment.
3. They are susceptible to abrasion, but steps can be taken to reduce the effects.
4. The existence of shear may sometimes turn into a disadvantage because flocculation cannot be used to enhance the separation as in the case of gravity thickeners.

Particle–fluid interaction (Chapter 1) is obviously the heart of the separation process. The separation is due to particle inertia and a dimensionless inertial parameter called the Stokes number is one of the most important dimensionless groups needed for description of hydrocyclone performance and scale-up.

Another fundamental subject, discussed in Chapter 2, is the way to describe the separation efficiency. The total mass recovery of the solids is obviously needed for mass balances in process flowsheets but the most objective way to describe the separation performance of a hydrocyclone is by the grade efficiency curve. From it, two other parameters may be derived: the cut size and the sharpness index, both being important measures of the classification performance of a cyclone.

There is obviously a strong vortex flow inside a hydrocyclone and studies of the flow patterns have been made by many workers, as reviewed in Chapter 3. From the point of view of a user, however, there are only a few important observations regarding the flow. Firstly and most importantly, for the solid particles to separate under the effect of the centrifugal force, there has to be a finite density difference between the solid and the liquid. Secondly, the flow must be steady, free of any disturbances or fluctuations. Thirdly, the vortex has a depression in the centre and any dispersed or dissolved gas in the feed tends to migrate to the centre and form a gas core. This is usually a desirable phenomenon as long as the gas core is straight. It gets distorted by sharp bends immediately above the vortex finder and these have to be avoided. The fourth important observation is that there are very steep velocity profiles in the flow, thus leading to high shearing forces which tend to break any loose flocs, agglomerates or droplets in the flow. In general, therefore, it would be a waste of money to use any artificial chemical flocculants to improve the separation performance of hydrocyclones. In fact, in classification duties, the steep velocity profiles are a positive advantage because the classification is then according to the size of individual particles rather than that of agglomerates.

Lastly and contrary to common belief, the discharge of the separated particles into the underflow orifice is due to the flow itself, which pushes the particle layer at the wall down into the apex. Gravity therefore does not significantly affect the separation, except in very large cyclones which tend to be used for separation of large particles. It also follows that hydrocyclones do not necessarily have to be operated in a vertical position and can be inclined or, in case of the smaller ones, even inverted if necessary.

There has been a lot published about the relative proportions of the cyclone dimensions and their effect on separation efficiency and pressure drop (Chapter 7). A user need not be concerned with this aspect except for two observations. Firstly, there is a rule by which every measure which increases resistance to flow improves separation efficiency and vice versa. This applies to all proportions of the cyclone body, within certain reasonable limits, except for the length of the cyclone. Thus, for example, a cyclone with relatively small inlet and outlet openings is expected to give higher mass recovery but will offer higher resistance to flow and therefore have lower capacity. There have been several 'optimum' or recommended designs published (Chapter 5) and, as those have been well tested and enough is known about their performance, they may be adopted if needed.

Secondly, one dimension of a cyclone should be made variable and that is the underflow orifice diameter. Correct adjustment of this is vital for the best operation of the cyclone, since the optimum size of the opening cannot be reliably predicted. It is for this reason that the underflow orifice diameter is regarded as an operating (rather than design) variable and is therefore discussed in Chapter 6 rather than Chapter 7. The orifice diameter is best adjusted after the start-up of the plant and also during the operation whenever some operating conditions change. Several possible designs are available for this: replaceable nozzles, mechanically adjustable orifices, pneumatically operated adjustable orifices or even self-adjusting devices, which maintain a constant underflow density or concentration (see Section 5.2).

There is a whole host of operating conditions that affect the performance of hydrocyclones (Chapter 6). Perhaps the most important are the operating pressure drop and the feed concentration. With increasing pressure drop the efficiency of separation increases but the law of diminishing returns applies here. There is little point in increasing the pressure beyond 5 or 6 bar, the typical operating pressures for large cyclones being between 1 and 2 bar. Within increasing feed concentration the efficiency of separation rapidly falls off and hydrocyclones are therefore operated with dilute feeds whenever high total mass recoveries are sought.

The scale-up of hydrocyclones (Chapter 8) is based on the concept of cut size, defined as the particle size at 50% on the grade efficiency curve, because the shape of the grade efficiency curve remains similar for a given family of geometrically similar cyclones. The theories proposed for the separation process inside a hydrocyclone (reviewed in Chapter 4) provide a good qualitative basis for hydrocyclone scale-up at low solids concentrations. The most reliable fundamental route to hydrocyclone design is via the scale-up model presented in Chapter 8 which is based on theory but uses experimentally obtained dimensionless constants.

At higher concentrations, the scale-up becomes affected by the nature of the solids as much as by the operating conditions and this complicates matters considerably. Empirical models exist of varying complexity and applicability (Chapters 6 and 8), and more work still has to be done in this area.

In order to make full use of the advantages of the hydrocyclone it is often best to use multiple units, connected either in series or in parallel. In clarification duties for example, the parallel connections (Chapter 5) allow the more efficient, smaller diameter units to be used to treat large flowrates. The series connections (Chapter 9) on the other hand, are used to improve overall recoveries, to produce thicker underflows and clearer overflows simultaneously, to wash solids or to sharpen the classification or sorting.

As can be seen from this introduction, the scope of the hydrocyclone and its application is very wide, providing that its performance is well understood and the hardware is properly installed and operated. It is hoped that this book will prove to be a useful tool for such understanding.

1 Elements of Particle–Fluid Interaction

This chapter is devoted to the fundamentals of particle dynamics, understanding of which is necessary for the sections of this book dealing with hydrocyclone scale-up. As it is written with particular relevance to hydrocyclones, it is not an exhaustive review. The emphasis is on fine particles (in the sub-sieve range) and the main aim is to derive or present the most important dimensionless groups affecting separation in hydrocyclones.

The chapter is divided into two sections, according to the concentration of solids: in the first instance the case when particles are so far apart that they behave as if each is on its own, and in the second instance of higher concentrations when particle–particle interaction becomes important.

1.1 Particle–fluid interaction at low concentrations

At concentrations of less than about 0.5% by volume, the individual particles are on average so far apart that they do not affect each other in their movement through the fluid. Particle separation can be achieved by putting a force on the particles which will move them towards a surface where they will separate. The magnitude of the force can usually be easily expressed but the force opposing particle motion, the drag force, is more difficult to establish.

The conventional way to express the drag force F_D is according to Newton:

$$F_D = C_D \cdot A \frac{\rho u^2}{2} \tag{1.1}$$

where u is the relative velocity between the fluid and the particle, ρ is the fluid density, A is the projected area of the particle (in the direction of the particle motion) and C_D is the drag coefficient. For coarse particles moving fast, the drag force is mostly due to the inertia of the fluid and C_D is then constant. Fine particles move more slowly and the drag force becomes affected by viscous forces; the drag coefficient is then dependent on the particle Reynolds number Re_P which characterizes the flow round the particle, and is given by

$$Re_P = \frac{u \cdot x \cdot \rho}{\mu} \tag{1.2}$$

where μ is liquid viscosity and x is particle diameter. The particle does not necessarily have to be spherical; for irregularly shaped particles x is preferably

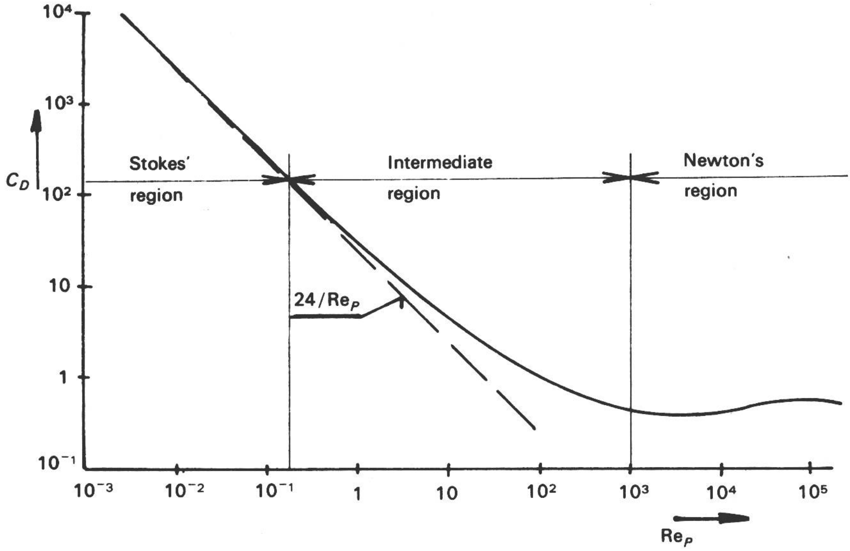

Fig. 1.1 Drag coefficient versus particle Reynolds number for spherical particles.

determined by sedimentation or elutriation methods of particle size measurement where it represents the equivalent diameter of a sphere which has the same settling velocity as the actual particle.

Figure 1.1 shows how the drag coefficient C_D depends on the particle Reynolds number Re_P for rigid spherical particles in a log–log plot. At low Reynolds numbers, under laminar flow conditions when viscous forces prevail, C_D can be determined theoretically from Navier–Stokes equations and the solution is known as Stokes' law:

$$F_D = 3\pi \cdot \mu \cdot u \cdot x \tag{1.3}$$

This is an approximation which becomes less accurate with increasing Re_P. The upper limit of its validity depends on the error which can be accepted: the usually quoted limit for the Stokes region is $Re_P = 0.2$ and this is based on an error of about 2% in the terminal settling velocity.

Equations (1.1), (1.2) and (1.3) combined give another form of Stokes' law as follows:

$$C_D = \frac{24}{Re_P} \qquad (\text{for } Re_P < 0.2) \tag{1.4}$$

and this is represented by a straight line in Fig. 1.1. The region between $Re_P = 0.2$ and $Re_P = 1000$ is known as the transition region where C_D is described in a graph or by one or more empirical equations. For Reynolds numbers greater than 1000 (Newton's region) the flow is fully turbulent, with inertial forces prevailing, and C_D becomes constant (equal to about 0.44).